Module 3

Supplemental Materials

Example:

LED

QUESTION

Show that FWHM power spectral width of LED becomes wider at larger wavelength.

SOLUTION

$$E = \frac{hc}{\lambda}$$

$$\Delta E = -\frac{hc\Delta\lambda}{\lambda^2}$$

Therefore: $\Delta\lambda \approx \lambda^2$

For λ= 870 nm, $\Delta\lambda$ = 47 nm

For λ= 1550 nm, $\Delta\lambda$ = 149 nm

For the same energy differences: $$\frac{\Delta\lambda_1}{\Delta\lambda_2} = \frac{\lambda_1^2}{\lambda_2^2}$$

Example:

LED Quantum Efficiency

QUESTION

An LED with driving current at 30 mA, τ_{nr} =100 ns, τ_r =25 ns, is emitting a light with λ = 1300 nm.

- What are the internal and external quantum efficiency?
- What are the internal and output optical power?

SOLUTION

$$\eta_{\text{int}} = \frac{R_r}{R_r + R_{nr}} = \frac{\frac{1}{\tau_r}}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}} = \frac{100}{125}$$

If light emitted to air, *n* is the index of refraction of LED material: *n* = 3.5 for GaAs

$$\eta_{\text{int}} = \frac{P_{\text{int}} / h\nu}{I / q}$$

$$\eta_{ext} = \frac{1}{n(n+1)^2}$$

$$P_{\text{int}} = \frac{100}{125} \times \frac{30\times10^{-3}}{1.6\times10^{-19}} \times \frac{6.625\times10^{-34}\times3\times10^{8}}{1.3\times10^{-6}}$$

$$P_{out} = \eta_{ext} P_{\text{int}}$$

Example:

LED Spectral Linewidth

QUESTION

We know that a spread in the output wavelengths is related to a spread in the emitted photon energies. The emitted photon energy $h\upsilon = hc/\lambda$. Assume that the spread in the photon energies $\Delta(h\upsilon) \approx 3k_BT$ between the half intensity points. Show that the corresponding linewidth $\Delta\lambda$ between the *half intensity points* in the output spectrum is:

$$\Delta\lambda = \lambda_o^2 \frac{3k_BT}{hc}$$

LED spectral linewidth

where λ_o is the peak wavelength

What is the spectral linewidth of an optical communications LED operating at 1310 nm and at 300 K?

SOLUTION

First, consider the relationship between the photon frequency υ and λ:

$$\lambda = \frac{c}{\upsilon} = \frac{hc}{h\upsilon}$$ in which $h\upsilon$ is the photon energy

We can differentiate this: $$\frac{d\lambda}{d(h\upsilon)} = -\frac{hc}{(h\upsilon)^2} = -\frac{\lambda^2}{hc}$$

The negative indicates that increasing the photon energy decreases the wavelength.

SOLUTION (CONT'D)

We are only interested in changes, thus $\Delta\lambda/\Delta(h\upsilon) \approx |d\lambda/d(h\upsilon)|$, and this spread should be around $\lambda = \lambda_o$, so

$$\Delta\lambda = \frac{\lambda_o^2}{hc}\Delta(h\upsilon) = \lambda_o^2\frac{3k_BT}{hc}$$

where we used $\Delta(h\upsilon) = 3k_BT$

We can substitute $\lambda = 1310$ nm, and $T = 300$ K to calculate the linewidth of the 1310 nm LED.

$$\Delta\lambda = \lambda^2\frac{3k_BT}{hc} = (1310\times10^{-9})^2\frac{3(1.38\times10^{-23})(300)}{(6.626\times10^{-34})(3\times10^{8})}$$

$$= 1.07\times10^{-7}\text{ m or }107\text{ nm}$$

SOLUTION (CONT'D)

The spectral linewidth of an LED output is due to the spread in the photon energies, which is fundamentally about $3k_BT$.

The only option for decreasing $\Delta\lambda$ at a given wavelength is to reduce the temperature.

The output spectrum of a laser, on the other hand, has a much narrower linewidth.

Example:

Energy Levels in the Quantum Well

QUESTION

Consider a GaAs *QW* sandwiched between two $Al_{0.40}Ga_{0.60}As$ layers. Suppose that the barrier height ΔE_c is 0.30 eV, the electron effective mass in the well is $0.067m_e$ and the width of the *QW (d)* is 12 nm.

- Calculate the energy levels E_1 and E_2 from the bottom of the well (E_c) assuming an infinite PE well as in Equation (1).

- Compare these with the calculations for a finite PE well that give 0.022 eV, 0.088 and 0.186 for n = 1, 2 and 3.

SOLUTION

We use Eq. (1) with $m_e^* = 0.067m_e$, $d = 12\times10^{-9}$ nm, so that for $n = 1$,

$$\Delta E_n = E_n - E_c = \frac{h^2 n^2}{8m_e^* d^2} = \frac{(6.624\times10^{-34}\ \text{J s})^2(1)^2/(1.602\times10^{-19}\ \text{J eV}^{-1})}{8(0.067\times9.1\times10^{-31}\ \text{kg})(12\times10^{-9}\ \text{m})^2} = 0.039\ \text{eV}$$

We can repeat the above calculation for n = 2 and 3 to find $\Delta E_2 = 0.156$ eV and $\Delta E_3 = 0.351$ eV. The third level will be above the well depth ($\Delta E_c = 0.3$ eV). Clearly, the infinite *QW* predicts higher energy levels, by a factor of 1.8, and puts the third level inside the well not outside. The finite *QW* calculation is not simple, and involves a numerical solution.