

PART A (18 marks)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN PART A MUST BE INDICATED ON THE SCANTRON SHEET. YOU SHOULD ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

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1. Which of the following equations is/are linear in the unknowns x , y and z ?

(i) $\frac{x}{3} + \frac{3}{y} + \frac{z}{3} = 6$ (ii) $x(1 + y + z) = 3$ (iii) $\sqrt{5}x + 5^2y + \frac{z}{5} = 11$

A: (i) and (ii) only	B: (i) and (iii) only	C: (ii) and (iii) only
D: (iii) only	E: none of (i), (ii) or (iii)	

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2. Find the augmented matrix corresponding to the following system of linear equations:

$$\begin{aligned} x &= y + 1 \\ y &= -z \end{aligned}$$

A: $\left[\begin{array}{ccc c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right]$	B: $\left[\begin{array}{ccc c} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right]$	C: $\left[\begin{array}{ccc c} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right]$
D: $\left[\begin{array}{ccc c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array} \right]$	E: $\left[\begin{array}{ccc c} 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{array} \right]$	

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3. Which of the following matrices is/are in row-reduced echelon form?

(i) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A: (i), (ii) and (iii) only	B: (iv), (v) and (vi) only	C: (i) and (vi) only
D: (v) and (vi) only	E: None of A, B, C or D.	

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4. Find the solution to the system of linear equations whose augmented matrix is $\left[\begin{array}{cc|c} 2 & 2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$.

A: (2, 1, 0)	B: (1, 1, 0)	C: (0, 1)	D: (1, 0)	E: There is no solution.
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5. Find all solutions to the system of linear equations whose augmented matrix is $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array} \right]$.

A: No solutions.	B: (5, 0) only	C: (5 + 3t, 0, t)	D: (5 - 3t, 0, t)	E: (5 - 3t, t, 0)
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Use the following information for questions 6 and 7.

Consider the system of linear equations:

$$\begin{aligned}x - 3y &= 1 \\ kx + 6y &= 1\end{aligned}$$

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6. For what value of k does the system have no solutions?

A: 1	B: -1	C: 2	D: -2	E: no value of k
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7. For what value of k does the system have infinitely many solutions?

A: 1	B: -1	C: 2	D: -2	E: no value of k
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8. Find all values of c for which the solution to the system of linear equations corresponding to the augmented matrix below has exactly one parameter.

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & c^2 - 1 & c + 1 \end{array} \right]$$

A: $c = 0$ only	B: $c = 1$ only	C: $c = -1$ only	D: all $c \neq \pm 1$	E: all values of c
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9. The equations shown below represent planes in \mathbb{R}^3 . Describe the region, if any, that is the intersection of these planes.

$$\begin{aligned}x - y + 2z &= 1 \\ -x + y - 2z &= -1 \\ 2x - y + 5z &= 3\end{aligned}$$

A: Exactly one point.	B: Exactly three points.	C: A line.
D: The planes do not intersect.	E: None of A, B, C or D.	

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10. You are given that A is a 2×3 matrix, B is a 3×2 matrix and C is a 3×3 matrix. Which one of the following operations is defined?

A: $A + C + B$	B: $BA + C^2$	C: CBA^T	D: AB^T	E: $AB - C$
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11. If $3 \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} - 5 \begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find the value of b .

A: -4	B: -3	C: -2	D: -1	E: None of A, B, C or D.
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12. If $A = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 1 \\ 1 & -3 \end{bmatrix}$, find the (1, 2)-entry of A .

A: 8	B: -4	C: -2	D: -10	E: 5
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13. Find the value of c_{32} if $C = [c_{ij}] = AB^T$ where A and B are the matrices shown below.

$$A = \begin{bmatrix} -1 & 1 & 5 \\ 4 & -2 & 1 \\ 2 & 1 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 & -3 \\ 1 & -2 & 5 \\ -1 & 3 & 1 \end{bmatrix}$$

A: -9	B: -15	C: 5	D: 3	E: None of A, B, C or D.
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14. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, find $A^3 + 3A$.

A: $4A$	B: $5A$	C: $6A$	D: $7A$	E: $8A$
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15. For what value of k does the matrix $\begin{bmatrix} 2 & 5 \\ 3 & k \end{bmatrix}$ have no inverse?

A: 0	B: $-\frac{2}{15}$	C: $\frac{2}{15}$	D: $-\frac{15}{2}$	E: $\frac{15}{2}$
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16. Let A and B be any $n \times n$ invertible matrices. Which of the following statements is/are **always true**?

- (i) $A + B = B + A$ (ii) $A - B = B - A$ (iii) $AB = BA$
 (iv) $(AB)^T = B^T A^T$ (v) $ABA^{-1} = B$

A: (i) only	B: (i) and (iv) only	C: (i), (iii) and (v) only
D: (i), (iii) and (iv) only	E: None of A, B, C or D.	

17. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, what is the first row of $C = AA^{-1}$?

A: $[1 \ 0]$	B: $[1 \ 1]$	C: $[d \ -b]$
D: $[a^2 + b^2 \ ac + bd]$	E: Cannot be determined.	

18. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 3 & 5 \\ 2 & 7 \end{bmatrix}$, find the value of x in the solution to the system of linear equations shown below.

$$\begin{aligned} ax + by &= -2 \\ cx + dy &= 3 \end{aligned}$$

A: -2	B: 1	C: 5	D: 8	E: 9
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PART B (7 marks)

SHOW YOUR WORK FOR ALL QUESTIONS IN PART B

- 4 marks* 19. In each of the following, find the row-reduced echelon form of the given augmented matrix and then state all solutions to the system of linear equations represented by the given matrix.

(a)
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ -1 & 1 & 2 & 3 \\ 2 & 0 & 1 & -1 \end{array} \right]$$

(b)
$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & -1 & -1 & -1 & -1 \end{array} \right]$$

3 marks 20. Let $A = \begin{bmatrix} 1 & -1 & -3 \\ 2 & -1 & -6 \\ -1 & 1 & 4 \end{bmatrix}$.

Use the method of row reduction to find A^{-1} , the inverse of A .