1. **Question 1**: What is the big-Oh (O) time complexity for the following algorithm (shown in pseudocode) in terms of input size n? Show all necessary steps:

(a) Algorithm MyAlgorithm (A,B):

```
Input: Arrays A and B each storing n >= 1 integers.
Output: What is the output? (Refer to part b below)
Start:
count = 0
for i = 0 to n-1 do {
sum = 0
for j = 0 to n-1 do {
sum = sum + A[0]
for k = 1 to j do
sum = sum + A[k]
}
if B[i] == sum then count = count + 1
}
return count
```

(b) Document a hand-run on MyAlgorithm for input arrays A = [1 2 5 9] and B = [2 29 40 57] and show the final output.

Solution:

(a) The big-Oh (O) time complexity of the algorithm MyAlgorithm (A,B):

`count = 0`	**1**
`for i = 0 to n-1 do {`	
`  sum = 0`	**n**
`  for j = 0 to n-1 do {`	
`  sum = sum + A[0]`	**n^2**
`  for k = 1 to j do`	
`  sum = sum + A[k]`	
`  }`	
`  if B[i] == sum then count = count + 1`	**n^3**
`  }`	
`  return count`	
The big-Oh time complexity is $O(n^3)$.	

(b) Hand-run the function MyAlgorithm MyAlgorithm for input arrays A = [1 2 5 9] and B = [2 29 40 57]

Step 1	count= 0			Step 45	j= 2		Step 87	j= 3			
Step 2	i= 0			Step 46	A[0]= 34		Step 88	A[0]= 71			
Step 3		j= 0		Step 47	sum= 34		Step 89	sum= 71			
Step 4		A[0]= 1		Step 48		k= 1	Step 90			k= 1	
Step 5		sum= 1		Step 49		A[k]= 1	Step 91			A[k]= 1	
Step 6		j= 1		Step 50		sum= 1	Step 92			sum= 1	
Step 7		A[0]= 2		Step 51		k= 2	Step 93			k= 2	
Step 8		sum= 2		Step 52		A[k]= 2	Step 94			A[k]= 2	
Step 9			k= 1	Step 53		sum= 2	Step 95			sum= 2	

```
Step 10                                A[k]= 1
Step 11                                sum= 1
Step 12                  j= 2
Step 13                  A[0]= 5
Step 14                  sum= 5
Step 15                                k= 1
Step 16                                A[k]= 1
Step 17                                sum= 1
Step 18                                k= 2
Step 19                                A[k]= 2
Step 20                                sum= 2
Step 21                  j= 3
Step 22                  A[0]= 13
Step 23                  sum= 13
Step 24                                k= 1
Step 25                                A[k]= 1
Step 26                                sum= 1
Step 27                                k= 2
Step 28                                A[k]= 2
Step 29                                sum= 2
Step 30                                k= 3
Step 31                                A[k]= 3
Step 32                                sum= 3
Step 33  B[i] = 2and the sum now =
Step 34  Is B[i] == sum: false
Step 35  i= 1
Step 36                  j= 0
Step 37                  A[0]= 30
Step 38                  sum= 30
Step 39                  j= 1
Step 40                  A[0]= 31
Step 41                  sum= 31
Step 42                                k= 1
Step 43                                A[k]= 1
Step 44                                sum= 1
Step 54                  j= 3
Step 55                  A[0]= 42
Step 56                  sum= 42
Step 57                                k= 1
Step 58                                A[k]= 1
Step 59                                sum= 1
Step 60                                k= 2
Step 61                                A[k]= 2
Step 62                                sum= 2
Step 63                                k= 3
Step 64                                A[k]= 3
Step 65                                sum= 3
Step 66  B[i] = 29and the sum now =
Step 67  Is B[i] == sum: false
Step 68  i= 2
Step 69                  j= 0
Step 70                  A[0]= 59
Step 71                  sum= 59
Step 72                  j= 1
Step 73                  A[0]= 60
Step 74                  sum= 60
Step 75                                k= 1
Step 76                                A[k]= 1
Step 77                                sum= 1
Step 78                  j= 2
Step 79                  A[0]= 63
Step 80                  sum= 63
Step 81                                k= 1
Step 82                                A[k]= 1
Step 83                                sum= 1
Step 84                                k= 2
Step 85                                A[k]= 2
Step 86                                sum= 2
Step 96                                k= 3
Step 97                                A[k]= 3
Step 98                                sum= 3
Step 99  B[i] = 40and the sum now =
Step 100 Is B[i] == sum: false
Step 101 i= 3
Step 102                 j= 0
Step 103                 A[0]= 88
Step 104                 sum= 88
Step 105                 j= 1
Step 106                 A[0]= 89
Step 107                 sum= 89
Step 108                               k= 1
Step 109                               A[k]= 1
Step 110                               sum= 1
Step 111                 j= 2
Step 112                 A[0]= 92
Step 113                 sum= 92
Step 114                               k= 1
Step 115                               A[k]= 1
Step 116                               sum= 1
Step 117                               k= 2
Step 118                               A[k]= 2
Step 119                               sum= 2
Step 120                 j= 3
Step 121                 A[0]= 100
Step 122                 sum= 100
Step 123                               k= 1
Step 124                               A[k]= 1
Step 125                               sum= 1
Step 126                               k= 2
Step 127                               A[k]= 2
Step 128                               sum= 2
Step 129                               k= 3
Step 130                               A[k]= 3
Step 131                               sum= 3
Step 132 B[i] = 57and the sum now =
Step 133 Is B[i] == sum: false
Step 135 final count variable is 0
Step 136 final sum variable is 116
```

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2. **Question 2:** Consider the following code fragments (a), (b) and (c) where n is the variable specifying data size and C is a constant. What is the big-Oh time complexity in terms of n in each case? Show all necessary steps.

(a)

```
for (int i = 0; i < n; i = i + C)
for (int j = 0; j < 10; j++)
Sum[i] += j * Sum[i];
```

(b)

```
for (int i = 1; i < n; i = i * C)
for (int j = 0; j < i; j++)
Sum[i] += j * Sum[i];
```

(c)

```
for (int i = 1; i < n; i = i * 2)
for (int j = 0; j < n; j = j + 2)
Sum[i] += j * Sum[i];
```

Solution:

(a)

`for (int i = 0; i < n; i = i + C);`	**n/c**
`for (int j = 0; j < 10; j++);`	**10*n/c**
`Sum[i] += j * Sum[i];`	**10*n/c +n**
The big-Oh time complexity is O(n).	

(b)

`for (int i = 1; i < n; i = i * C)`	**log(n)**
`for (int j = 0; j < i; j++)`	**nlog(n)**
`Sum[i] += j * Sum[i];`	**nlogn + n**
The big-Oh time complexity is O(nlogn).	

(c)

`for (int i = 1; i < n; i = i * 2)`	**log(n)**
`for (int j = 0; j < n; j = j + 2)`	**(n/2)log(n)**
`Sum[i] += j * Sum[i];`	**(n/2)log(n) + n**
The big-Oh time complexity is O(nlogn).	

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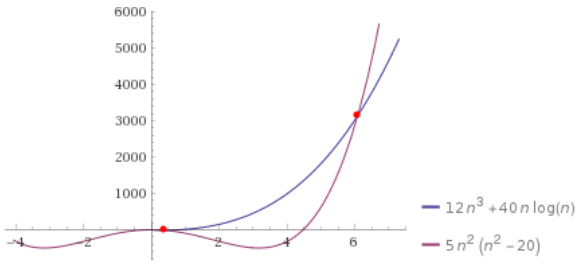
3. **Question 3:** The number of operations executed by algorithms A and B are $12n^3 + 40n \log n$ and $5n^4 - 100n^2$ respectively. Determine an n_0 such that B is greater than A for $n \geq n_0$.

Solution:



The graphs above are showing the behavior of these algorithms beginning with A higher (slower) than B, and finally cross. After the crossing point, B is always higher than A. Therefore we need to find the point where they cross.
That is the value where $12n^3 + 40n \log n = 5n^4 - 100n^2$.

Applying algebra we get:
$12n^3 + 40n \log n = 5n^4 - 100n^2$

- ⇨ $+ 40\log n =$
- ⇨ $40 \log n = 5n^3 - 12n^3 - 100n^2$

- ⇨ Solving for n we get $n \approx 6.08006101894724....$ Thus $n_0 = 7$, since for all $n >= 7$, A will be faster than B (at 6.0800… they're equal.)

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4. **Question 4:** Answer the following questions:
   (a) Show that if *d(n)* is *O(f(n))* and *e(n)* is *O(g(n))*, then *d(n) + e(n) is O(f(n) + g(n)).*

   (b) Show that if *d(n)* is *O(f(n))* and *e(n)* is *O(g(n))*, then *d(n) – e(n)* is not necessarily *O(f(n) – g(n))*.
   (c) Show that $2^{n+1} + n^2$ is $O(2^n)$
   (d) Show that $f(n) = \sum_{i=1}^{n} i^2$ is $O(n^3)$

Solution:

(a) True.

```
d(n) is O(f(n)), and e(n) is O(g(n))
  -> d(n) <= K f(n) for n >= N, and e(n) <= L g(n) for n >= M
  -> d(n) + e(n) <= K f(n) + L g(n)
                 <= (K + L)(f(n) + g(n)), for n >= max(N,M)
```

(b) False.

```
We have d(n)=2n and e(n)=n. So, d(n)-e(n)=n.
➔ d(n)-e(n)=n.
Because d(n)=O(n) and e(n)=O(n), so we have:
   f(n)=g(n)=n,
so
O(f(n)-g(n))= O(n-n) = O(1),
and obviously n ≠ O(1). Contradict to the question.
```

(c) True

$2^{n+1}=O(2^n)$, but $2^{2n} \neq O(2^n)$. To show that $2^{n+1}=O(2^n)$, we must find constants c; $n_0>0$ such that $0 \leq 2^{n+1}c2^n$ for all $n \geq n_0$. Since $2^{n+1}= 2*2^n$ for all n, we can satisfy the definition with $c = 2$ and $n_0= 1$.

To show that $2^{2n} \neq O(2^n)$, assume there exist constants c; $n_0>0$ such that $0 \leq 2*2^n \leq c*2^n$ for all $n \geq n_0$. Then $2*2^n = 2^n \times 2^n \leq c*2^n$. But no constant is greater than 2^n for all n, and so the assumption leads to a contradiction.

(d) Choose k=1
Assuming n>1, then

$$\frac{f(n)}{g(n)} = \frac{\sum_{i=1}^{n} i^2}{n^3} \leq \frac{\sum_{i=1}^{n} n^2}{n^3} = \frac{n^3}{n^3} = 1$$

Choose c=1. Note that i ≤ n because n is the upper limit. Hence, $\sum_{i=1}^{n} i^2$ is $O(n^3)$ because $\sum_{i=1}^{n} i^2 \leq n^3$ whenever n>1.

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5. **Programming Questions**: Referring to the slides from text book, Chapter 5, also added to moodle, there are two versions of Fibonacci number calculators: BinaryFib(n) and LinearFibonacci(n). The first algorithm has exponential time complexity, while the second one is linear.

(a) In this programming assignment, you will implement in Java both the versions of Fibonacci calculators and experimentally compare their runtime performances. For that, with each implemented version you will calculate Fibonnaci (5), Fibonacci (10), etc. in increments of 5 up to Fibonacci (100) (or higher value if required for your timing measurement) and measure the corresponding run times. You need to use Java's built-in time function for this purpose. You should redirect the output of each program to an out.txt file. You should write about your observations on timing measurements in a separate text or pdf file. You are required to submit the two fully commented Java source files, the compiled executables, and the text/pdf files.The set of strings over {0; 1}, such that there are two 0's separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.

(b) Briefly explain why the first algorithm is of exponential complexity and the second one is linear (more specifically, how the second algorithm resolves some specific bottleneck(s) of the first algorithm). You can write your answer in a separate file and submit it together with the other submissions.

(c) Do any of the previous two algorithms use tail recursion? Why or why not? Explain your answer. If your answer is ``No'' then:

i. design the pseudo code for a tail recursive version of Fibonacci calculator;
ii. implement the corresponding Java program and repeat the same experiments as in part (a) above. You will need to submit both the pseudo code and the Java program, together with your experimental results.

Solution:

(a) See the attachment.

(b) The first algorithm has the exponential complexity of $O(3^n)$ because it calls itself not only recursively but it calls itself recursively within itself. A call to a loop inside the method would raise the power x of n^x. However, a call to itself recursively raises the power of x in x^n. The second algorithm calls itself O(n) times by reducing the amount of times it calls back to itself, once instead of three times.

(c) The first algorithm uses tail recursion, because the last thing the function does before it returns is recursively call itself. The second algorithm does not use tail-recursion. A tail recursive method could not be implemented because it is printing out the results and manipulating data after each recursive call. This could not be changed.

i. design the pseudo code for a tail recursive version of Fibonacci calculator

```
# A tail recursive function to calculate nth fibonacci number
function tail fibonacci (n, a = 0, b = 1):
    if(n == 0):
        return a
    if(n == 1):
        return b
    return fibonacci(n-1,b,a+b);
```

ii. implement the corresponding Java program and repeat the same experiments as in part (a) above. You will need to submit both the pseudo code and the Java program, together with your experimental results.
See the attachment of .java file.
