Module 2

Supplemental Materials

Example:

A Multimode Fiber

QUESTION

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 μm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

SOLUTION

Substitute, a = 50 μm, λ = 0.850 μm, n_1 = 1.468, n_2 = 1.447 into the expression for the V-number,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi 50/0.850)(1.468^2 - 1.447^2)^{1/2}$$

$$= 91.44.$$

Since $V >> 2.405$, the number of modes is

$$M \approx V^2/2 = (91.44)^2/2 = 4181$$

which is large.

Example:

A Single Mode Fiber

QUESTION

What should be the core radius of a single mode fiber which has a core of n_1 = 1.4680, cladding of n_2 = 1.447 and it is to be used with a source wavelength of 1.3 μm?

SOLUTION

For single mode propagation, $V \leq 2.405$. We need,

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$

or

$[2\pi a/(1.3\ \mu\text{m})](1.468^2 - 1.447^2)^{1/2} \leq 2.405$

which gives $\boldsymbol{a \leq 2.01\ \mu\text{m}}$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; a is comparable to λ which means that the geometric ray picture, strictly, cannot be used to describe light propagation.

Example:

Single Mode Cut-off Wavelength

QUESTION

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 μm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the V-number and the mode field diameter (MFD) for operation at $\lambda = 1.31$ μm?

SOLUTION

For single mode operation,

$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$

Substituting for a, n_1 and n_2 and rearranging we get,

$\lambda > [2\pi(4.1\ \mu m)(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251\ \mu m$

Wavelengths shorter than 1.251 μm give multimode propagation.

At $\lambda = 1.31\ \mu m$,

$V = 2\pi[(4.1\ \mu m)/(1.31\ \mu m)](1.4532^2 - 1.4485^2)^{1/2} = 2.30$

Mode field diameter MFD

SOLUTION (CONT'D)

Mode field diameter MFD from the Marcuse Equation is

$$2w = 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6})$$
$$= 2(4.1)[0.65 + 1.62(2.30)^{-3/2} + 2.88(2.30)^{-6}]$$

$2w$ = 9.30 μm   86% of total power is within this diameter.

$2w = (2a)(2.6/V) = 2(4.1)(2.6/2.30) = 9.28$ μm

$2w = 2a[(V+1)/V] = 11.8$ μm   This is for a planar waveguide, and the definition is different than that for an optical fiber.

Example:

A Multimode Fiber and Total Acceptance Angle

QUESTION

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Normalized refractive index

$\Delta = (n_1 - n_2) / n_1 =$ **0.0135 or 1.35%**

SOLUTION

The numerical aperture is

$NA = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} =$ **0.2425 or 24.3%**

From, $\sin\alpha_{max} = NA/n_o = 0.2425/1$

Acceptance angle $\alpha_{max} =$ **14°**

Total acceptance angle $2\alpha_{max} =$ **28°**

V-number in terms of the numerical aperture can be written as,

$V = (2\pi a/\lambda)NA = [(2\pi 50\ \mu m)/(0.85\ \mu m)](0.2425) = 89.62$

The number of modes, $M \approx V^2/2 =$ **4016**

Example:

A Single Mode Fiber and Cut-off Wavelength

QUESTION

A typical single mode optical fiber has a core of diameter 8 μm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 μm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency λ_c of the fiber?

SOLUTION

The numerical aperture

$NA = (n_1{}^2 - n_2{}^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$NA \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} =$ **0.113 or 11.3 %**

The acceptance angle is given by

$\sin\alpha_{max} = NA/n_o = 0.113/1$ or $\alpha_{max} =$ **6.5°**, and $2\alpha_{max} =$ **13°**

SOLUTION (CONT'D)

The condition for single mode propagation is $V \leq 2.405$. At cut-off,

$V = (2\pi a/\lambda_c)(n_1^2 - n_2^2)^{1/2} = 2.405$

$\therefore \lambda_c = [2\pi a\text{NA}]/2.405 = [(2\pi)(4\ \mu\text{m})(0.113)]/2.405 = \mathbf{1.18\ \mu m}$

Wavelengths shorter than 1.18 μm will result in multimode operation.

Dispersion = Spread of Information

DISPERSION = SPREAD OF INFORMATION

- Intermode (Intermodal) Dispersion: Multimode fibers only
- Material Dispersion
  - Group velocity depends on N_g and hence on λ
- Waveguide Dispersion
  - Group velocity depends on waveguide structure
- Chromatic Dispersion
  - Material dispersion + Waveguide Dispersion
- Polarization Dispersion
- Profile Dispersion
  - Like material and waveguide dispersion. Add all 3: Material + Waveguide + Profile
- Self phase modulation dispersion

Example:

Combining Intermodal and Intramodal Dispersions

QUESTION

Consider a graded index fiber with a core diameter of 30 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient D_m at 1300 nm is –7.5 ps nm^{-1} km^{-1}.

SOLUTION

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 =$ $(1.474 - 1.453)/1.474 = 0.01425$. Modal dispersion for 1 km is

$\sigma_{intermode} = Ln_1\Delta^2/[(20)(3^{1/2})c] = 2.9\times10^{-11}$ s ^{1} or 0.029 ns.

The material dispersion is

$\Delta\tau_{1/2} = LD_m\,\Delta\lambda_{1/2} = (1\ \text{km})(7.5\ \text{ps nm}^{-1}\ \text{km}^{-1})(3\ \text{nm}) = 0.0225\ \text{ns}$

Assuming a Gaussian output light pulse shape,

$\sigma_{intramode} = 0.425\Delta\tau_{1/2} = (0.425)(0.0225\ \text{ns}) = 0.0096\ \text{ns}$

SOLUTION (CONT'D)

Total dispersion is

$$\sigma_{\text{rms}} = \sqrt{\sigma_{\text{intermode}}^2 + \sigma_{\text{intramode}}^2} = \sqrt{0.029^2 + 0.0096^2} = 0.030 \text{ ns}$$

Assume $L = 1$ km

$B = 0.25/\Delta\tau_{\text{rms}}$ = **8.2 Gb**

Example:

Power Attenuation

QUESTION

A 12-km long optical fiber has a loss of 1.5 dB/km. What is the minimum optical power level that must be launched into the fiber to maintain an optical power level of 0.3 μW at the receiving end? How about if the loss is 2.5 dB/km.

SOLUTION

$$dB = -10\log_{10}\frac{P_{out}}{P_{in}}$$

$$P_{in} = P_{out}10^{\alpha L/10} = (0.3\mu W)10^{(1.5)(12)/10} = 18.9\mu W$$

$$P_{in} = P_{out}10^{\alpha L/10} = (0.3\mu W)10^{(2.5)(12)/10} = 300\mu W$$