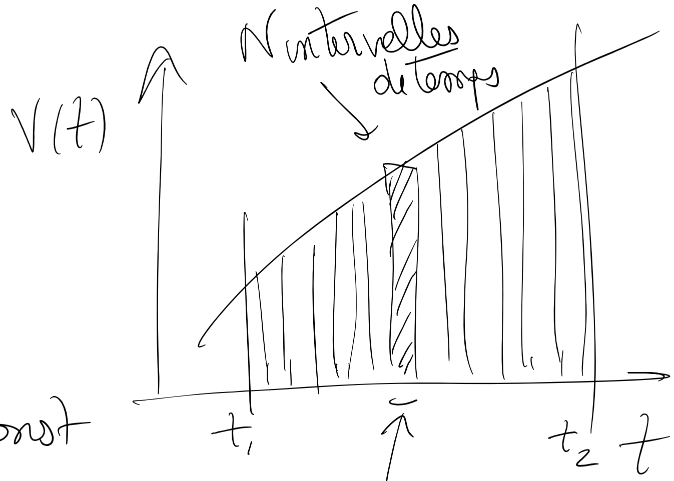
Jeudi 13 septembre 2018

September 13, 2018 10:28 AM



$$V = \frac{\Delta x}{\Delta t}$$

$$\Rightarrow \Delta x = v \Delta t$$

$$x - x_0 = v(t - t_0)$$

si $V = \text{const}$

si $V \neq \text{const}$

$\Delta x_n = v(t_n)\Delta t_n$

$$\Delta x = x_2 - x_1 = \sum_{n=1}^{N} \Delta x_n = \sum_{n=1}^{N} v(t_n)\Delta t_n = \lim_{\Delta t_n \to 0} \sum v(t_n)\Delta t_n$$

somme

= aire sous la courbe $v(t)$ entre t_1 et t_2

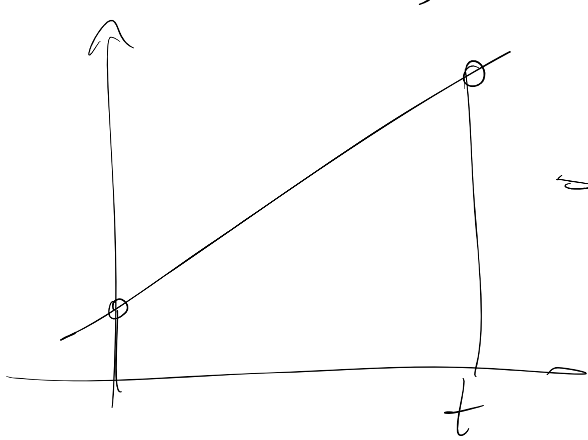
$$= \int_{t_1}^{t_2} v(t)\,dt$$

si $a = \text{const}$

$$\Delta V = a \Delta t$$

$$V - V_0 = a(t - t_0) \rightarrow \text{si } t_0 = 0 \quad V = V_0 + at \quad (1)$$

$\Rightarrow$



$$\rightarrow \boxed{x(t) = x_0 + V_0 t + \tfrac{1}{2} a t^2} \quad (2)$$

$$(1) \Rightarrow t = V - V_0$$

$$(3) \rightarrow (2) \quad x = x_0 + V_0\left(\frac{V - V_0}{a}\right) + \frac{1}{2} a \left(\frac{V - V_0}{a}\right)^2$$

$$\Delta x = x - x_0 = \left(\frac{v-v_0}{a}\right)\left[\underbrace{v_0 + \frac{1}{2}a\left(\frac{v-v_0}{a}\right)}_{\frac{1}{2}(v+v_0)}\right]$$

$$= \frac{1}{2a}(v-v_0)(v+v_0) = \frac{1}{2a}(v^2 - v_0^2)$$

$$\Rightarrow \boxed{v^2 - v_0^2 = 2a\,\Delta x}$$

$$\boxed{\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ v^2 - v_0^2 &= 2a(x - x_0) \end{aligned}}$$

Ex1 ~~Accélération~~ d'une voiture

$v_0 = 0$

en 6s $a = \text{const}$

$0\,km/h \to 100\,km/h$

(1) accélération $a = \dfrac{v - v_0}{t}$

j'ai besoin de v en m/s

$$v = 100\,km/h = \frac{100 \times 10^3}{3600}\,m/s = \frac{100}{3{,}6} = 27{,}8\,m/s$$

précision 1%

$$\Rightarrow a = \frac{27{,}8 - 0}{6} = 4{,}63\,m/s^2$$

(2) Déplacement

$$x - x_0 = \frac{1}{2}at^2 = \frac{1}{2} \times 4{,}63 \times 6^2 = 83{,}34\,m$$

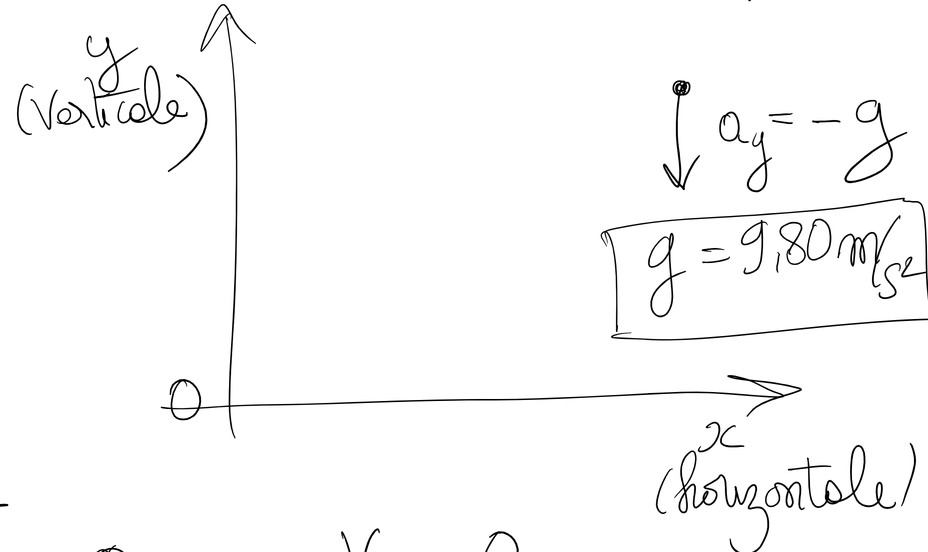
$$x - x_0 = \frac{v^2}{2a} = \frac{27{,}8^2}{2 \times 4{,}63} = 83{,}46\,m$$

$\equiv$ à 1% près

$x - x_0 = \frac{v}{2a} = \frac{27,8}{2 \times 4,63} = 83,46\,m$ à 1cm près

Chute libre

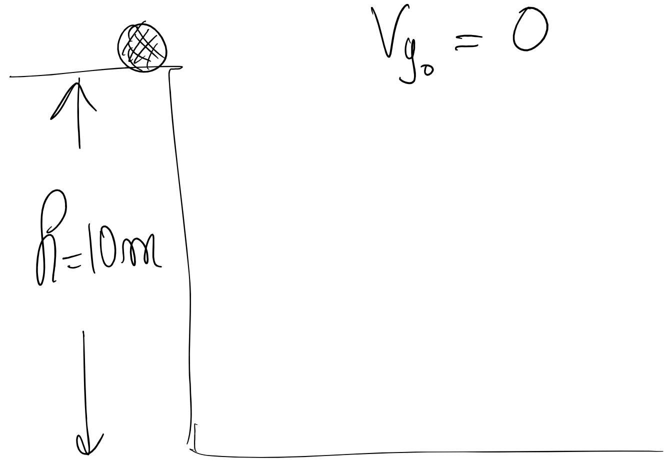
$$\boxed{\begin{aligned} V_y &= V_{y_0} - gt \\ y &= y_0 + V_{y_0}t - \tfrac{1}{2}gt^2 \\ V_y^2 - V_{y_0}^2 &= -2g(y - y_0) \end{aligned}}$$



$a_y = -g$

$\boxed{g = 9,80\,m/s^2}$

$V_{y_0} = 0$



$V_y^2 - 0 = -2g(0 - h)$

$V_y^2 = 2gh$

$= 2 \times 9,8 \times 10$

$= 196\,m^2/s^2$

$V_y = \pm\sqrt{196}\,m/s$

$= -14\,m/s$

Chap. 4 Mouvement en 2 dimensions

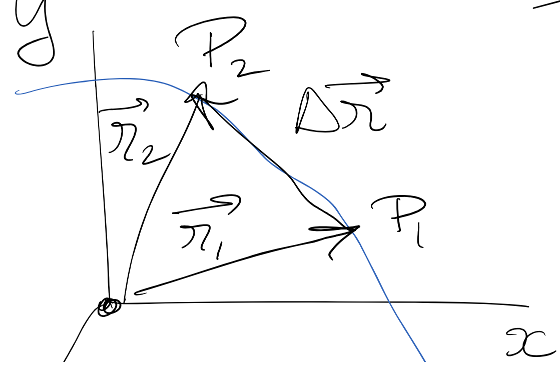
① Mouvement dans l'espace (3 dimensions)

à t_1 . $\vec{r}_1$ au point P_1

à t_2 $\vec{r}_2$ " P_2

déplacement

$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$

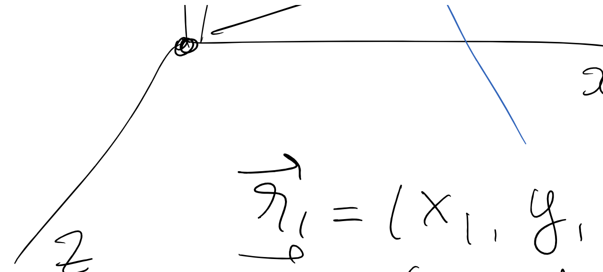


Déplacement

$$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$$

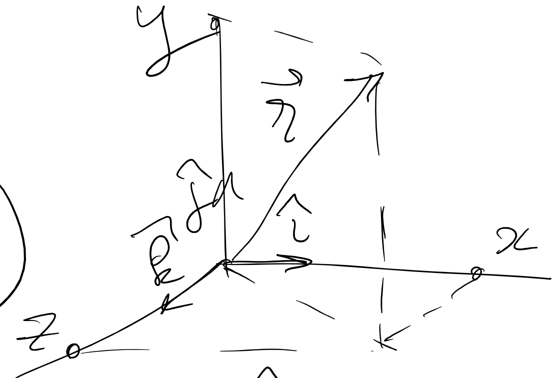
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta t = t_2 - t_1$$



$$\vec{r}_1 = (x_1, y_1, z_1)$$
$$\vec{r}_2 = (x_2, y_2, z_2)$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k})$$
$$-(x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$



$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

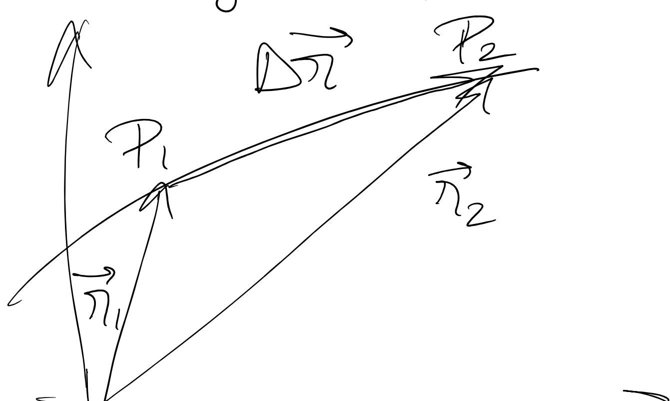
Vitesse moyenne

$$\vec{v}_{moy} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

Vitesse instantanée

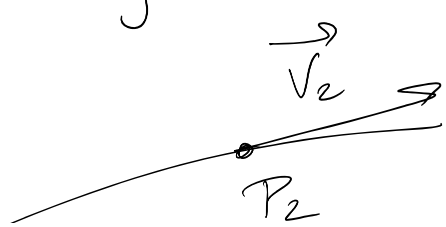
$$= \bar{v}_x\,\hat{i} + \bar{v}_y\,\hat{j} + \bar{v}_z\,\hat{k}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$



$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$
$$= v_x\,\hat{i} + v_y\,\hat{j} + v_z\,\hat{k}$$

$P_2 \to P_1$: $\vec{v}$ est tangente à la trajectoire



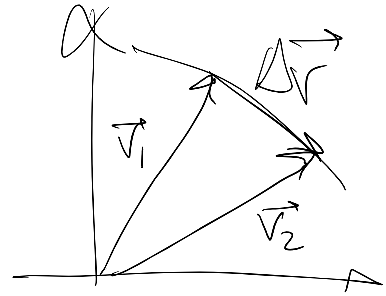
Accélération

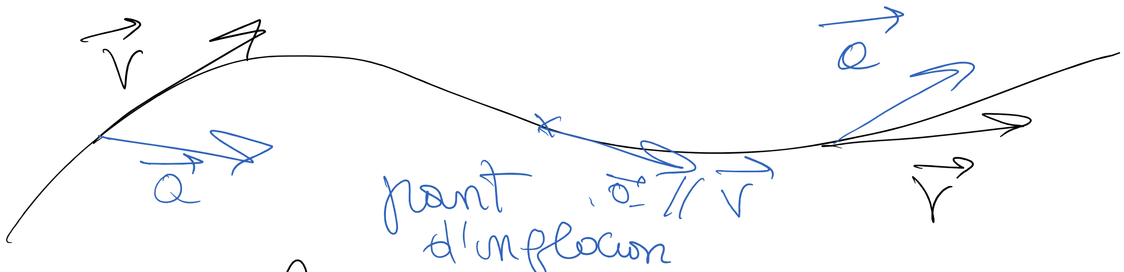
$\vec{\ }$ $\quad \vec{v}_2 - \vec{v}_1 \quad \Delta\vec{v}$

$$\vec{a}_{moy} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

tangent à la courbe décrite par $\vec{v}(t)$

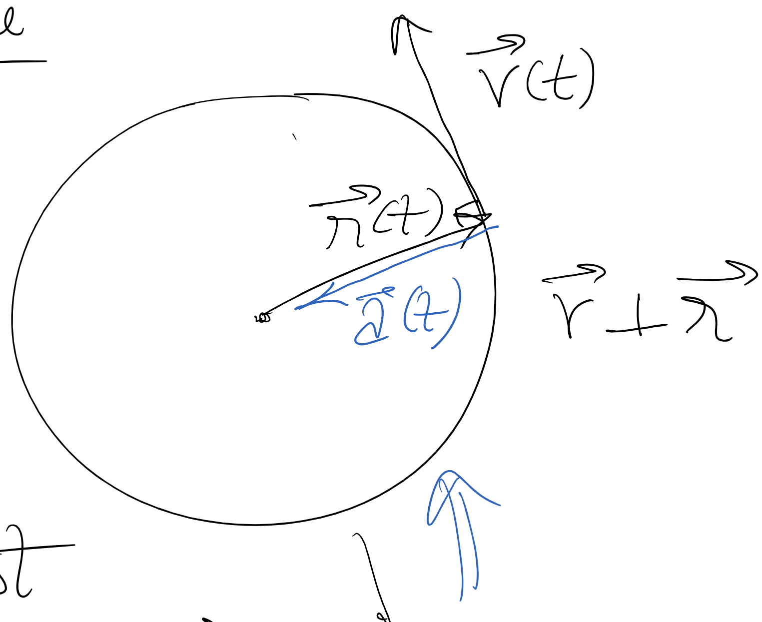




Mouvement circulaire

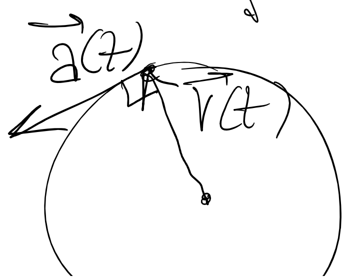
$|\vec{r}(t)| = r$ constant

$\Rightarrow$ particule décrit un cercle



spécifique : circulaire uniforme

$|\vec{v}| = \text{const}$

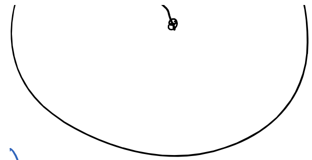


$\vec{a}$ est tangente au cercle décrit par cercle décrit par

$\vec{v}(t)$

$\Longrightarrow \vec{a}(t) \propto -\vec{r}(t)$

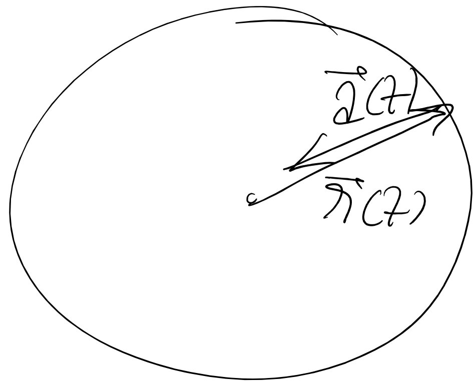
accélération centripète



$\propto$ Symbole de Proportionnalité

$\vec{a}(t) = -a_r \hat{r}$

radiale



$$\hat{r} = \frac{\vec{r}(t)}{|\vec{r}(t)|}$$