MAT 2384 A
DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS
TEST #2
November 20, 2015

Instructor: Dr. Steve Desjardins

Duration: 80 minutes

Name:_____

Student Number:_____

Instructions:
- Print your name and student number on this page.
- Verify that your copy of the exam has all 6 pages.
- You must answer all questions.
- Write your answers in the spaces below the questions. You may use the backs of the pages if necessary.
- Basic scientific calculators only - graphing and/or programmable calculators are NOT permitted.
- Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam. By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature:_____

Question 1 (5 marks) Find the general solution of

$$y''' - 2y'' - 3y' = 6 + 10\cos x\,.$$

Question 2 (5 marks) Find the general solution of

$$y'' - 2y' + y = 2x^{3/2}\, e^x \,.$$

Question 3 (5 marks) Find the general solution of the homogeneous system

$$\begin{aligned} y_1' &= 3y_1 - y_2 \\ y_2' &= y_1 + y_2 \end{aligned} .$$

Question 4 (5 marks) Use Gaussian Quadrature with 4 steps to approximate $\int_0^1 \cos(x^2)\,dx$ to 6 decimal places.

Formulas

$$\int_a^b f(x)dx = h\sum_{j=1}^{n} f(x_j^*)\,, \qquad |\epsilon| \le \frac{1}{24}\,M\,(b-a)\,h^2\,, \qquad M = max_{a\le x\le b}\,|f''(x)|$$

$$\int_a^b f(x)dx = \frac{h}{2}\sum_{j=1}^{n}\left(f(x_{j-1}) + f(x_j)\right), \qquad |\epsilon| \le \frac{1}{12}\,M\,(b-a)\,h^2\,, \qquad M = max_{a\le x\le b}\,|f''(x)|$$

$$\int_a^b f(x)dx = \frac{h}{3}\sum_{j=0}^{n-1}\left(f(x_{2j}) + 4f(x_{2j+1}) + f(x_{2j+2})\right), \qquad |\epsilon| \le \frac{1}{180}\,M\,(b-a)\,h^4\,,$$

$$M = max_{a\le x\le b}\,|f^{(4)}(x)|$$

$$\int_{-1}^{1} f(t)\,dt \approx \sum_{j=1}^{n} A_j\,f(t_j)\,, \qquad x = \frac{1}{2}\left(a\,(1-t) + b\,(t+1)\right)$$

Order n	Nodes t_j	Coefficients A_j
2	-0.5773502692	1.0
	0.5773502692	1.0
3	-0.7745966692	0.555555556
	0.0	0.888888889
	0.7745966692	0.555555556
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451
5	-0.9061798459	0.2369268850
	-0.5384693101	0.4786286705
	0.0	0.5688888889
	0.5384693101	0.4786286705
	0.9061798459	0.2369268850