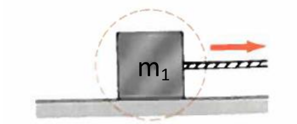
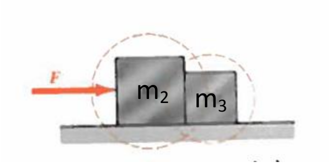
ENGR 290 – Homework #1

Draw free-body diagrams for the listed objects:

2.

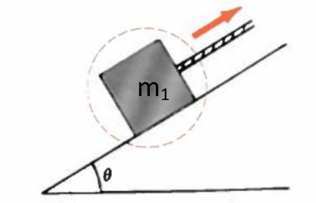


a) m_1 assuming a smooth surface (i.e. neglect friction)
b) m_1 assuming a rough surface (i.e. with friction)

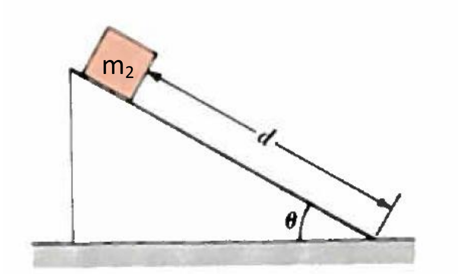


c) m_2 assuming a smooth surface
d) m_3 assuming a smooth surface

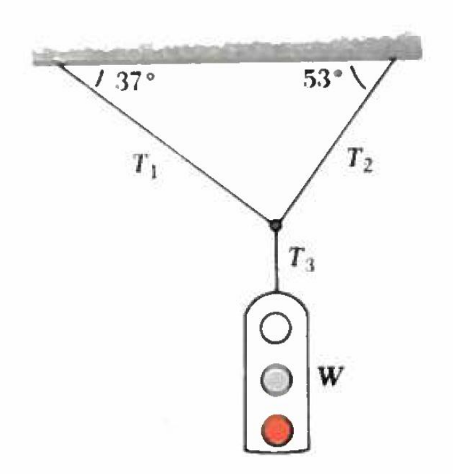
3.



a) m_1 assuming a rough incline



b) m_2 assuming a smooth incline

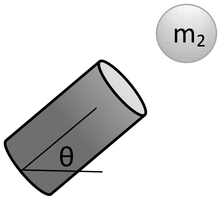


c) the traffic light

d) the knot at the juncture of cables T_1, T_2, and T_3. Express the forces acting on the knot in terms of W (i.e. cW where c is a numerical coefficient).

4.

a) the metal ball m_1, assuming it was dropped from the tower of Pisa (don't forget about air resistance, though)



b) the metal ball m_2, assuming it has just been shot out of a cannon

5. Show how $v(t) = v_0 + a_0 t$ and $x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

can be combined to yield:

$$x(t) = x_0 + \frac{(v(t)^2 - v_0^2)}{2a_0}$$

6. Your 1.5 kg hovercraft is drifting toward a barrier that you don't want to crash into. It is 1 m away and you are moving toward it at 2 m/s. You have a thruster on the craft that you can point in the opposite direction and fire with a constant force of 2.5 N.

If you turn the lift fan off, the coefficient of friction between the hovercraft and the floor is 0.05.

- Would the thruster be enough to stop you before hitting the barrier?
- Would friction alone be enough to stop you before hitting the barrier?
- What should you do to avoid crashing?

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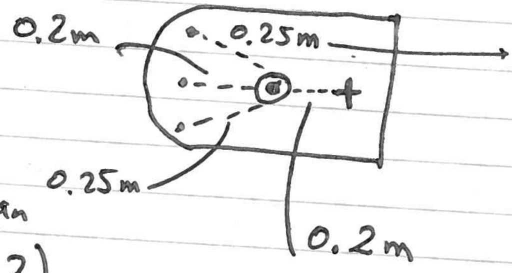
Sol. to Set #3

Problem 1

a)



$$\left(\sqrt{(.2)^2+(.15)^2} = 0.25\right.$$

$$I = I_{\text{lift fan}} + \sum m r^2$$

$$= \underbrace{(0.0005)}_{\text{lift fan}} + \underbrace{(0.2)^2 (0.2)}_{\text{thrust fan}}$$

$$+ \underbrace{(0.2)^2 (0.01)}_{\text{Servo}} + \underbrace{(0.2)^2 (0.15)}_{\text{controller}} + \underbrace{2(0.25)^2 (0.03)}_{\text{batteries}} = 0.01865 \ \text{kg}\cdot\text{m}^2$$

b) Total mass $m = 0.2 + 0.2 + 0.15 + 2(0.03) + 0.01 = 0.62 \ \text{kg}$

Linear Acceleration: $\sum F = ma$

$$\sum F_y = F_{\text{thrust}} = m a_y \Rightarrow a_y = \frac{1}{0.62} = 1.61 \ \text{m/s}^2$$

$$\sum F_x = 0 \Rightarrow a_x = 0$$

$\sum T = I \alpha$ where $\sum T = F_{\text{thrust}} * d = 1(0.2) = 0.2 \ \text{Nm}$

$$\therefore \alpha = \frac{0.2}{0.01865} = 10.7 \ \text{rad/s}^2$$

c) Slightly more angular rotation than 10.7 rad/s^2 due to conservation of angular momentum. There will be a slight higher and additional counter clockwise rotation of the hovercraft to balance the clockwise rotation of the fan blades.

Problem 2

a) $\mathcal{F}_b = |F| \left(\cos\varphi \ \hat{x}_b + \sin\varphi \ \hat{y}_b \right)$

b) $\mathcal{F}_i = |F| \left(\cos(\varphi - \theta) \ \hat{x}_i + \sin(\varphi - \theta) \ \hat{y}_i \right)$

Problem 3

a) Mass concentrated at the center of hovercraft yields:

1. Easier to rotate the craft about its center of mass
2. Lift fan would generate a torque which will rotate the hovercraft

Mass concentrated at the edges of hovercraft yields:

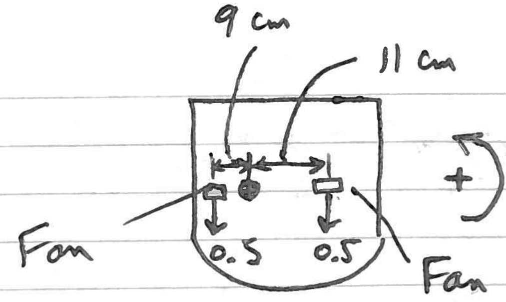
1. Harder to rotate the craft about its COM
2. Lift fan makes the hovercraft less stable

b) When both fans are ON there will be NO angular acceleration; however when only ONE fan is ON there will be the maximum angular acceleration that is given by

$$\alpha_{max} = \frac{\Gamma_{max}}{I} = \frac{(1)(0.5\,N)(0.1\,m)\sin(\pi/2)}{0.1\ kg.m^2} = 0.5\ rad/s^2$$

c)

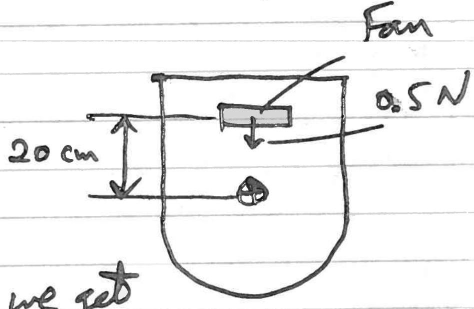
$$\sum \Gamma_{net} = (0.5N)(0.09m) - (0.5N)(.11m) = -0.01\ N.m$$



or equivalently $\sum \Gamma_{net} = 0.01\ N.m$ in CW direction

$$\therefore \quad \alpha = \frac{0.01\ N.m}{0.1\ kg.m^2} = 0.1\ rad/s^2$$

d) There will be no angular acceleration in this case!



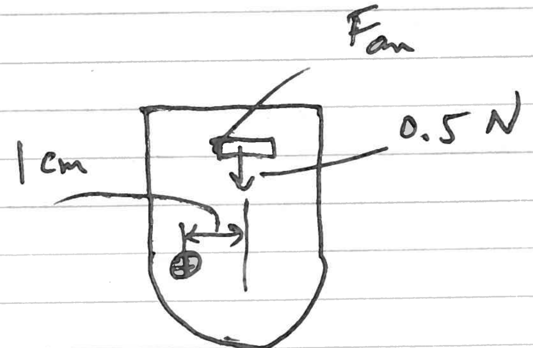
Now if we rotate the fan by 90° we get

$$T = (0.5N)(0.2\ m)\ \sin \pi/2 = 0.1\ N.m.\ rad.$$

$$\therefore \quad \alpha = \frac{0.1\ N.m.rad}{0.1\ kg.m^2} = 1\ rad/s^2$$

e)

$$\Gamma = (0.5N)(0.01m)\ \sin \pi/2 = 0.005\ N.m.\ rad$$



$$\therefore \quad \alpha = \frac{0.005\ N.m.rad}{0.1\ kg.m^2} = 0.05\ rad/s^2$$

f) - The further the fan is from the COM ⇒ easier to rotate

- Deviation of COM from center make the craft more unstable
- To counter balance torque generated by the lift fan one needs to determine the optimal location of the thrust fan w.r.t. COM.

Problem 4.

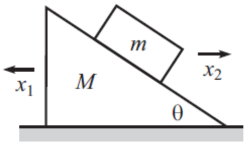


Figure 6.37

Let x_1 be the horizontal coordinate of the plane (with positive x_1 to the left), and let x_2 be the horizontal coordinate of the block (with positive x_2 to the right); see Fig. 6.37. The relative horizontal distance between the plane and the block is $x_1 + x_2$, so the height fallen by the block is $(x_1 + x_2)\tan\theta$. The Lagrangian is therefore

$$L = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_2^2 + (\dot{x}_1 + \dot{x}_2)^2\tan^2\theta\right) + mg(x_1 + x_2)\tan\theta. \tag{6.99}$$

The equations of motion obtained from varying x_1 and x_2 are

$$\begin{aligned} M\ddot{x}_1 + m(\ddot{x}_1 + \ddot{x}_2)\tan^2\theta &= mg\tan\theta, \\ m\ddot{x}_2 + m(\ddot{x}_1 + \ddot{x}_2)\tan^2\theta &= mg\tan\theta. \end{aligned} \tag{6.100}$$

Note that the difference of these two equations immediately yields conservation of momentum, $M\ddot{x}_1 - m\ddot{x}_2 = 0 \Longrightarrow (d/dt)(M\dot{x}_1 - m\dot{x}_2) = 0$. Eqs. (6.100) are two linear equations in the two unknowns, $\ddot{x}_1$ and $\ddot{x}_2$, so we can solve for $\ddot{x}_1$. After a little simplification, we arrive at

$$\ddot{x}_1 = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}. \tag{6.101}$$

Problem 5.

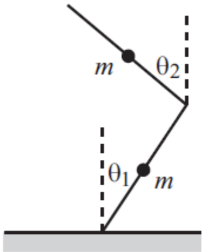


Figure 6.38

Let $\theta_1(t)$ and $\theta_2(t)$ be defined as in Fig. 6.38. Then the position of the bottom mass in Cartesian coordinates is $(r\sin\theta_1, r\cos\theta_1)$, and the position of the top mass is $(2r\sin\theta_1 - r\sin\theta_2, 2r\cos\theta_1 + r\cos\theta_2)$. So the potential energy of the system is

$$V(\theta_1,\theta_2) = mgr(3\cos\theta_1 + \cos\theta_2). \tag{6.102}$$

The kinetic energy is somewhat more complicated. The kinetic energy of the bottom mass is simply $mr^2\dot{\theta}_1^2/2$. Taking the derivative of the top mass's position given above, we find that the kinetic energy of the top mass is

$$\frac{1}{2}mr^2\Big((2\cos\theta_1\dot{\theta}_1 - \cos\theta_2\dot{\theta}_2)^2 + (-2\sin\theta_1\dot{\theta}_1 - \sin\theta_2\dot{\theta}_2)^2\Big). \tag{6.103}$$

We can simplify this, using the small-angle approximations. The terms involving $\sin\theta$ are fourth order in the small θ's, so we can neglect them. Also, we can approximate $\cos\theta$ by 1, because this entails dropping only terms of at least fourth order. So the top mass's kinetic energy becomes $(1/2)mr^2(2\dot{\theta}_1 - \dot{\theta}_2)^2$. In retrospect, it would have been easier to obtain the kinetic energies of the masses by first applying the small-angle approximations to the positions, and then taking the derivatives to obtain the velocities. This strategy shows that both masses move essentially horizontally (initially). You will probably want to use this strategy when solving Exercise 6.28.

Using the small-angle approximation $\cos\theta \approx 1 - \theta^2/2$ to rewrite the potential energy in eq. (6.102), we have

$$L \approx \frac{1}{2}mr^2\left(5\dot{\theta}_1^2 - 4\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) - mgr\left(4 - \frac{3}{2}\theta_1^2 - \frac{1}{2}\theta_2^2\right). \tag{6.104}$$

The equations of motion obtained from varying θ_1 and θ_2 are, respectively,

$$\begin{aligned} 5\ddot{\theta}_1 - 2\ddot{\theta}_2 &= \frac{3g}{r}\theta_1 \\ -2\ddot{\theta}_1 + \ddot{\theta}_2 &= \frac{g}{r}\theta_2. \end{aligned} \tag{6.105}$$

At the instant the sticks are released, we have $\theta_1 = 0$ and $\theta_2 = \epsilon$. Solving eq. (6.105) for $\ddot{\theta}_1$ and $\ddot{\theta}_2$ gives

$$\ddot{\theta}_1 = \frac{2g\epsilon}{r}, \qquad \text{and} \qquad \ddot{\theta}_2 = \frac{5g\epsilon}{r}. \tag{6.106}$$