Module 1

Supplemental Materials

Example:

Reflection at Normal Incidence (Internal and External Reflection)

QUESTION

Consider the reflection of light at normal incidence on a boundary between a glass medium of refractive index 1.5 and air of refractive index 1.

A. If light is traveling from air to glass, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

B. If light is traveling from glass to air, what is the reflection coefficient and the intensity of the reflected light with respect to that of the incident light?

SOLUTION: FOR QUESTION (A)

The light travels in air and becomes partially reflected at the surface of the glass, which corresponds to external reflection. Thus $n_1 = 1$ and $n_2 = 1.5$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.5}{1+1.5} = -0.2$$

This is negative, which means that there is a 180° phase shift.

The reflectance (R), which gives the fractional reflected power, is:

$$R = r_{//}^2 = 0.04 \text{ or } 4\%$$

SOLUTION: FOR QUESTION (B)

The light travels in glass and becomes partially reflected at the glass-air interface, which corresponds to internal reflection. Thus $n_1 = 1.5$ and $n_2 = 1$. Then,

$$r_{//} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1.5 - 1}{1.5 + 1} = 0.2$$

There is no phase shift. The reflectance is again 0.04 or 4%.

In both cases (A) and (B), the amount of reflected light is the same.

Example:

Reflection and Transmission

QUESTION

A light beam traveling in air is incident on a glass plate of refractive index 1.50.

- What is the Brewster or polarization angle?
- What are the relative intensities of the reflected and transmitted light for the polarization perpendicular and parallel to the plane of incidence at the Brestwer angle of incidence?

SOLUTION:

Light is traveling in air, and is incident on the glass surface at the polarization angle θ_p. Here n_1 = 1 and n_2 = 1.5 and $\tan\theta_p = (n_2/n_1) = 1.5$, so that $\theta_p = 56.31°$. We now have to use Fresnel's equations to find the reflected and transmitted amplitudes. For the perpendicular polarization:

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - [n^2 - \sin^2\theta_i]^{1/2}}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$r_\perp = \frac{\cos(56.31^o) - [1.5^2 - \sin^2(56.31^o)]^{1/2}}{\cos(56.31^o) + [1.5^2 - \sin^2(56.31^o)]^{1/2}} = -0.385$$

SOLUTION: (CONT'D)

On the other hand, $r_{//} = 0$. The reflectances $R_\perp = |r_\perp|^2 = 0.148$ and $R_{//} = |r_{//}|^2 = 0$, so that $R = 0.074$ and has no parallel polarization in the plane of incidence. Notice the negative sign in $r_\perp$, which indicates a phase change of π.

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_{//} = \frac{E_{t0,//}}{E_{i0,//}} = \frac{2n\cos\theta_i}{n^2\cos\theta_i + [n^2 - \sin^2\theta_i]^{1/2}}$$

$$t_\perp = \frac{2\cos(56.31^\circ)}{\cos(56.31^\circ) + [1.5^2 - \sin^2(56.31^\circ)]^{1/2}} = 0.615$$

$$t_{//} = \frac{2(1.5)\cos(56.31^\circ)}{(1.5)^2\cos(56.31^\circ) + [1.5^2 - \sin^2(56.31^\circ)]^{1/2}} = 0.667$$

Notice that $r_{//} + nt_{//} = 1$ and $r_\perp + 1 = t_\perp$, as we expect.

SOLUTION: (CONT'D)

To find the transmittance for each polarization, we need the refraction angle θ_t. From Snell's law, $n_1 \sin\theta_i = n_t \sin\theta_t$.

Example: (1)sin(56.31°) = (1.5)sinθ_t, we find **θ_t = 33.69°**.

$$T_{//} = \frac{n_2 \left|E_{to,//}\right|^2}{n_1 \left|E_{io,//}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{//}\right|^2 \qquad T_{\perp} = \frac{n_2 \left|E_{to,\perp}\right|^2}{n_1 \left|E_{io,\perp}\right|^2} = \left(\frac{n_2}{n_1}\right)\left|t_{\perp}\right|^2$$

$$T_{//} = \left[\frac{(1.5)\cos(33.69^o)}{(1)\cos(56.31^o)}\right](0.667)^2 = 1 \qquad T_{\perp} = \left[\frac{(1.5)\cos(33.69^o)}{(1)\cos(56.31^o)}\right](0.615)^2 = 0.852$$

Clearly, light with polarization parallel to the plane of incidence has greater intensity.

SOLUTION: (CONT'D)

If we were to reflect light from a glass plate, keeping the angle of incidence at 56.3°, then the reflected light will be polarized with an electric field component perpendicular to the plane of incidence. The transmitted light will have the field greater in the plane of incidence, that is, it will be partially polarized.

By using a stack of glass plates, one can increase the polarization of the transmitted light. (This type of *pile-of-plates polarizer* was invented by Dominique F.J. Arago in 1812.)

Example:

Reflection of Light from a Less Dense Medium (Internal Reflection)

QUESTION

A ray of light, which is traveling in a glass medium of refractive index n_1 = 1.460, becomes incident on a less dense glass medium of refractive index n_2 = 1.440. The free space wavelength (λ) of the light ray is 1300 nm.

A. What should be the minimum incidence angle for *TIR*?

B. What is the phase change in the reflected wave, when $\theta_i = 87°$ and $\theta_i = 90°$?

C. What is the penetration depth of the evanescent wave into medium 2, when $\theta_i = 87°$ and $\theta_i = 90°$?

SOLUTION: FOR QUESTION (A)

The critical angle θ_c for *TIR* is given by $\sin\theta_c = n_2/n_1 = 1.440/1.460$, so that

$\theta_c = 80.51°$.

SOLUTION: FOR QUESTION (B)

Since the incidence angle $\theta_i > \theta_c$, there is a phase shift in the reflected wave.

The phase change in $E_{r,\perp}$ is given by $\phi_\perp$. Using $n_1 = 1.460$, $n_2 = 1.440$ and $\theta_i = 87°$:

$$\tan\left(\tfrac{1}{2}\varphi_\perp\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{\cos\theta_i} = \frac{\left[\sin^2(87^\circ) - \left(\dfrac{1.440}{1.460}\right)^2\right]^{1/2}}{\cos(87^\circ)}$$

$$= 2.989 = \tan\left[\frac{1}{2}\left(143.0^\circ\right)\right]$$

so that the phase change **$\phi_\perp = 143°$.**

For the $E_{r,//}$ component, the phase change is:

$$\tan\left(\tfrac{1}{2}\phi_{//} + \tfrac{1}{2}\pi\right) = \frac{\left[\sin^2\theta_i - n^2\right]^{1/2}}{n^2\cos\theta_i} = \frac{1}{n^2}\tan\left(\tfrac{1}{2}\phi_\perp\right)$$

SOLUTION: FOR QUESTION (B) (CONT'D)

So that: $\tan(\frac{1}{2}\phi_{//} + \frac{1}{2}\pi) = (n_1/n_2)^2\tan(\phi_{\perp}/2) = (1.460/1.440)^2\tan(\frac{1}{2}143°)$

which gives $\boldsymbol{\phi_{//} = 143.95° - 180°}$ or $\boldsymbol{-36.05°}$

Repeat with $\theta_i = 90°$ to find $\boldsymbol{\phi_{\perp} = 180°}$ and $\boldsymbol{\phi_{//} = 0°}$

Note: As long as $\theta_i > \theta_c$, the magnitude of the reflection coefficients are unity. Only the phase changes.

SOLUTION: FOR QUESTION (C)

The amplitude of the evanescent wave as it penetrates into medium 2 is $E_{t,\perp}(y,t) \propto E_{to,\perp}\exp(-\alpha_2 y)$.

The field strength drops to e^{-1} when $y = 1/\alpha_2 = \delta$, which is called the ***penetration depth***.

The attenuation constant α_2 is: $$\alpha_2 = \frac{2\pi n_2}{\lambda}\left[\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

SOLUTION: FOR QUESTION (C) (CONT'D)

Example: $$\alpha_2 = \frac{2\pi(1.440)}{(1300\times10^{-9}\ \text{m})}\left[\left(\frac{1.460}{1.440}\right)^2 \sin^2(87^\circ) - 1\right]^{1/2}$$ **= 1.10×10⁶ m⁻¹**

The penetration depth is

$$\delta = 1/\alpha_2 = 1/(1.104\times10^6\ \text{m}) = 9.06\times10^{-7}\ \text{m, or } \mathbf{0.906\ \mu m}$$

For 90°, repeating the calculation:

$\alpha_2 = 1.164\times10^6\ \text{m}^{-1}$, so that $\delta = 1/\alpha_2 =$ **$0.859\ \mu m$**

The penetration is greater for smaller incidence angles.

Group Velocity and Group Index

GROUP VELOCITY

The *group velocity* defines the speed with which energy or information is propagated.

$$v_g = \frac{d\omega}{dk}$$

$\omega = 2\pi c/\lambda_o$ and $k = 2\pi n/\lambda_o$, λ_o is the free space wavelength.

Differentiate the above equations in red $d\omega = -(2\pi c/\lambda_o^2)d\lambda_o$

$$dk = 2\pi n(-1/\lambda_o^2)d\lambda_o + (2\pi/\lambda_o)\left(\frac{dn}{d\lambda_o}\right)d\lambda_o$$

$$dk = -(2\pi/\lambda_o^2)\left(n - \lambda_o \frac{dn}{d\lambda_o}\right)d\lambda_o$$

$$\therefore v_g = \frac{d\omega}{dk} = \frac{-(2\pi c/\lambda_o^2)d\lambda_o}{-(2\pi/\lambda_o^2)\left(n - \lambda_o \frac{dn}{d\lambda_o}\right)d\lambda_o} = \frac{c}{n - \lambda_o \frac{dn}{d\lambda_o}}$$

GROUP VELOCITY (CONT'D)

Where $n = n(\lambda)$ is a function of the wavelength. The group velocity v_g in a medium is given by:

$$v_g(\text{medium}) = \frac{d\omega}{dk} = \frac{c}{n - \lambda \frac{dn}{d\lambda}}$$

This can be written as:

$$v_g(\text{medium}) = \frac{c}{N_g}$$

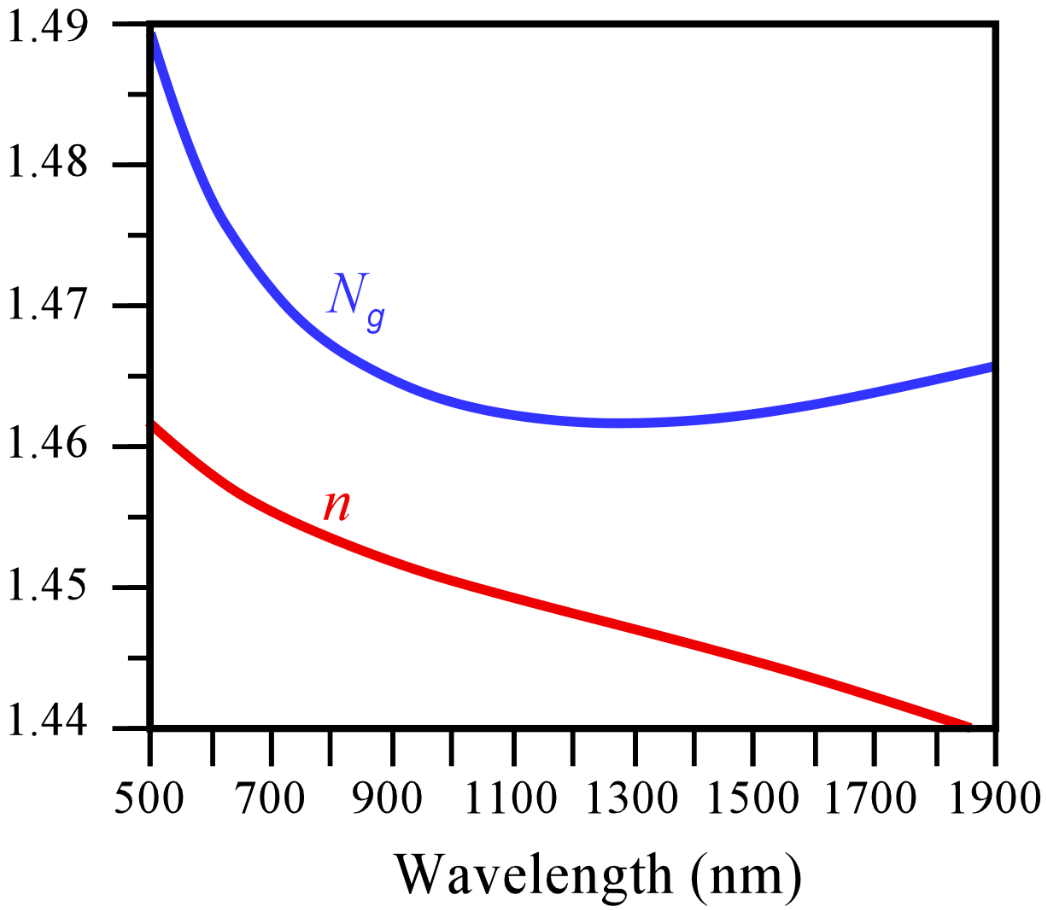
GROUP INDEX

$$N_g = n - \lambda \frac{dn}{d\lambda}$$ is defined as the *group index of the medium*.

In general, for many materials, the refractive index *n* and hence the group index N_g depend on the wavelength of light. Such materials are called *dispersive*.

GROUP INDEX (CONT'D)

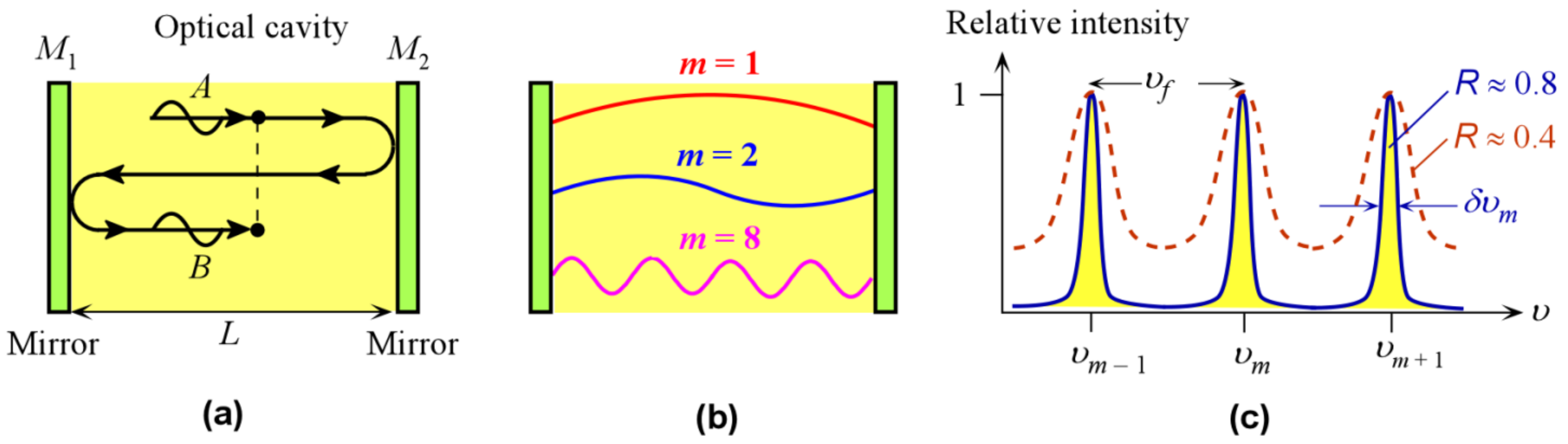
Refractive Index and Group Index



Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

GROUP INDEX (CONT'D)

Optical Resonator and Fabry-Perot Optical Cavity



Schematic illustration of the Fabry-Perot optical cavity and its properties.

a) Reflected waves interfere.

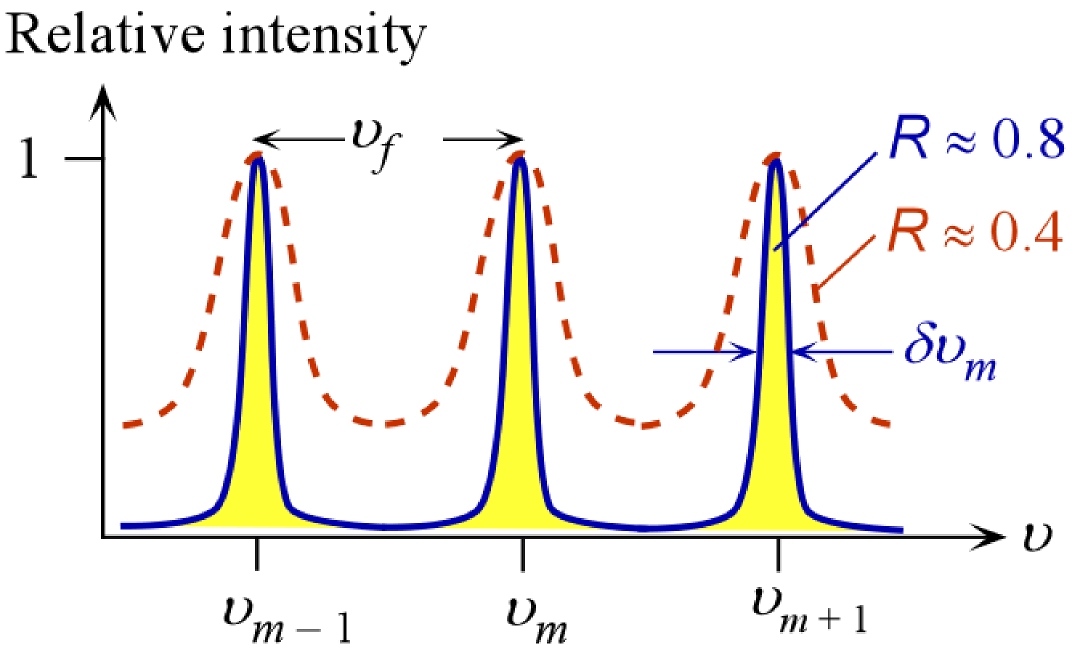
b) Only standing EM waves, modes, of certain wavelengths are allowed in the cavity.

c) Intensity vs. frequency for various modes. *R* is mirror reflectance, and lower *R* means higher loss from the cavity.

Note: The two curves are sketched, so that the maximum intensity is unity.

GROUP INDEX (CONT'D)

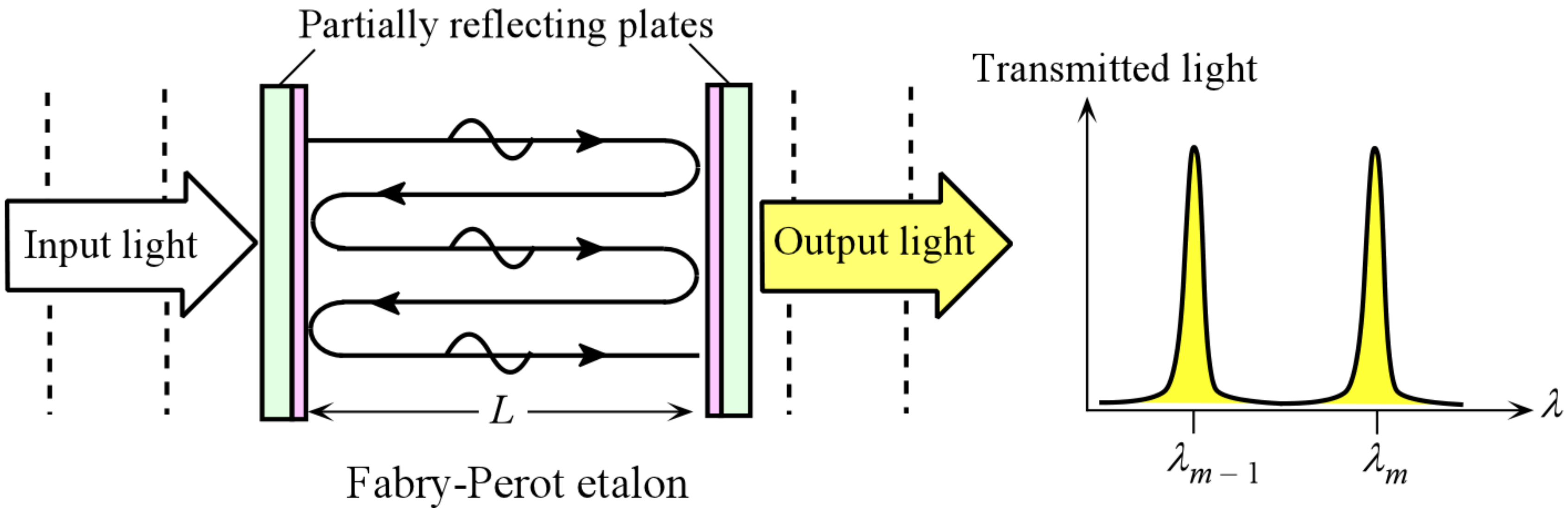
Quality factor Q is similar to the Finesse F.



$$Q = \frac{\text{Resonant frequency}}{\text{Spectral width}} = \frac{\upsilon_m}{\delta\upsilon_m} = mF$$

GROUP INDEX (CONT'D)

- *Optical resonator is also an optical filter.*
- *Only certain wavelengths (cavity modes) are transmitted.*



$$I_{\text{transmitted}} = I_{\text{incident}} \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)}$$

Example:

An Optical Resonator in Air

QUESTION

Consider a Fabry-Perot optical cavity in air of length 100 microns with mirrors that have a reflectance of 0.90.

- Calculate the cavity mode nearest to the wavelength 900 nm, and corresponding wavelength.
- Calculate the separation of the modes, the finesse, the spectral width of each mode and the *Q*-factor.

SOLUTION:

Find the mode number *m* corresponding to 900 nm, and then take the integer:

$$m = \frac{2L}{\lambda} = \frac{2(100\times10^{-6})}{(900\times10^{-9})} = 222.2 \qquad \lambda_m = \frac{2L}{m} = \frac{2(100\times10^{-6})}{(222)} = 900.9\ \text{nm}$$

Thus, $m = 222$ (must be an integer), $\lambda_m = 900.90$ nm $\approx$ 900 nm (very close).

The frequency corresponding to λ_m is:

$$\upsilon_m = c/\lambda_m = (3\times10^8)/(900.9\times10^{-9}) = 3.33\times10^{14}\ \text{Hz}$$

SOLUTION: (CONT'D)

υ_f = $c/2L$ = Separation of modes = $(3 \times 10^8)/[2(100 \times 10^{-6})]$ = 1.5×10^{12} Hz.

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.90^{1/2}}{1-0.90} = 29.8 \qquad \delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.5\times 10^{12}}{29.8} = 50.3\,\text{GHz}$$

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times 10^8)}{(3.33\times 10^{14})^2}(5.03\times 10^{10}) = 0.136\,\text{nm}$$

The Q-factor is:

$$Q = mF = (222)(29.8) = 6.6 \times 10^3$$

Example:

Semiconductor Optical Cavity

QUESTION

Consider a Fabry-Perot optical cavity of a semiconductor material of length 250 microns with mirrors, each with a reflectance of 0.90.

- Calculate the cavity mode nearest to 1310 nm.
- Calculate the separation of the modes, finesse, the spectral width of each mode, and the *Q*-factor.

Take $n = 3.6$ for the semiconductor medium.

SOLUTION:

Given: L = 250×10^{-6} m, n = 3.6, R = 0.90

$$\Delta\upsilon_m = \upsilon_f = c/2nL = \text{Separation of modes} = 1.67\times10^{11}\ \text{Hz}$$

$$F = \frac{\pi R^{1/2}}{1-R} = \frac{\pi 0.9^{1/2}}{1-0.9} = 29.8$$

$$\delta\upsilon_m = \frac{\upsilon_f}{F} = \frac{1.67\times10^{11}}{29.8} = 5.59\,\text{GHz}$$

Mode number m corresponding to 1310 nm is:

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(250\times10^{-6})}{(1310\times10^{-9})} = 1374.05$$ which must be an integer (1374).

SOLUTION: (CONT'D)

The actual mode wavelength is:

$$\lambda_m = \frac{2nL}{m} = \frac{2(3.6)(250\times10^{-6})}{(1374)} = 1310.04 \text{ nm}$$

For all practical purposes, the mode wavelength is 1310 nm.

Mode frequency is: $$\upsilon_m = \frac{c}{\lambda_m} = \frac{(3\times10^{8})}{(1310\times10^{-9})} = 2.3\times10^{14} \text{ Hz}$$

SOLUTION: (CONT'D)

Spectral width of a mode in wavelength is:

$$\delta\lambda_m = \left|\delta\left(\frac{c}{\upsilon_m}\right)\right| = \left|-\frac{c}{\upsilon_m^2}\right|\delta\upsilon_m = \frac{(3\times10^8)}{(3.33\times10^{14})^2}(5.03\times10^{10}) = 0.136\ \text{nm}$$

The *Q*-factor is:

$$Q = mF = (1374)(29.8) = 4.1 \times 10^4$$