

QUESTION 3. Follow the steps below to graph the function

$$f(x) = e^{-\frac{x^2-2x+1}{2}}.$$

a) Find the domain of $f(x)$.

Answer:

b) Give all vertical asymptotes:

c) Give all horizontal asymptotes:

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} e^{-\frac{x^2-2x+1}{2}} = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} e^{-\frac{x^2-2x+1}{2}} = 0$$

Answer: $y = \input{text}{0}$ is a horizontal asymptote.

d) Find the zeros of the function and the y -intercept.

Answer: The zeros are . Answer: y -intercept = .

e) Find the critical points. Indicate in the table where $f(x)$ is increasing or decreasing, and where the sign of $f'(x)$ is positive or negative:

Find the first derivative and simplify: $f'(x) = \input{text}{(1-x)e^{-\frac{x^2-2x+1}{2}}}$.

Find the critical points. Answer: $x = \input{text}{1}$.

| x | $-\infty$ | 1 | $+\infty$ |
|---------|-----------|---|-----------|
| $f'(x)$ | + | 0 | - |
| $f(x)$ | ↗ | | ↘ |

f) Find the inflection points. Indicate in the table where $f(x)$ is concave up or down, and where the sign of $f''(x)$ is positive or negative: Find the second derivative and simplify

$$f''(x) = \input{text}{x(x-2)e^{-\frac{x^2-2x+1}{2}}}$$

| | | | | |
|----------|-----------|--------|--------|-----------|
| x | $-\infty$ | 0 | 2 | $+\infty$ |
| $f''(x)$ | $+$ | 0 | $-$ | $+$ |
| $f(x)$ | \cup | \cap | \cup | |

Find the inflection points of f .

Answer: $x_1 = \boxed{x = 0}$ and $x_2 = \boxed{x = 2}$.

g) Graph the function $f(x)$:

