5. RC Circuits

Purpose

In this experiment, you will explore the behavior of resistor-capacitor (RC) and resistor-inductor-capacitor (RLC) circuits. You will experimentally determine the time constant (τ) of an RC circuit and study the oscillation and resonance effects of an RLC circuit.

Apparatus

The following instruments will be used in this experiment:

- Function Generator (BK Precision 5 MHz)
- Differential Voltage Probe (6V)
- Resistor
- Capacitors

Theory

In studying RC and RLC circuits, you will strengthen your understanding of resistors and capacitors and learn about a new electronics component, the inductor. In an RC circuit, the capacitor stores charge in order to resist changes in voltage, then undergoes exponential decay with a distinctive time constant. In an RLC circuit, the inductor acts to maintain a constant current, which can result in a resonance effect when combined with the charge storage effects of a capacitor.

RC Circuits

The circuit shown in Figure 1 provides a simple setup for studying an RC circuit. When the power supply is turned on for a sufficiently long period of time, the capacitor will reach an equilibrium maximum voltage. This voltage is achieved by storing an electric charge on the internal plates of the capacitor.

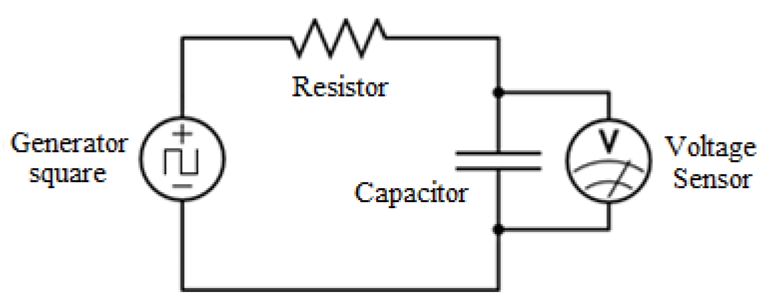

Figure 1. Diagram of an RC circuit.

We want to explore what happens to that charge when the power supply is suddenly cut off. A *square wave* function generator approximates this well enough. The capacitor will alternate between charging and discharging cycles in each period of the square wave as shown in Figure 2. If the frequency is sufficiently low, the capacitor will reach maximum charge before the square wave oscillates to the trough of the cycle. However, if the frequency is too high and the period is too short, the oscillation will occur before the capacitor will have time to charge or discharge completely.

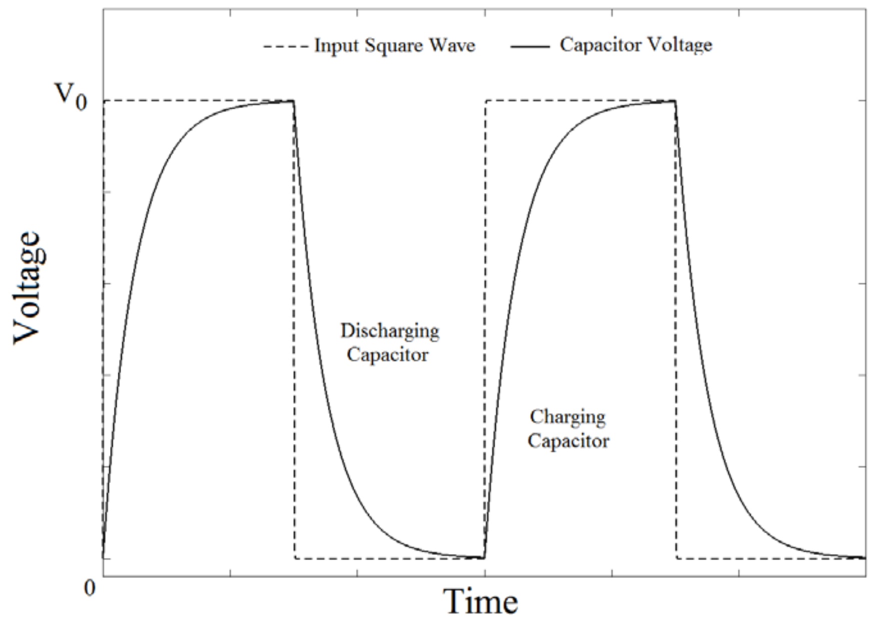

Figure 2. Charging/discharging cycles of a capacitor in an RC circuit.

Charging the capacitor

When the function generator supplies a voltage V_0 to the RC circuit, a current I will flow through the resistor and the capacitor. This causes the capacitor to charge until its voltage (V_C) reaches that of the generator. We wish to find how I and V_C vary with time.

Let the capacitor be uncharged when the voltage is initially applied at $t = 0$. The voltage (V_C) on a capacitor of capacitance C is directly related to how much charge (Q) is stored on its plates. On the other hand, the voltage (V_R) across a resistor of resistance R is related to the current flowing through it. We have that V_C and V_R are given by,

$$V_C(t) = \frac{Q(t)}{C} \qquad ; \qquad V_R(t) = R\,I(t) \tag{5.1}$$

where $Q(t)$ and $I(t)$ are the charge on the capacitor and current through the resistor at time t, respectively.

Using Kirchhoff's voltage law for the circuit, the relation between the power supply voltage $V_S(t)$ and the voltage on the components is given by,

$$V_S(t) = V_C(t) + V_R(t) \qquad \rightarrow \qquad V_S(t) = \frac{Q(t)}{C} + R\,I(t) \tag{5.2}$$

As we know the behaviour of the voltage source, the current in the capacitor is given by the change in the amount of charge stored in the component. Moreover, since the voltage from the power supply, $V_S(t)$, is switched on to a constant V_0, the time derivative of the power supply voltage is equal to zero. This is,

$$I(t) = \frac{dQ}{dt} \qquad ; \qquad \frac{dV_S}{dt} = 0 \tag{5.3}$$

If we differentiate each component in Eq. 5.2, and combine it with Eq. 5.3, we get that

$$\begin{aligned} \cancelto{0}{\frac{dV_S}{dt}} &= \frac{1}{C}\frac{dQ}{dt} + R\frac{dI}{dt} \\ 0 &= \frac{1}{C}I + R\frac{dI}{dt} \end{aligned} \tag{5.4}$$

This forms a first order differential equation that we can rearrange to look like

$$\frac{dI}{I} = -\frac{dt}{RC} \tag{5.5}$$

We solve this by integrating both sides by their respective differential element (dI and dt):

$$\int_{I_o}^{I} \frac{dI}{I} = -\int_0^t \frac{dt}{RC} \quad \rightarrow \quad \ln\left(\frac{I}{I_0}\right) = -\frac{t}{RC} \tag{5.6}$$

where I_0 is the initial current at $t = 0$. We can isolate the current, I, by taking the exponential of each side to get

$$I(t) = I_0 e^{-\frac{t}{RC}} \tag{5.7}$$

While the voltage from the power supply rises rapidly from 0 V to V_0, the current flowing through the circuit decays exponentially from its initial value of $I_0 = V_0/R$. This happens until the capacitor becomes fully charged to a value of V_0, and the voltage difference across the resistor therefore drops to 0 V (remember: $I_R = V_R/R$, so if $V_R = 0$, then the current is also 0!). Comparing back to Eq. 4.2, solving for the voltage on the capacitor we see that

$$V_C(t) = V_0 - R\left(I_0 e^{-\frac{t}{RC}}\right)$$

Or equivalently, (5.8)

$$V_C(t) = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

Discharging the capacitor

After the capacitor is fully charged and the power supply goes back to neutral (0 V), the capacitor will discharge through the resistor. Integrating the Kirchhoff's voltage law equation gives the same result for discharging current as for charging current, but with the current flowing in the opposite direction. We have that

$$I = -I_o e^{-\frac{t}{RC}} \tag{5.9}$$

This time, since the voltage of the power supply is 0 V, the voltage across the capacitor will be

$$V_C = V_0 e^{-\frac{t}{RC}} \tag{5.10}$$

RC Time Constant

In each of the previous equations, the time varying exponential decay is proportional to the *characteristic time constant* (τ), the amount of time needed for the current/voltage to change by a factor of $1/e = 1/2.71828 \ldots \approx 1/3$. In this case, we can see that

$$\tau = RC \tag{5.11}$$

which has units of time (Ohms (Ω) $\times$ Farads (F) = Seconds (s)).

Note: the above description assumes the use of an ideal capacitor, when fully charged, which does not allow any current to flow in the circuit once it has reached steady state (i.e. $t \gg \tau$). The capacitors used in this experiment usually have a small *leakage current*. That means that even when the capacitor is fully charged, there will still be a small current flowing in the circuit. This is usually small enough to ignore, however for some capacitors (especially in aging ones) it will have to be taken into account.

Complete cycle

Once the function generator goes back to V_0, the current will recharge the capacitor. If the period of the signal from the function generator is sufficiently long, the voltage on the capacitor should look like Figure 3a. However, if the frequency is too high and the period is too short, the oscillation will occur before the capacitor will have time to charge or discharge completely and the waveform would appear as shown in Figure 3b.

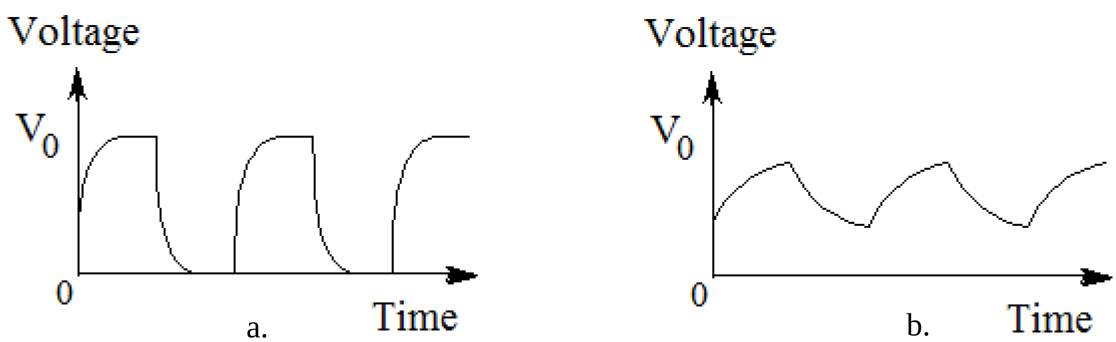

Figure 3. Waveform of the charge/discharge cycle of a capacitor when the period of the function generator is smaller (**a**) or higher (**b**) than the RC time constant.

Equivalent Capacitance

In this lab, you will be combining various capacitors in series and parallel configurations. For reference, capacitors combine oppositely to resistors, and so the equivalent capacitance is given by

$$C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} \tag{5.12}$$

$$C_{parallel} = C_1 + C_2 \tag{5.13}$$

Procedure RC Circuits

RC Time Constant

1. Download the *"RC Time Constant"* Logger Pro file from CuLearn.
2. Build the RC circuit as shown in Fig. 7.
   - ✓ *Use C_1 as your capacitor.*
   - ✓ *Set the function generator to a* ***60 Hz square wave.***
   - ✓ *Zero the Vernier voltage sensor and measure the OUTPUT voltage of the generator. Set it at ~4V peak-to-peak.*
   - ✓ *Connect the voltage sensor across the capacitor.*

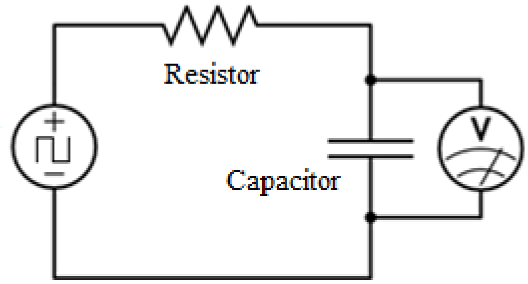

Figure 7. Diagram of an RC circuit.

Verify your circuit with a demonstrator before proceeding further!

3. Record the nominal values of your resistor and capacitor C_1 in your write-up and calculate the expected RC time constant, τ, of the circuit.
   - ✓ *Use the colour code on the resistor to determine its nominal value.*
   - ✓ $\boldsymbol{C_1 = (0.98 \pm 0.02)\ \mu F}$.
4. Measure the RC time constant for the charge (τ_{charge}) and discharge ($\tau_{discharge}$) cycles.
   - ✓ *Press the* Collect *button in Logger Pro to begin data collection.*
   - ✓ *Highlight the range of data corresponding to a single charge/discharge cycle.*
   - ✓ *Go to* Analyze > Curve Fit *and select the appropriate exponential decay curve (eq. 5.8 and 5.10).*
   - ✓ *Check the* Time Offset *option, and then click* Try Fit *and* OK.
   - ✓ *Identify which parameter corresponds to the RC time constant and record the value in your write-up.*
5. Compare the measured values of τ_{charge} vs. $\tau_{discharge}$ using the statistical t-test.
6. Add a descriptive title to your graph and print it. Make sure that both analysis panels are shown and not covering your data. Print the graph in landscape form.

t-test
Consider two quantities with their respective errors ($x_1 \pm \sigma_{x_1}$ and $x_2 \pm \sigma_{x_2}$). $$t = \frac{\lvert x_1 - x_2 \rvert}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$ ▪ If $t \leq 2$, x_1 and x_2 are **consistent** with each other.   ▪ If $t > 2$, x_1 and x_2 are **inconsistent** with each other.

RC Time Constant for Capacitors in Parallel

In this part of the experiment, you will determine the capacitance of an unknown capacitor (C_2) by measuring the time constant of an RC circuit with two capacitors connected in parallel.

1. Build the RC circuit shown in Figure 8. by connecting capacitors C_1 and C_2 in parallel.

   Verify your circuit with a demonstrator before proceeding further!

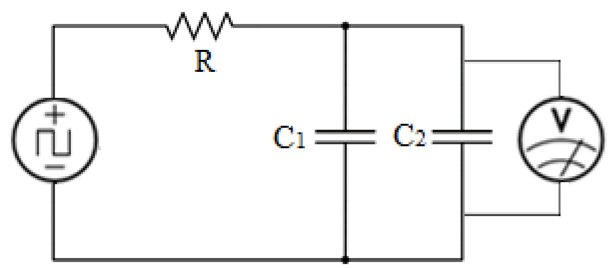

Figure 8. RC Circuit with parallel capacitors.

2. Measure the time constant ($\tau_p \pm \sigma_{\tau_p}$) of the circuit using the same method as in step 4 of the previous section.
   - ✓ *You can choose whether to use a charging or discharging cycle for this measurement.*
   - ✓ *If necessary, increase the displayed precision of your values by double clicking on the fit panel and selecting a higher precision.*
3. Calculate the unknown capacitance, $C_2 \pm \sigma_{C_2}$, from the measured time constant, $\tau_p \pm \sigma_{\tau_p}$.
   - ✓ *Use the definition of τ (eq. 5.11) and the general relationship for combining capacitors in parallel (eq. 5.13).*
   - ✓ *Use error propagation to find the corresponding uncertainty, σ_{C_2}.*
4. A printout of this graph is not required.

Error Propagation
Consider two quantities with their respective errors ($x \pm \sigma_x$ and $y \pm \sigma_y$). The error on a function $f(x, y, ...)$ is given by, $$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \cdots}$$ where $\frac{\partial f}{\partial x}$ denotes the partial derivative of the function f with respect to x.

Questions: Many answers are possible; the explanation/justification you provide is more important than the actual answer.

1. Do you think it is possible to have a circuit where the RC time constant is different when charging than when discharging?
2. How would you increase the RC time constant?
3. Provide an application of RC circuits.
4. Does adding capacitors in series generally increases or decrease the RC time constant, or does it depend on the value of the capacitor added?
5. How many charges were stored in one of your capacitor? (pick any part of the experiment and chose your favorite capacitor, provide enough information in your answer).