

Tutorial Session 10 – Fall 2017

CAPM & Market Efficiency (Ch. 9 & 10)

Agenda

- A. Quick Formula Review of the following topics:
 - 1) CAPM
 - 2) Market Efficiency

- B. Problem Solving related to Risk and Return Measures

Quick review of formulas:

$$\text{Expected Return} = ER = \sum_{i=1}^n (r_i \text{Prob}_i)$$

$$\sigma_{\text{ExPost}} = \sqrt{\frac{\sum_{i=1}^n (r_i - \bar{r})^2}{n - 1}}$$

$$\sigma_{\text{ExAnte}} = \sqrt{\sum_{i=1}^n (\text{Prob}_i) (r_i - ER)^2}$$

Quick review of formulas:

$$COV_{AB} = \sum_{i=1}^n Prob_i (r_{A,i} - \bar{r}_A)(r_{B,i} - \bar{r}_B)$$

$$COV_{AB} = \rho_{A,B} \sigma_A \sigma_B$$

$$ER_p = \sum_{i=1}^n w_i ER_i$$

$$\sigma_{P,AB} = \sqrt{w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B COV_{AB}}$$

Portfolio Return and Risk

- If we invest a portion w of our wealth in the risky portfolio A and the remainder $(1 - w)$ in the **risk-free asset**

- The expected return on our portfolio will be:

$$ER_P = RF + w(ER_A - RF)$$

Risk-free return Expected return on risky portfolio

Weight placed in risky portfolio

- The standard deviation of our portfolio will be:

$$\sigma_P = \sqrt{w^2 \sigma_A^2} = w \sigma_A$$

Weight invested in the risky portfolio Std dev. of risky portfolio returns

CAPM Related Formulas

$$k_i = RF + \beta_i (ER_M - RF)$$

Risk-free rate of return

Expected return on market

Required return on asset/portfolio i

Beta of asset or portfolio i

Market Risk Premium

$$\beta_i = \frac{COV_{i,M}}{\sigma_M^2} = \frac{\rho_{i,M} \sigma_i}{\sigma_M}$$

$$\beta_P = w_A \beta_A + w_B \beta_B + \dots + w_n \beta_n$$

Problem

Consider the following information about two stocks:

State of Economy	Probability of state	Stock A return	Stock B return
Boom	0.20	0.24	0.36
Normal	0.55	0.17	0.13
Bust	0.25	0.00	-0.28

- A) If the portfolio is invested 40 percent in A and 60 percent in B, what is the portfolio expected return? The variance? The standard deviation?
- B) If the expected T-bill rate is 3.80 percent, what is the expected risk premium on the portfolio?
- C) If beta of stock A is 0.87 and beta of stock B is 1.2, what is the portfolio beta?

Solution

A) Before calculating expected return of portfolio, we need to calculate expected return of each stock separately:

$$\text{Expected return}(A) = \sum_{i=1}^3 \text{Prob}_i \times r_{iA} = (0.20 * 0.24) + (0.55 * 0.17) + (0.25 * 0.00) = 0.1415 = 14.15\%$$

$$\text{Expected return}(B) = \sum_{i=1}^3 \text{Prob}_i \times r_{iB} = (0.20 * 0.36) + (0.55 * 0.13) + (0.25 * -0.28) = 0.0735 = 7.35\%$$

Weights are Given: $w_A = 0.40$ $w_B = 0.60$

$$ER_{\text{portfolio}} = w_A \cdot ER_A + w_B \cdot ER_B$$

$$ER_{\text{portfolio}} = (0.40 \times 0.1415) + (0.60 \times 0.0735) = 0.1007 = 10.07\%$$

Solution(continue)

To calculate variance of portfolio we need weights (which we have), variance of each stock and the covariance(A,B).

Lets first calculate **variance of each single stock** (variance is squared of standard deviation, so the formula is standard deviation formula without the square root), and also the **covariance between A and B**:

$$\text{Variance: } \sigma^2_{\text{ExAnte}} = \sqrt{\sum_{i=1}^n (\text{Prob } i) (r_i - ER)^2}$$

$$\sigma_A^2 = 0.20 * (0.24 - 0.1415)^2 + 0.55 * (0.17 - 0.1415)^2 + 0.25 * (0.00 - 0.1415)^2 = 0.007392 = 0.7392\%$$

$$\sigma_B^2 = 0.20 * (0.36 - 0.0735)^2 + 0.55 * (0.13 - 0.0735)^2 + 0.25 * (-0.28 - 0.0735)^2 = 0.04941 = 4.941\%$$

$$\text{Covariance(A,B): } Cov_{A,B} = \sum_{i=1}^n prob_i * (r_{A,i} - ER_A)(r_{B,i} - ER_B)$$

$$Cov_{A,B}: 0.20 * (0.24 - 0.1415)(0.36 - 0.0735) + 0.55 * (0.17 - 0.1415)(0.13 - 0.0735) + 0.25 * (0 - 0.1415)(-0.28 - 0.0735)$$

$$= 0.0190 = 1.90\%$$

Solution

Now we have everything to calculate the variance of the portfolio!

The formula for variance of portfolio is:

$$\sigma_{portfolio}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B Cov_{A,B} = 0.0280 = 2.80\%$$

Standard deviation of portfolio is square root of variance of portfolio, so:

$$\sigma_{portfolio} = \sqrt{0.0280} = 0.1673 = 16.73\%$$

Solution(continue)

B) Expected risk **premium** is expected return minus risk free rate:

So, expected risk premium on this portfolio = $ER_{portfolio} - RF = 10.07\% - 3.80\% = 6.27\%$

We got the 10.07% in part a. Also, T-bill rate = risk free rate.

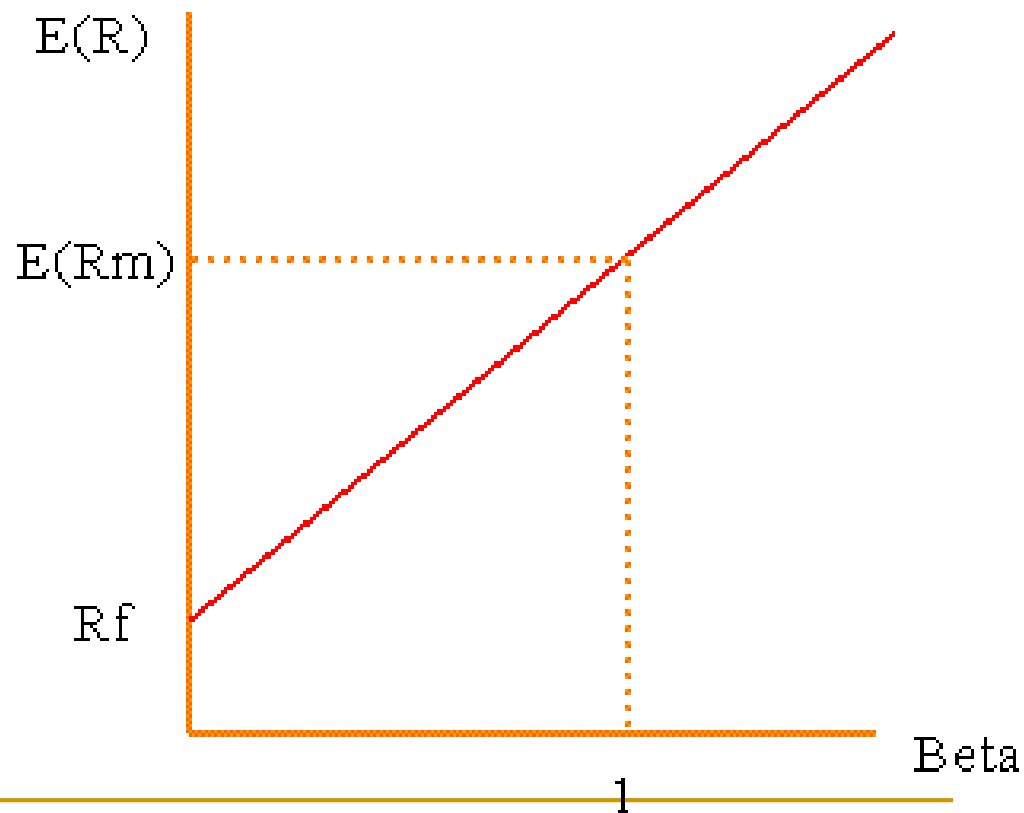
C) The formula for beta of portfolio is:

$$\beta_{portfolio} = w_A \cdot \beta_A + w_B \cdot \beta_B$$

$$\beta_{portfolio} = 0.40 * 0.87 + 0.60 * 1.2 = 1.068$$

SML: Overpriced Vs. Underpriced

- Any return you compute using CAPM will fall on the SML.
- For a given beta, Any actual return **above** the SML means the stock is **undervalued**. And **below** is **overvalued**



Fall 2013, Q8

Security A currently trades on the Toronto stock exchange. In your opinion security A has an expected rate of return of 16.5%. It has a beta of 1.5. The yield on three month treasury bill is currently 3% and the rate of return on S&P TSX index is 11%. According to the Capital Asset Pricing Model, this security is _____

- A. underpriced.
- B. overpriced.
- C. fairly priced.
- D. cannot be determined from data provided.
- E. none of the above.

Answer: A

- $E(R) = (0.03) + 1.5 * 0.08 = 15\%$ (what CAPM says) $< 16.5\%$ (actual)
- So, it is above SML, so it is underpriced

Final exam 2009, Question 11

You are considering two stocks: ABC and XYZ. ABC has a beta of 1.25 while XYZ has a beta of .65. The risk free rate is 3% and the expected return on the market is 10%. You have analyzed the two companies and believe that ABC will earn a return of 10% next year while XYZ will earn a return of 7%. Based on your analysis, which of the stocks are overvalued?

- a) ABC
- b) XYZ
- c) Both ABC and XYZ are overvalued
- d) Neither ABC nor XYZ are overvalued

Answer: C $ER(ABC) = 3\% + 1.25 \cdot (10\% - 3\%) = 11.75\% > 10\%(\text{actual})$
 $ER(XYZ) = 3\% + 0.65 \cdot (10\% - 3\%) = 7.75\% > 7\%(\text{actual})$

Problem

Assume the risk-free rate is 3.5%. The expected return on the market is 12% and has a standard deviation of 23.5%. Determine the required rate of return necessary for investors to hold an efficient portfolio with a standard deviation of 27%.

Answer: $\beta_p = \frac{\rho_{p,m} \sigma_p}{\sigma_m}$

$$k_P = RF + \left[\frac{ER_M - RF}{\sigma_M} \right] \sigma_P$$

Efficient portfolio: $\rho_{p,m} = 1$

$$= 3.5 + \left[\frac{12 - 3.5}{23.5} \right] (27) = 13.27\%$$

Winter 2014

FarNorth Airlines is expected to pay a dividend of \$8 in the coming year. Dividends are expected to decline at the rate of 2% per year. The risk-free rate of return is 6% and the expected return on the market portfolio is 14%. The stock of Far North Airlines Company has a beta of -0.25. The price of the stock is _____.

- A. \$80.00
- B. 133.33
- C. \$200.00
- D. \$400.00
- E. none of the above

Answer: B

$$E(K) = 0.06 + (-0.25) * (0.14 - 0.06) \\ = \underline{\underline{4\%}}$$

$$P_0 = \frac{8}{0.04 + 0.02} = \underline{\underline{\$133.33}}$$

Winter 2014

Suppose the economy can be in one of the following two states: (i) Boom or “good” state and (ii) Recession or “bad” state. Each state can occur with an equal opportunity (50%). The annual return on the market and a certain security X in the two states of the economy are as follows:

- *Market*: at the end of the year, the market is expected to yield a return of 30% in the good state and a return of (-10%) in the bad state;
- *Security X*: at the end of the year, the security is expected to yield a return of 40% in the good state and a return of (-35%) in the bad state;

Furthermore, assume that annual risk-free rate of return is 6% in all states of the market.

Assuming CAPM is the correct model:

- i. (7 points) Calculate the beta of security X relative to the market.
- ii. (2 points) Calculate the alpha of security X.

i) beta of security X relative to the market:

(i) β of security X is:

$$\text{Market: } E(r_m) = 0.5 \times 0.3 + 0.5 \times (-0.1) = 0.1 = 10\%$$

$$\sigma_{r_m}^2 = 0.5 \times (0.3 - 0.1)^2 + 0.5 \times (-0.1 - 0.1)^2 = 0.04$$

$$\text{Security X: } E(r_x) = 0.5 \times 0.4 + 0.5 \times (-0.35) = 0.025 = 2.5\%$$

$$\begin{aligned} \text{Cov}(r_m, r_x) &= 0.5 \times (0.3 - 0.1)(0.4 - 0.025) + 0.5 \times (-0.1 - 0.1)(-0.35 - 0.025) \\ &= 0.0750 \end{aligned}$$

$$\beta_x = \frac{\text{Cov}(r_m, r_x)}{\sigma_m^2} = \frac{0.0750}{0.04} = \underline{\underline{1.875}}$$

ii) alpha of security X :

(ii) Security α is :

$$\begin{aligned}\alpha &= E(r_x) - \left[r_f + \beta_x * (r_m - r_f) \right] \\ &= 0.025 - \left[0.06 + 1.875 * (0.1 - 0.06) \right] \\ &= -11\%\end{aligned}$$

Winter 2015-Q2

Q2. (5 Points):

It is given that assets A and B are priced in the market by the CAPM, with the following data:

Stock	Expected Return	Beta
A	25%	1.5
B	-10%	-1

*Expected returns are based on CAPM

Assets C and D are priced in the market by the dividend growth model, with the following data:

Stock	Expected Return	Beta
C	28.6%	2
D	16.8%	0.75

**Expected returns are based on the dividend growth model

Assume that the correct asset-pricing model is the CAPM. Based on the above data, show that Asset C and D are Overpriced/Underpriced/correctly priced.

Winter 2015-Q2-Sol.

$$0.25 = r_f + 1.5 \times (r_m - r_f)$$

$$\frac{-0.1}{+} = \frac{-r_f}{+} + \frac{1.5}{+} (r_m - r_f)$$

$$0.35 = 2.5 (r_m - r_f) \Rightarrow (r_m - r_f) = \underline{\underline{0.14}} \rightarrow \underline{\underline{1 \text{ pt}}}$$

$$\therefore \underline{\underline{r_f}} = -0.1 + 0.14 = \underline{\underline{4\%}} \rightarrow \underline{\underline{1 \text{ pt}}}$$

$$\text{CAPM} \Rightarrow r_c = 0.04 + 2 \times 0.14 = 0.32 = 32\% > 23.6\% \rightarrow \underline{\underline{1 \text{ pt}}}$$

\therefore Asset C is overpriced.

$$1.5 \times 2 \rightarrow \underline{\underline{1 \text{ pt}}}$$
$$r_d = 0.04 + 0.75 \times 0.14 = 14.5\% < 16.8\% \rightarrow \underline{\underline{1 \text{ pt}}}$$

\Rightarrow Asset D is underpriced.

Blue Version

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for decisions

Fall 2013, Question 13

Which of the following is true regarding the concept of beta?

- A. To benefit from an upcoming bull market (rising market), you need to invest in high beta stocks.
- B. Treasury bills have a beta of 1.
- C. Total risk equals beta risk plus systematic risk.
- D. You can form a zero-beta portfolio by investing in as many stocks as you can afford.
- E. All assets with beta of 1 may be used as a market proxy.

Answer: A

Fall 2013, Question 15

Which of the following statements is most correct?

- A. Risk refers to the chance that some unfavorable event will occur.
- B. Portfolio diversification reduces the variability of returns on an individual stock.
- C. When company-specific risk has been diversified, the inherent risk that remains is market risk, which is constant for all securities in the market.
- D. A stock with a beta of -1.0 has zero market risk.
- E. None of the above.

Answer: E

Fall 2013, Question 22

Which of the following statements is true?

- A. All assets with beta equal to zero are risk free.
- B. Any asset with beta equal to one may be used as a market proxy.
- C. All risk free assets have beta equal to zero.
- D. Statements A and C are true.
- E. Statements A, B, and C are true.

Answer: C

Summer II 2013, Question 19

If a company's shares have a β of 1.0 this means that:

- A. The volatility of the share is greater than or equal to the market volatility.
- B. The volatility of the share is equal to the market volatility.
- C. The volatility of the share is less than or equal to the market volatility.
- D. The systematic risk of the share will be zero.
- E. None of the above statements are correct.

Answer: A

Stock Volatility = Market Volatility of Stock + Idiosyncratic Volatility

Summer II 2013, Question 20

Stock A has a beta of 0.8, while stock B has a beta of 1.6. Which of the following statements is most correct?

- A. Stock B's required return is double that of stock A's.
- B. An equally weighted portfolio of stock A and stock B will have a beta less than 1.2.
- C. A will outperform B when the market is rising.
- D. B will always outperform A.
- E. None of the above statements are correct.

Answer: E

Summer II 2013, Question 27

Suppose you hold a diversified portfolio consisting of a \$10,000 investment in each of 12 different common stocks. The portfolio's beta is 1.25. Now suppose you decided to sell one of your stocks that has a beta of 1.00 and to use the proceeds to buy a replacement stock with a beta of 1.34. What would the portfolio's new beta be? Assume equally weighted portfolio. (Assume equally weighted)

- A) 1.15
- B) 1.21
- C) 1.28
- D) 1.34
- E) None of the above

Answer: C

$$1.25 = \frac{\sum_{i=1}^{12} \beta_i}{12} \rightarrow \sum_{i=1}^{12} \beta_i = 15$$

$$\sum_{i=1}^{12} \beta_{new,i} = 15 - 1 + 1.34 = 15.34 \rightarrow \beta_{portfolio,new} = \frac{15.34}{12} = 1.278$$

Summer2-2015-Q5

You want your portfolio beta to be 1.20. Currently, your portfolio consists of \$100 invested in stock A with a beta of 1.4 and \$300 in stock B with a beta of 0.6. You have another \$400 to invest and want to divide it between stock C with a beta of 1.6 and a risk-free asset. How much should you invest in stock C?

\$0

\$140

\$200

\$320

\$400

$$1.20 = \frac{100}{800} (1.40) + \frac{300}{800} (0.60) + w_c (1.60) + w_d * (0)$$

$$\underline{w_c = 0.50}$$

$$\rightarrow w_d = 1 - (0.50 + (300/800) + (100/800)) = 0$$

$$\text{Amount of money in C} = (0.50) * 800 = 400$$

Summer II 2013, Problem 2

XYZ Inc.'s stock has a beta of 1.40, and its required return is 12.00%. ABC Inc.'s stock has a beta of 0.80. If the risk-free rate is 4.75%, what is the required rate of return on ABC stock?

$$E(r_{XYZ}) = 0.12 = r_f + 1.4 * (E(r_m) - r_f)$$

$$r_f = 4.75\% \quad \text{--- 1 Point}$$

$$\begin{aligned} \therefore E(r_m) &= \frac{(0.12 - 0.0475)}{1.4} + 0.0475 \\ &= \underline{\underline{9.929\%}} \quad \text{--- 1 Point} \end{aligned}$$

$$\begin{aligned} E(r_{ABC}) &= 0.0475 + 0.8 * (0.09929 - 0.0475) \\ &= \underline{\underline{8.8932\%}} \quad \text{--- 1 Point} \end{aligned}$$

2012, Question 6-Review market efficiency

Casey, Chuck and Sarah are three fund managers sitting in a coffee shop. To pass the time, they talk about investing in stock markets. Casey started to brag about how his stock investments have consistently beaten the market, and how he is very skilled at picking winning stocks because he knows what to look for in the financial statements of publicly traded companies. Chuck couldn't understand why Casey went to all trouble of reading the financial statements of firms; because he thought picking winning stocks only required an analysis of past stock prices. Sarah, on the other hand, was skeptical of Casey's skill at picking stocks. She said that, if he really had consistently beaten the market returns, he was either extremely lucky or he used insider information to make his high returns.

- State and explain how each person (Chuck, Sarah, Casey) views the efficiency of financial markets (Weak form, semi-strong form, strong form or completely inefficient?).

Answer

Chuck:

Markets are not efficient even in weak form. Therefore, markets are not efficient at-all (believes that by analyzing past prices, can find a pattern to predict future!!!, so believes prices do not follow random walk)

Sarah:

Believes that markets are semi-strong form efficient. However, they are not strong form efficient (money can be made from insider information, but not from publicly available information).

Casey:

Markets are not semi-strong form efficient because he think public info. Is useful (either thinks that markets are weak efficient or not efficient at all).

Fall 2012, Question 10

In a portfolio of three different stocks, which of the following could not be true? Assume that the portfolio weight of each stock is less than 1.

- A. The riskiness of the portfolio is less than the riskiness of each of the stocks if they were held in isolation.
- B. The riskiness of the portfolio is greater than the riskiness of one or two of the stocks.
- C. The beta of the portfolio is less than the beta of each of the individual stocks.
- D. The beta of the portfolio is greater than the beta of one or two of the individual stock's betas.
- E. None of the above

Answer: C

Fall 2012, Question 11

Which of the following statements is correct?

- A. If you add enough randomly selected stocks to a portfolio, you can completely eliminate all the market risk from the portfolio.
- B. If you form a large portfolio of stocks each with a beta greater than 1.0, this portfolio will have more market risk than a single stock with beta of 0.8.
- C. Company specific (or unsystematic) risk can be reduced by forming a large portfolio, but normally even highly diversified portfolios are subject to market (or systematic) risk.
- D. Choices A, B, and C are correct
- E. Choices B and C are correct.

Answer: E

Winter 2013, Question 23

The market risk premium is:

- I. The slope of the security market line
- II. The percentage of the portfolio consisting of risk-free assets divided by the percentage of the portfolio consisting of risky assets
- III. The difference between the expected rate of return from the market portfolio and the expected rate of return from the risk-free asset

- A. I only
- B. II only
- C. III only
- D. I and III only
- E. II and III only

Answer : D

Problem

A stock has an expected return of 10.2 percent, the risk-free rate is 4.5 percent, and the market risk premium is 7.5 percent. What must the beta of this stock be?

Solution

$$ER = RF + \beta \cdot (ER_{market} - RF)$$

Careful! Don't mistake M.P. for ER_{market}

$(ER_{market} - RF)$ is also called: Market Risk premium(M.P.).
so we can re-write the formula as:

$$ER = RF + \beta \cdot (M.P.)$$

Here: $0.102 = 0.045 + \beta \cdot 0.075$ So:

$$0.075 \beta = 0.102 - 0.045$$

$$0.075 \beta = 0.057 \rightarrow \beta = 0.76$$

Winter 2013, Question 24

An investor invests \$800 in a risky asset with an expected rate of return of 18% and a standard deviation of 25%. The investor also invests \$200 in a Treasury bill with a 4% rate of return. Her portfolio's expected rate of return and standard deviation are _____ and _____, respectively.

- A. 14.4%; 20.0%
- B. 18.4%; 20.8%
- C. 15.2%; 20.0%
- D. 15.2%; 44.7%
- E. Insufficient information

$$E(r_p) = 0.8 \times 0.18 + 0.2 \times 0.04 = 0.152$$

$$\sigma_p = 0.8 \times 0.25 = 0.2$$

Answer : C

Winter 2013, Question 28

In the capital asset pricing model, if β of a stock is greater than 1 then:

- A. The rate of return on an individual stock is less volatile than the rate of return on a total stock market portfolio.
- B. The rate of return on an individual stock is more volatile than the rate of return on a total stock market portfolio.
- C. The rate of return on an individual stock is as volatile than the rate of return on a total stock market portfolio.
- D. The required rate of return on a stock the same as the rate of return on the total stock market portfolio.
- E. The required rate of return on a stock is less than the rate of return on the total stock market portfolio.

Answer: B

Summer I 2013, Question 19

For markets to be in equilibrium, that is, for there to be no strong pressure for prices to depart from their current levels,

- A. The expected rate of return must be equal to the required rate of return.
- B. Ex-ante return must equal ex-post return.
- C. The past realised return must equal the expected rate of return.
- D. All three A, B, C must hold.
- E. None of the above statements are correct.

Answer: A

Summer 2 2014 - Q19

According to the Capital Asset Pricing Model, fairly priced securities could _____.

- A. have positive betas
- B. have negative betas
- C. have zero betas
- D. have non zero alphas
- E. have positive, negative or zero beta

Answer: E

Good luck 😊