Module 4

Supplemental Materials

Example:

Peak Emission

QUESTION

For a $Ga_{1-x}Al_xAs$ laser with x = 0.07, determine the band gap of the materials, and the peak emission wavelength of the laser.

SOLUTION

From:

$$E_g = 1.424 + 1.266 + 0.266x^2$$

We find E_g = 1.51 eV

From:

$$E_g(eV) = \frac{1.24}{\lambda(\mu m)}$$

We find λ = 820 nm

Example:

An Optical Resonator in Air

QUESTION

Consider a Fabry-Perot optical cavity in air of length 100 microns with mirrors that have a reflectance of 0.90. Calculate the cavity mode nearest to the wavelength 900 nm, and corresponding wavelength. Calculate the separation of the modes, the finesse, the spectral width of each mode and the *Q*-factor.

SOLUTION

Find the mode number *m* corresponding to 900 nm and then take the integer

$$m = \frac{2L}{\lambda} = \frac{2(100\times10^{-6})}{(900\times10^{-9})} = 222.2 \rightarrow \lambda_m = \frac{2L}{m} = \frac{2(100\times10^{-6})}{(222)} = 900.9\ \text{nm}$$

Thus, m = 222 (must be an integer)

λ_m = 900.90 nm ≈ 900 nm (very close)

The frequency corresponding to λ_m is

$\upsilon_m = c/\lambda_m = (3\times10^{8})/(900.9\times10^{-9}) = 3.33\times10^{14}$ Hz

SOLUTION (CONT'D)

$\upsilon_f = c/2L$ = separation of modes

$= (3\times10^8) / [2(100\times10^{-6})] = 1.5\times10^{12}$ Hz

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.90^{1/2}}{1-0.90} = 29.8 \qquad \delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.5\times10^{12}}{29.8} = 50.3\,\text{GHz}$$

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times10^8)}{(3.33\times10^{14})^2}(5.03\times10^{10}) = 0.136\,\text{nm}$$

The Q-factor is

$Q = mF = (222)(29.8) = 6.6\times10^3$

Example:

Semiconductor Optical Cavity

QUESTION

Consider a Fabry-Perot optical cavity of a semiconductor material of length 250 microns with mirrors, each with a reflectance of 0.90. Calculate the cavity mode nearest to 1310 nm. Calculate the separation of the modes, finesse, the spectral width of each mode, and the *Q*-factor. Take $n = 3.6$ for the semiconductor medium.

SOLUTION

Given, $L = 250\times10^{-6}$ m, $n = 3.6$, $R = 0.90$

$\Delta\upsilon_m = \upsilon_f = c/2nL$ = Separation of modes = 1.67×10^{11} Hz

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.9^{1/2}}{1-0.9} = 29.8 \qquad \delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.67\times10^{11}}{29.8} = 5.59\,\text{GHz}$$

SOLUTION (CONT'D)

Mode number *m* corresponding to 1310 nm is:

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(250\times10^{-6})}{(1310\times10^{-9})} = 1374.05$$

which must be an integer (1374) so that the actual mode wavelength is

$$\lambda_m = \frac{2nL}{m} = \frac{2(3.6)(250\times10^{-6})}{(1374)} = 1310.04 \text{ nm}$$

For all practical purposes the mode wavelength is 1310 nm

Mode frequency is $\upsilon_m = \frac{c}{\lambda_m} = \frac{(3\times10^{8})}{(1310\times10^{-9})} = 2.3\times10^{14} \text{ Hz}$

SOLUTION (CONT'D)

Spectral width of a mode in wavelength is

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times10^8)}{(3.33\times10^{14})^2}(5.03\times10^{10}) = 0.136\,\text{nm}$$

The Q-factor is

$Q = mF = (1374)(29.8) = 4.1\times10^4$

Example:

Modes in a Laser and the Optical Cavity Length

QUESTION

Consider an AlGaAs based heterostructure laser diode that has an optical cavity of length 200 μm. The peak radiation is at 870 nm and the refractive index of GaAs is about 3.6. What is the mode integer *m* of the peak radiation and the separation between the modes of the cavity? If the optical gain vs. wavelength characteristics has a FWHM wavelength width of about 6 nm how many modes are there within this bandwidth? How many modes are there if the cavity length is 20 μm?

SOLUTION

The wavelength λ of a cavity mode and length *L* are related by, ***m*(1/2)(λ/*n*) = *L***, where *n* is the refractive index of the semiconductor medium, so that

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(200 \times 10^{-6})}{(870 \times 10^{-9})} = 1655.1$$

or 1655 (integer).

The wavelength separation $\Delta\lambda_m$ between the adjacent cavity modes *m* and (*m*+1) is

$$\Delta\lambda_m = \frac{2nL}{m} - \frac{2nL}{m+1} \approx \frac{2nL}{m^2} = \frac{\lambda^2}{2nL}$$

where we assumed that the refractive index *n* does not change significantly with wavelength from one mode to another. Thus the separation between the modes for a given peak wavelength increases with decreasing *L*.

SOLUTION (CONT'D)

When L = 200 μm,

$$\Delta\lambda_m = \frac{(870\times10^{-9})^2}{2(3.6)(200\times10^{-6})} = 5.26\times10^{-10}\ \text{m or } 0.526\ \text{nm}$$

If the optical gain has a bandwidth of $\Delta\lambda_{1/2}$, then there will be $\Delta\lambda_{1/2}/\Delta\lambda_m$ number of modes, or (6 nm)/(0.526 nm), that is 11 modes.

When L = 20 μm, the separation between the modes becomes,

$$\Delta\lambda_m = \frac{(870\times10^{-9})^2}{2(3.6)(20\times10^{-6})} = 5.26\ \text{nm}$$

Then $(\Delta\lambda_{1/2})/\Delta\lambda_m$ = 1.14 and there will be one mode that corresponds to about 870 nm. In fact m must be an integer so that choosing the nearest integer, m = 166, gives λ = 867.5 nm (choosing m = 165 gives 872.7 nm) It is apparent that reducing the cavity length suppresses higher modes. Note that the optical bandwidth depends on the diode current.

Example:

External Quantum Efficiency - 1

QUESTION

Find the external quantum efficiency for a $Ga_{0.997}Al_{0.03}As$ laser diode, which has an optical-power vs current relationship of 0.5 mW/mA. Assume output wavelength is 1300 nm.

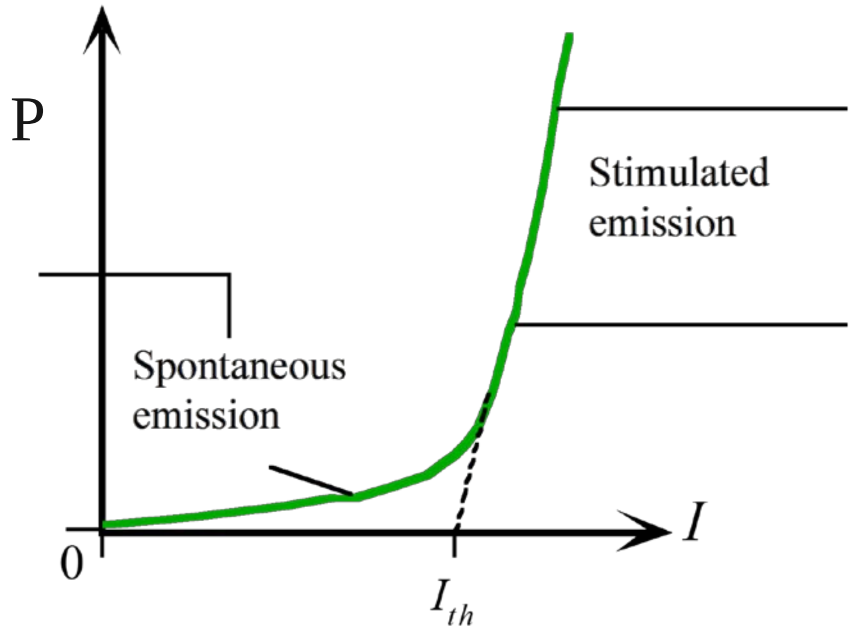
SOLUTION

$$\eta_{ext} = \frac{q}{h\nu}\frac{\Delta P}{\Delta I}$$

$$\frac{\Delta P}{\Delta I} = 0.5mW / mA$$

$$E_g(eV) = h\nu = \frac{1.24}{\lambda(\mu m)} = 0.95eV$$

$$\eta_{ext} = \frac{1}{0.95}(0.5) = 0.52$$



Example:

External Quantum Efficiency - 2

QUESTION

A GaAlAs laser has L = 500 μm with effective absorption coefficient of 10/cm. Assume $R_1 = R_2 = 0.32$ at each side, also assume confinement factor is equal to 1, what is:

a) Optical gain g_{th} at the lasing threshold?

b) What is external quantum efficiency if internal quantum efficiency is 0.65?

c) Calculate the above parameters if $R_1 = 0.9$ and $R_2 = 0.32$.

SOLUTION

a) $$\Gamma g_{th} = \bar{\alpha} + \frac{1}{2L}\ln\frac{1}{R_1R_2} = 10 + \frac{1}{2(500)\times10^{-4}}\ln\frac{1}{(0.32)^2} \cong 14.4/cm$$

b) $$\eta_{ext} = \eta_{\text{int}}\left(\frac{g_{th} - \bar{\alpha}}{g_{th}}\right) = 0.65\times\frac{14.4-10}{14.4} \cong 20\%$$

c) g_{th}= 22.4 /cm and η_{ext} = 36%

Example:

A GaAs Quantum Well

QUESTION

Consider a very thin GaAs quantum well sandwiched between two wider bandgap semiconductor layers of AlGaAs ($Al_{0.33}Ga_{0.67}As$ in present case). The QW depths from E_c and E_v are approximately 0.28 eV and 0.16 eV respectively. Effective mass m_e* of a conduction electron in GaAs is approximately $0.07m_e$ where m_e is the electron mass in vacuum. Calculate the first two electron energy levels for a quantum well of thickness 10 nm. What is the hole energy in the QW above E_v of GaAs, if the hole effective mass $m_h^* \approx 0.50m_e$? What is the change in the emission wavelength with respect to bulk GaAs, for which E_g = 1.42 eV? Assume infinite QW depths for the calculations.

SOLUTION

Suppose that ε_n is the electron energy with respect to E_c in GaAs, or $\varepsilon_n = E_n - E_c$
Then, the energy of an electron in a one-dimensional infinite potential energy well is

$$\varepsilon_n = \frac{h^2 n^2}{8 m_e^* d^2} = \frac{(6.626\times10^{-34})^2(1)^2}{8(0.07\times9.1\times10^{-31})(10\times10^{-9})^2} = 8.62\times10^{-21} \text{ J or } 0.0538 \text{ eV}$$

where n is a quantum number, 1, 2,…, and we have used $d = 10\times10^{-9}$ m, $m_e^* = 0.07m_e$ and $n = 1$ to find $\varepsilon_1 = 0.054$ eV. The next level from the same calculation with $n = 2$ is $\varepsilon_2 = 0.215$ eV.

The hole energy levels below E_v are given by $\varepsilon'_{n'} = \frac{h^2 n'^2}{8 m_h^* d^2}$

SOLUTION (CONT'D)

where n' is the quantum number for the hole energy levels above E_v. Using d = 10×10^{-9} m, $m_h{}^*$ ≈ 0.5m$_e$ and n' = 1, we find, ε'_1 = **0.0075 eV.**

The wavelength of emission from bulk GaAs with E_g = 1.42 eV is

$$\lambda_g = \frac{hc}{E_g} = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{(1.42)(1.602 \times 10^{-19})} = 874 \times 10^{-9} \text{ m (874 nm)}$$

In the case of QWs, we must obey the selection rule that the radiative transition must have $\Delta n = n' - n = 0$.

SOLUTION (CONT'D)

Thus, the radiative transition is from ε_1 to ε_1' so that the emitted wavelength is

$$\lambda_{QW} = \frac{hc}{E_g + \varepsilon_1 + \varepsilon_1'} = \frac{(6.626\times10^{-34})(3\times10^{8})}{(1.42+0.0538+0.0075)(1.602\times10^{-19})}$$

$$= 838\times10^{-9}\ \text{m}\ \ (838\ \text{nm})$$

The difference is λ_g - λ_{QW} = 36 nm. We note that we assumed an infinite PE well. If we actually solve the problem properly by using a finite well depth, then we would find $\varepsilon_1 \approx$ 0.031 eV, $\varepsilon_2 \approx$ 0.121 eV, $\varepsilon'_1 \approx$ 0.007 eV. The emitted photon wavelength is 848 nm and λ_g - λ_{QW} = 26 nm.

Example:

Threshold Current and Optical Output Power

QUESTION

Consider GaAs DH laser diode that lases at 860 nm. It has an active layer (cavity) length L of 250 μm. The active layer thickness d is 0.15 μm and the width W is 5 μm. The refractive index is 3.6, and the attenuation coefficient α_s inside the cavity is 10^3 m^{-1}. The required threshold gain g_{th} corresponds to a threshold carrier concentration $n_{th} \approx 2 \times 10^{18}$ cm^{-3}. The radiative lifetime τ_r in the active region can be found (at least approximately) by using $\tau_r = 1/Bn_{th}$, where B is the direct recombination coefficient, and assuming strong injection as will be the case for laser diodes. For GaAs, $B \approx 2 \times 10^{-16}$ m^3 s^{-1}. What is the threshold current density and threshold current? Find the output optical power at $I = 1.5I_{th}$, and the external slope efficiency η_{slope}. How would $\Gamma = 0.5$ affect the calculations?

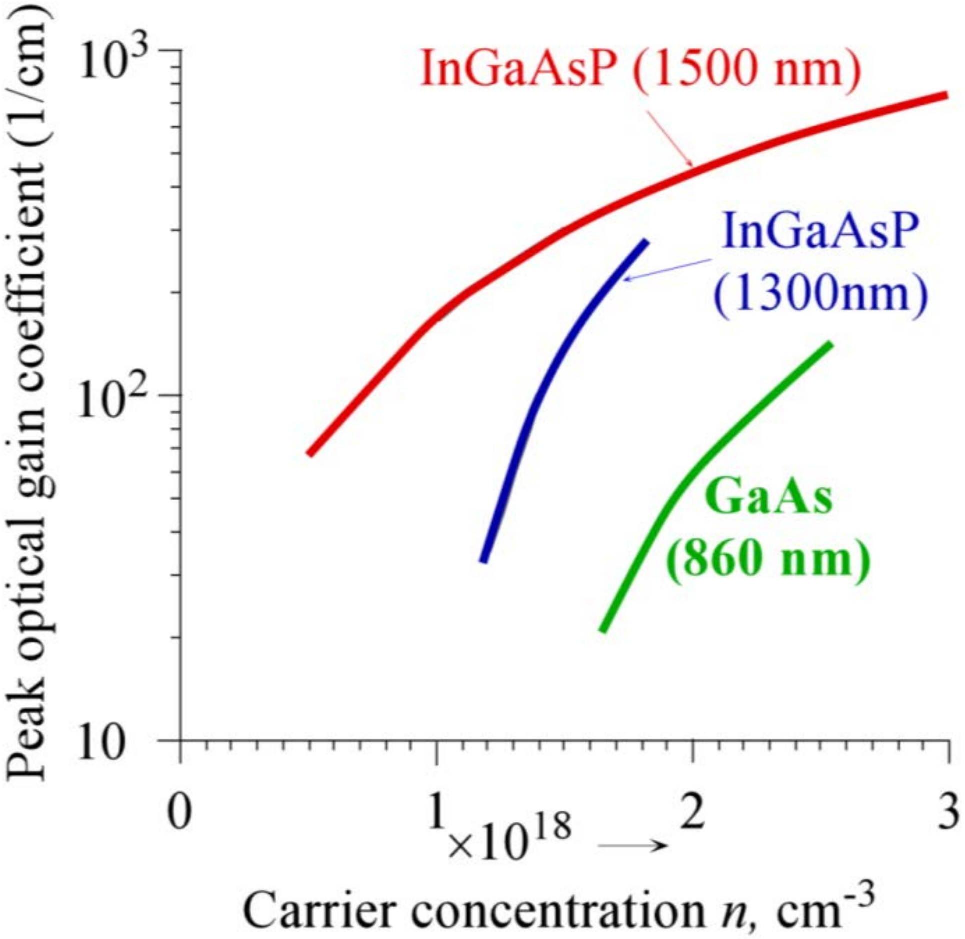
SOLUTION

The reflectances at the each end are the same (we assume no other thin film coating on the ends of the cavity) so that $R = (n-1)^2 / (n+1)^2 = 0.32$. The total attenuation coefficient α_t and hence the threshold gain g_{th}, assuming $\Gamma = 1$, is

$$g_{\text{th}} = \alpha_t = (10 \text{ cm}^{-1}) + \frac{1}{(2 \times 250 \times 10^{-4} \text{ cm})} \ln\left[\frac{1}{(0.32)(0.32)}\right] = 55.6 \text{ cm}^{-1}$$

From the figure on the next page, at this gain of 56 cm^{-1}, $n_{\text{th}} \approx 2 \times 10^{18} \text{ cm}^{-3}$. This is the threshold carrier concentration that gives the right gain under ideal optical confinement, with $\Gamma = 1$.

SOLUTION (CONT'D)



SOLUTION (CONT'D)

The radiative lifetime $\tau_r = 1/Bn_{\text{th}} = 1/[2\times10^{-16}\ \text{m}^3\ \text{s}^{-1})(\ (2\times10^{24}\ \text{m}^{-3})] =$ **2.5 ns**

Since $J = I/WL$, the threshold current density is

$$J_{\text{th}} = \frac{n_{\text{th}}ed}{\tau_r} = \frac{(2\times10^{24}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})(0.15\times10^{-6}\ \text{m})}{(2.5\times10^{-9}\ \text{s})}$$

$= 1.9\times10^{7}\ \text{A m}^{-2}$ or $1.9\ \text{kA cm}^{-2}$ or $19\ \text{A mm}^{-2}$.

The threshold current itself is

$I_{\text{th}} = (WL)J_{\text{th}} = (5\times10^{-6}\ \text{m})\ (250\times10^{-6}\ \text{m})(1.9\times10^{7}\ \text{A m}^{-2}) = 0.024$ A or **24 mA**

The photon cavity lifetime depends on α_t, and is given by

$\tau_{\text{ph}} = n/(c\alpha_t) = 3.6\ /\ [(3\times10^{8}\ \text{m s}^{-1})(5.56\times10^{3}\ \text{m}^{-1}) =$ **2.16 ps**

SOLUTION (CONT'D)

The laser diode output power is

$$P_o = \left[\frac{hc^2\tau_{ph}(1-R)}{2en\lambda L}\right](I - I_{th}) = \frac{(6.626\times10^{-34})(3\times10^{8})^2(2.16\times10^{-12})(1-0.32)}{2(1.6\times10^{-19})(3.6)(860\times10^{-9})(250\times10^{-6})}(I - I_{th})$$

That is **P_o = (0.35 W A^{-1})(I – I_{th}) = (0.35 mW mA^{-1})(I – 24 mA)**

When I = 1.5I_{th} = 36 mA,

P_o = (0.35 mW mA^{-1})(36 mA-24 mA) = **4.2 mW**

The slope efficiency is the slope of the P_o vs. I characteristic above I_{th}

$$\eta_{slope} = \frac{\Delta P_o}{\Delta I} = \left[\frac{hc^2\tau_{ph}(1-R)}{2en\lambda L}\right]$$ = **0.35 mW mA^{-1}**

SOLUTION (CONT'D)

We can now repeat the problem say for $\Gamma = 0.5$, which would give $\Gamma g_{th} = \alpha_t$, so that $g_{\text{th}} = 55.6\ \text{cm}^{-1} / 0.5 = 111\ \text{cm}^{-1}$. At this gain of 111 cm^{-1}, $n_{\text{th}} \approx 2.5\times10^{18}\ \text{cm}^{-3}$. The new radiative lifetime,

$$\tau_r = 1/Bn_{\text{th}} = 1/[2.0\times10^{-16}\ \text{m}^3\ \text{s}^{-1})(2.5\times10^{24}\ \text{m}^{-3})] = 2.0\ \text{ns}$$

The corresponding threshold current density is

$J_{\text{th}} = n_{\text{th}}ed/\tau_r = (2.5\times10^{24}\ \text{m}^{-3})(1.6\times10^{-19}\ \text{C})(0.15\times10^{-6}\ \text{m})/(2.0\times10^{-9}\ \text{s})$ **= 30 A mm^{-2}**

and the corresponding threshold current I_{th} is **37.5 mA**.

THERE ARE SEVERAL IMPORTANT NOTES TO THIS PROBLEM

- First, the threshold concentration $n_{th} \approx 2\times1018$ cm^{-3} was obtained graphically from the figure by using the g_{th} value we need.
- Second is that, at best, the calculations represent rough values since we also need to know how the mode spreads into the cladding where there is no gain but absorption and, in addition, what fraction of the current is lost to nonradiative recombination processes. We can increase α_s to account for absorption in the cladding, which would result in a higher g_{th}, larger n_{th} and greater I_{th}. If τ_{nr} is the nonradiative lifetime, we can replace τ_r by an effective recombination time τ such that $\tau^{-1} = \tau_r^{-1} + \tau_{nr}^{-1}$ which means that the threshold current will again be larger. We would also need to reduce the optical output power since some of the injected electrons are now used in nonradiative transitions.
- Third, is the low slope efficiency compared with commercial LDs. η_{slope} depends on τ_{ph}, the photon cavity lifetime, which can be greatly improved by using better reflectors at the cavity ends, e.g., by using thin film coating on the crystal facets to increase *R*.