FACULTY OF ENGINEERING AND COMPUTER SCIENCE
FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS
ENGR 213 - FALL 2007

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Material Allowed: Faculty Approved Calculators

DO ALL THE PROBLEMS`

Problem No. 1. (10 MARKS) Solve the following equations using separation of variables:

(*a*) $\frac{dy}{dx} + 2xy^2 = 0$

(*b*) $\frac{dy}{dx} = e^{3x+2y}$

Problem No. 2. (10 MARKS) NASA has decided to send an experimental probe to planet Venus. Its weight on earth is 400 Newtons (*Nt*). When the probe is near the planet it will be attracted by the its gravitational field. You are asked to provide the parachute's diameter so that the probe will touch the planet's surface with a velocity of 3 m/s. The equation that describes the free-fall of the equipment, including the air resistance is:

$$m\frac{dV}{dt} = -m\, g_{Venus} + 0.7\, \rho_{Venusian\ atmosphere}\, A_{chute}\, V^2$$

where g_{Venus} = 8.93 m/s^2, $A_{chute} = \pi D^2/4$ and the density of Venusian atmosphere is 65 kg/m^3. Assume that terminal velocity, V_t, (as $t \rightarrow \propto$ then $V \rightarrow V_t$ = constant) has been reached when the equipment touches the surface of the planet.

Problem No. 3. (15 MARKS) Solve the following equations using the exact differentials method:

(*a*) $\{5y - 2x\}\, dy - 2y\, dx = 0$

(*b*) $\{2x - 1\}\, dx + \{3y + 7\}\, dy = 0$

(*c*) $\{x^3 + 3x^2 y\}\, dx + \{x^3 + y^3\}\, dy = 0$

Problem No. 4. (10 MARKS) Solve the following Bernoulli equation

$$t^2 \frac{dy}{dt} + y^2 = t\, y$$

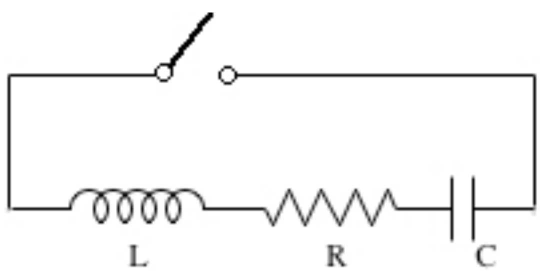
Problem No. 5. (10 MARKS) Solve the following differential equations using the integrating factor method:

$$(a) \qquad \frac{dy}{dx} - \frac{y}{x} = 1$$

$$(b) \qquad \frac{dy}{dx} - 2\,y = e^{x}$$

Problem No. 6. (20 MARKS) The equation describing an *RLC* (resistance-inductance-capacitance) system is:

$$L \frac{d^2 q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = 0$$



Where:

C the capacitance (in $F = s\, A / V$, in fundamental units: $s^4\, A^2\, m^{-2}\, Kg^{-1}$)
i is the current (in A)
L the inductance (in $H = V\, s\, A^{-1}$, in fundamental units: $m^2\, kg\, s^{-2}\, A^{-2}$)
q the electrical charge in the capacitor ($Q = A\, s$)
R the resistance (in $\Omega = V / A$, in fundamental units: $m^2\, kg\, s^{-3}\, A^{-2}$)
t the time (in s)

Let $L = 1\, m^2\, kg\, s^{-2}\, A^{-2}$, $R = 1\, m^2\, kg\, s^{-3}\, A^{-2}$, and $1/C = 1\, s^{-4}\, A^{-2}\, m^2\, Kg$

(a) (5 MARKS) Based on the auxiliary equation characterize the system (underdamped/critically damped/overdamped).

(b) (15 MARKS). Originally (electrical switch is open) the capacitor is charged with q_{in} = 0.2 $A\, s$ and then the electrical switch is closed.

The initial conditions are:

i. $t = 0,\ q = q_{in} = 0.2\ A\ s$

ii. $t = 0,\ i_{in} = dq/dt = 0$

Develop an equation that describes q as a function of time and describe briefly the behavior of the current ($i\ (t) = dq(t)/dt$) as a function of time.

Problem No. 7. (5 MARKS) Find the solution of the differential equations:

(a) $y'' + 4y = 8x^2$

(b) $y'' - 3y' + 2y = e^x$

(c) $y'' + 4y = e^x \cos x$

Problem No. 8. (5 MARKS) Solve the following differential equation by variation of parameters:

$$y'' + 2y' + y = x e^{-x}$$

Problem No. 9. (10 MARKS). Solve the following system of differential equations by either the matrix method or by systematic elimination:

$$\frac{dx}{dt} + 4y = 4e^{-t}$$

$$\frac{dy}{dt} + x = e^{-t}$$

Problem No. 10. (5 MARKS). Solve the following differential equation using the power series method about the ordinary point x= 0:

$$y'' + xy = 0$$

Provide only the first four terms of the solution.

USEFULL FORMULA

$$\int \frac{dx}{ax^2 - b} = \frac{1}{2\sqrt{ab}} \ln \frac{\sqrt{a}\,x - \sqrt{b}}{\sqrt{a}\,x + \sqrt{b}}$$