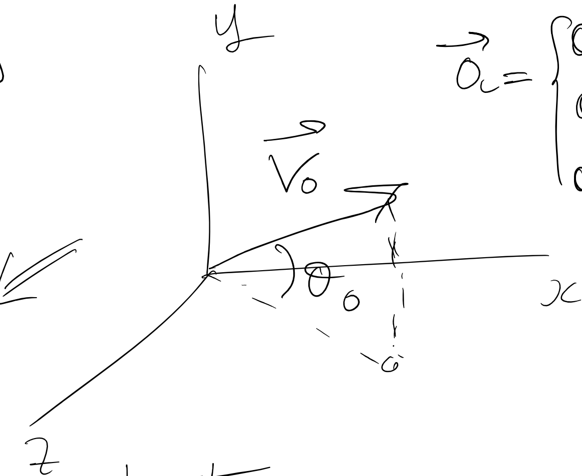
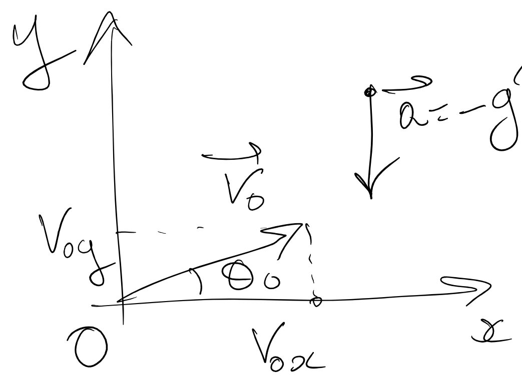
Les projectiles



$$\vec{a} = \begin{cases} a_x = 0 \\ a_y = -g \\ a_z = 0 \end{cases}$$

$\vec{a} = -g\hat{j}$



$V_{0x} = V_0 \cos\theta_0$

$V_{0y} = V_0 \sin\theta_0$

direction x

$a_x = 0$

$V_x = V_{0x} = \text{const}$

$x = x_0 + V_{0x}t$

direction y

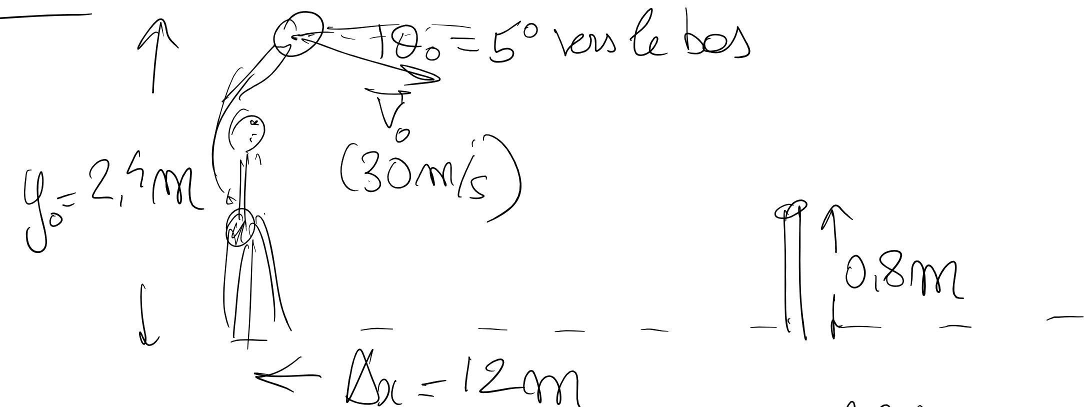
$a_y = -g$

$V_y = V_{0y} - gt$

$y = y_0 + V_{0y}t - \frac{1}{2}gt^2$

$V_y^2 - V_{0y}^2 = -2g(y - y_0)$

Exercice 1



direction x : temps pour atteindre le filet

$V_{0x} = V_0 \cos\theta_0 = 30 \cos 5° = 29,9$ m/s

$$\Delta t = \frac{\Delta x}{V_{0x}} = \frac{12}{29,9} = 0,401\,s$$

direction y — altitude de la balle après 0,401 s

$$y = y_0 + V_{0y} t - \tfrac{1}{2} g t^2 \qquad V_{0y} = -V_0 \sin 5° = -2{,}6 \text{ m/s}$$

$$= 2{,}4 + (-2{,}6)(0{,}401) - \tfrac{1}{2} \times 9{,}8 (0{,}401)^2$$

$$= 0{,}58 \text{ m} < 0{,}8 \text{ m}$$

la balle frappe le filet

Exercice 2 — Tir sur l'ourson

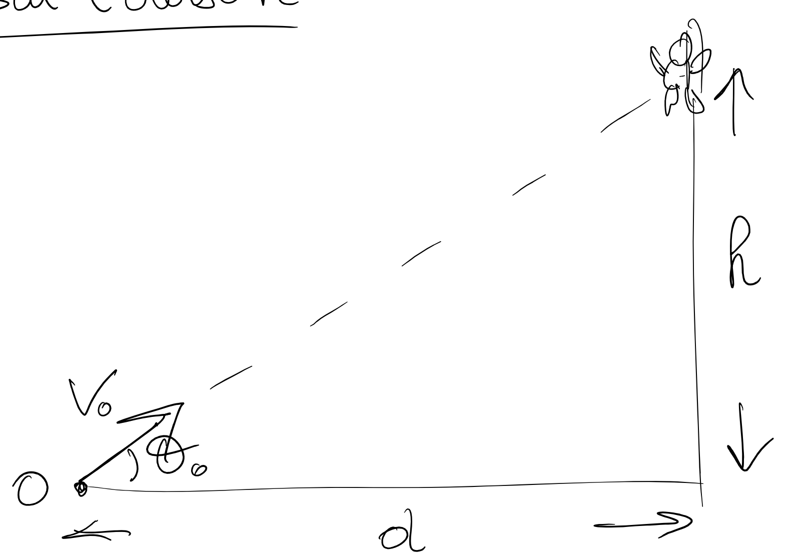
balle

$x_{b_0} = 0$

$y_{b_0} = 0$

$V_{bx_0} = V_0 \cos\theta_0$

$V_{by_0} = V_0 \sin\theta_0$



$$x_b = \cancel{x_{b_0}}^{\,0} + V_{bx_0} t = d$$

$$t = \frac{d}{V_{bx_0}} = \frac{d}{V_0 \cos\theta_0}$$

$$y_b = y_{b_0} + V_{by_0} t - \tfrac{1}{2} g t^2$$

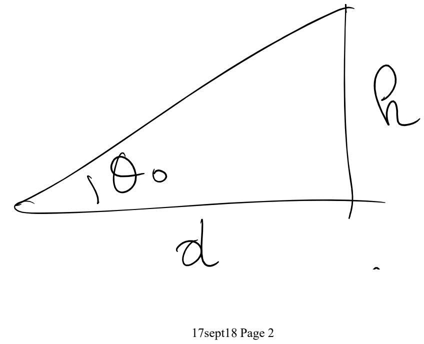
$$= 0 + V_0 \sin\theta_0 \left(\frac{d}{V_0 \cos\theta_0}\right) - \tfrac{1}{2} g \left(\frac{d}{V_0 \cos\theta_0}\right)^2$$

$$y_b = d \tan\theta_0 - \tfrac{1}{2} g \frac{d^2}{V_0^2 \cos^2\theta_0}$$

ourson (T : teddy bear)

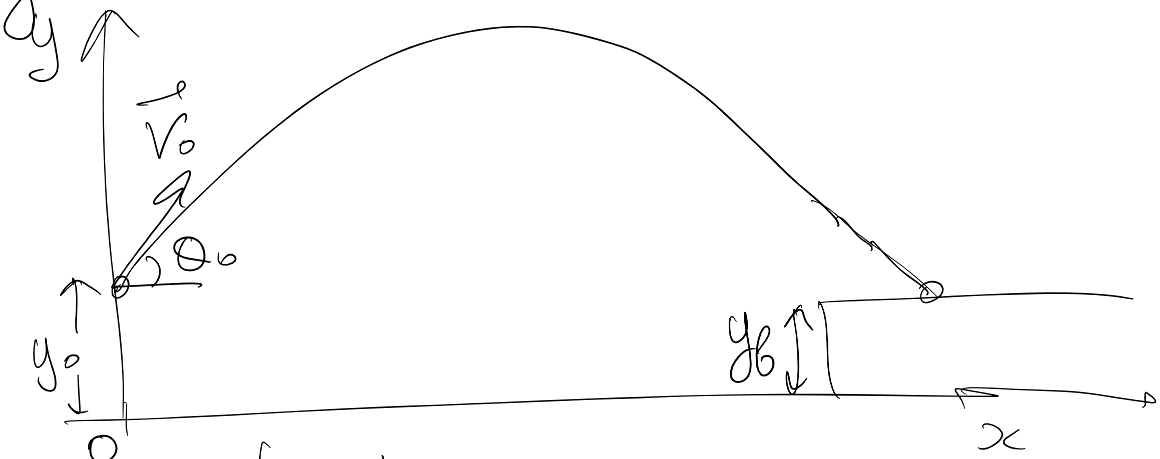
$$y_T = y_{T_0} + V_{Ty_0} t - \tfrac{1}{2} g t^2$$

$$= h + 0 - \tfrac{1}{2} g \left(\frac{d}{V_0 \cos\theta_0}\right)^2$$



$$\tan\theta_0 = \frac{h}{d} \Rightarrow y_b = y_T$$

Portée et la hauteur maximale et la forme de la trajectoire

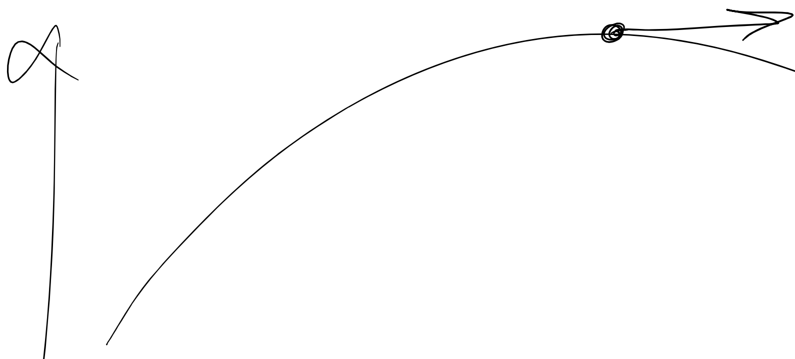


la forme de la trajectoire $x = V_{0x} t$ si $x_0 = 0$

$$y = y_0 + V_{0y} t - \frac{1}{2} g t^2$$

$$= y_0 + V_{0y}\left(\frac{x}{V_{0x}}\right) - \frac{1}{2} g \left(\frac{x}{V_{0x}}\right)^2$$

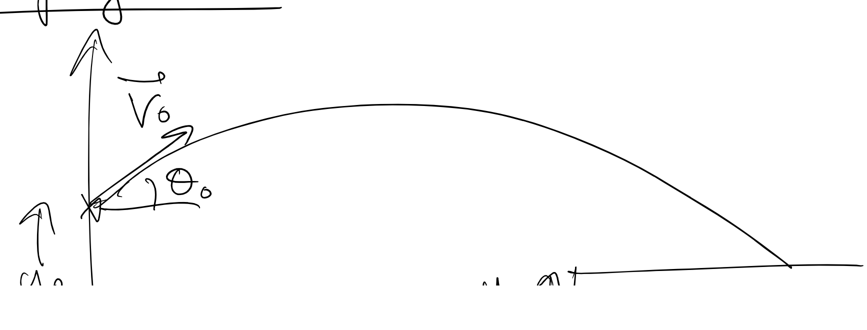
$$y - y_0 = \frac{V_{0y}}{V_{0x}} x - \frac{1}{2} g \frac{x^2}{V_{0x}^2}$$ eqn de la parabole



$\vec{V} = V_{0x} \hat{i}$

$(V_{0y} = 0)$

La portée d'un projectile

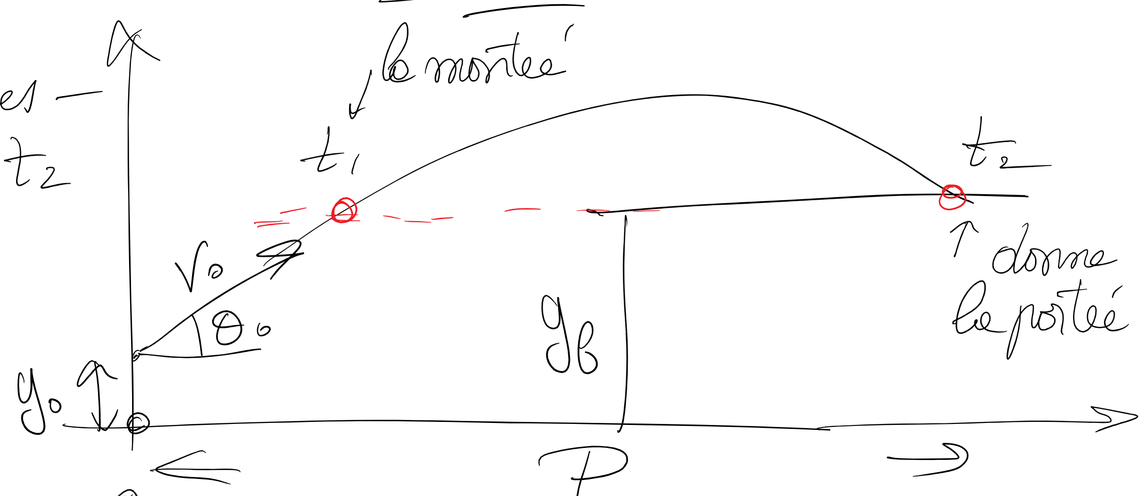


$P = (V_0 \cos\theta_0)\, t$

$\Delta y = y_f - y_0 = (V_0 \sin\theta_0)\, t - \frac{1}{2} g t^2$ ← eqn quadratique à résoudre pour t

si $y_0 > y_f$ 2 solns une < 0 et une > 0

si $y_f > y_0$ 2 solns positives t_1 et t_2



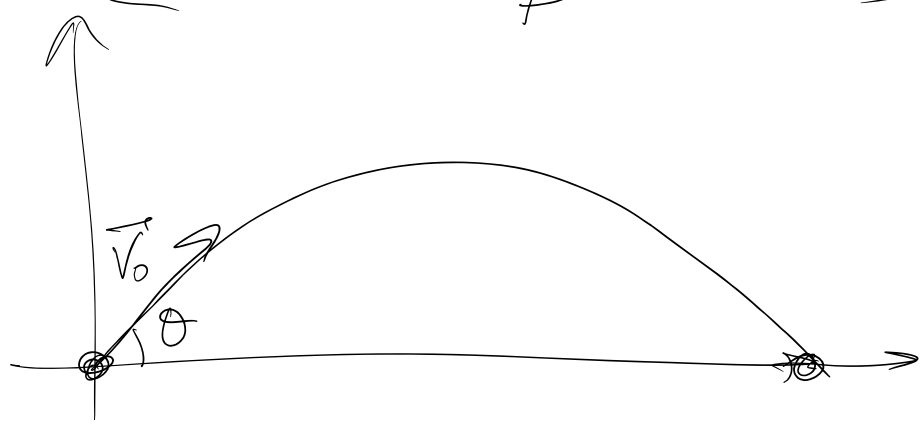
un cas spécial

si $y_f = y_0$

2 solns

$t_1 = 0$

$t_2 \neq 0$



$P = V_0 \cos\theta\, t$

$\Delta y = y_f - y_0 = 0 = V_0 \sin\theta_0\, t - \frac{1}{2} g t^2$

$= t\left(V_0 \sin\theta_0 - \frac{1}{2} g t\right)$

$t_1 = 0$ $\quad \Longrightarrow t_2 = \dfrac{2 V_0 \sin\theta_0}{g}$

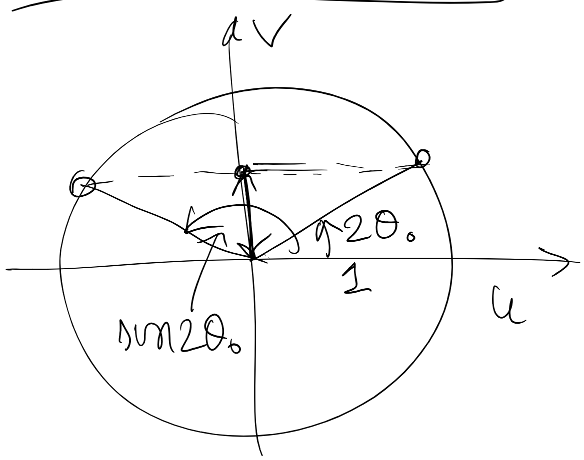
$P = 2V_0^2 \sin\theta_0 \cos\theta_0 \quad V_0^2 \sin 2\theta_0$

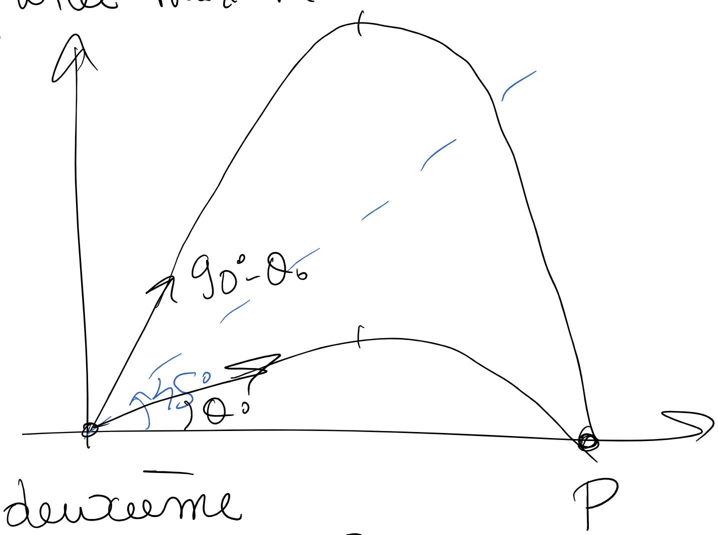
$$P = \frac{2V_0^2 \sin\theta_0 \cos\theta_0}{g} = \frac{V_0^2 \sin 2\theta_0}{g}$$

$$\boxed{P = \frac{V_0^2 \sin 2\theta_0}{g}}$$

$\sin 2\times 45^\circ = \sin 90^\circ = 1$

portée maximale





pour tout $2\theta_0$ il y a un deuxième angle avec le même sinus $180^\circ - 2\theta^\circ$

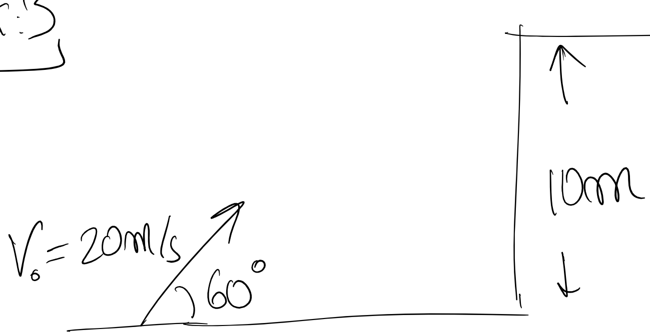
$$\sin 2\theta_0 = \sin(180^\circ - 2\theta_0) = \sin 2\theta_0'$$

$$\Rightarrow 2\theta_0' = 180^\circ - 2\theta_0$$

$$\boxed{\theta_0' = 90 - \theta_0}$$

quand $y_f = y_0$ le parabole est symmétrique
hauteur maximale est au milieu

Ex: 3



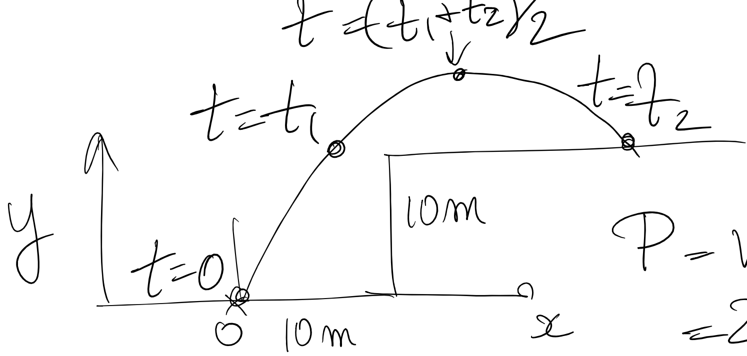
$$P = V_0 \cos\theta_0 t$$

$$y_f = y_0 + (V_0 \sin\theta_0)t - \frac{1}{2} g t^2$$

$\leftarrow$ 10m $\rightarrow$ $\quad y_0 = 0 \quad y_f = 10$

$$V_{y_0} = V_0 \sin\theta_0 = 20 \sin 60^\circ$$
$$= 20 \times 0{,}866$$
$$= 17{,}36 \text{ m/s}$$

$$10 = 17{,}36t - \tfrac{1}{2} \times 9{,}8\, t^2$$
$$\Rightarrow \begin{cases} t_1 = 0{,}73 \text{ s} \\ t_2 = 2{,}81 \text{ s} \end{cases}$$



$$\cos 60^\circ = \tfrac{1}{2}$$

$$P = V_0 \cos\theta_0\, t_2$$
$$= 20 \times \tfrac{1}{2} \times 2{,}81$$
$$= 28{,}1 \text{ m}$$

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Laquelle des trajectoires est correcte