1a

$V_i = 0.150\,m^3$ $\qquad$ $V_f = \ell\,\frac{4}{3}\pi r^3 + V_i$

$p_i = 120\,atm$ $\qquad$ $p_f = 1.20\,atm.$

$$V_i \cdot p_i = nRT = \text{CONST} = V_f\, p_f$$

$$V_f = \frac{p_i}{p_f} V_i$$

$$V_f = \frac{120}{1.2} V_i$$

$$\ell\,\frac{4}{3}\pi r^3 + V_i = 100\,V_i$$

$$\ell = (99 V_i) \Big/ \tfrac{4}{3}\pi r^3$$

$$\ell = \frac{99 \cdot (0.150)}{\frac{4}{3}\pi\,(0.075)^3} = 8403.3$$

ANS: One can inflate 8403 balloons

NOTE: 1) V_i has to be added to the volume of the inflated balloons, since there will be some gas left in the tent at $p_f = 1.2$ atm.

2) $T = \text{CONST}$ if balloons are filled slowly.
$n = \text{CONST}$ He gas is expensive!

1b $\quad n = 0.5$ mole $\qquad C_V = \frac{3}{2} R$ (monoatomic ideal)

$$V_A = V_i = 10\,\ell = 0.01\,m^3$$
$$p_A = p_i = 5\text{ atm} = 506500\,Pa$$
$$p_A V_A = nRT_A \;;\; \underline{T_A = 1219\,K}$$

$$V_B = 50\,\ell = 0.05\,m^3$$
$$p_B = 1\text{ atm} = 101300\,Pa$$
$$p_B V_B = nRT_B \;;\; T_B = 1219\,K$$

$$V_C = 10\,\ell = 0.01\,m^3$$
$$p_C = 1\text{ atm} = 101300\,Pa$$
$$p_C V_C = nRT_C \;;\; T_C = 243.8\,K$$

$$W_{net} = W_{AB} + W_{BC} = -nRT_A \ln\frac{V_B}{V_A} + \left(-p\left(V_C - V_B\right)\right)$$

$$W_{net} = \left[-(5065)\ln 5 - 101300\,(0.01 - 0.05)\right] J$$

$$W_{net} = (-5065)\ln 5 + 4052 = -4099.8\,J \simeq \underline{-4100\,J}$$

Net work done by the engine was 4100 J.

1b) ii $\qquad$ Heat: $\quad Q_{AB} = nRT\ln\frac{V_f}{V_i} = +5065\ln 5 =$

$$Q_{BC} = nC_p\,\Delta T = (0.5)\left(\tfrac{5}{2}\,8.31\right)(243.8 - 1219)$$
$$Q_{BC} = -10129.89\,J = -10130\,J$$

$$Q_{CA} = nC_V\,\Delta T = 0.5\,\tfrac{3}{2}(8.31)(1219 - 243.8)$$

$$Q_{CA} = 6077.9 = 6078\,J$$

$$Q_h = Q_{CA} + Q_{AB} = 6078\,J + 8152\,J = 14230\,J$$

$$Q_c = Q_{BC} = -10130\,J.$$

$$e = \frac{|W|}{|Q_h|} = \frac{4100}{14230} = \underline{0.288}$$

2

Rod

$l = 0.9\,m$

$r = 0.011\,m$

$\rho = 8.94\, g/cm^3$

$\Downarrow$

mass $= \rho V = 3.06\,kg$

Water

$m = 6\,kg$

$T_i = 15°C$

$T_{Cu} = 750°C = 1023\,K$

ANSWER (C) $P = \sigma A T^4$

$$P = 5.67\times10^{-8} \frac{W}{K^4 m^2} \cdot (2\pi r l + 2\pi r^2) \cdot (1023)^4$$

$$P = 62\,100\ (0.0629) = \underline{3910\ W}$$

ANS (A)

$$Q_{H_2O} + Q_{Cu} = 0$$

$$m_{H_2O} C_{H_2O} (T_f - 15) + m_{Cu} C_{Cu} (T_f - 750) = 0$$

$$(6)(4186)(T_f - 15) + (3.06)(385)(T_f - 750) = 0$$

$$25116(T_f - 15) + 1178.1(T_f - 750) = 0$$

$$(25116 + 1178.1)T_f = 25116 \cdot 15 + 1178.1 \cdot 750$$

$$(26{,}294)\ T_f = 1260315$$

$$\underline{T_f = 47.93\ °C}$$

2 B

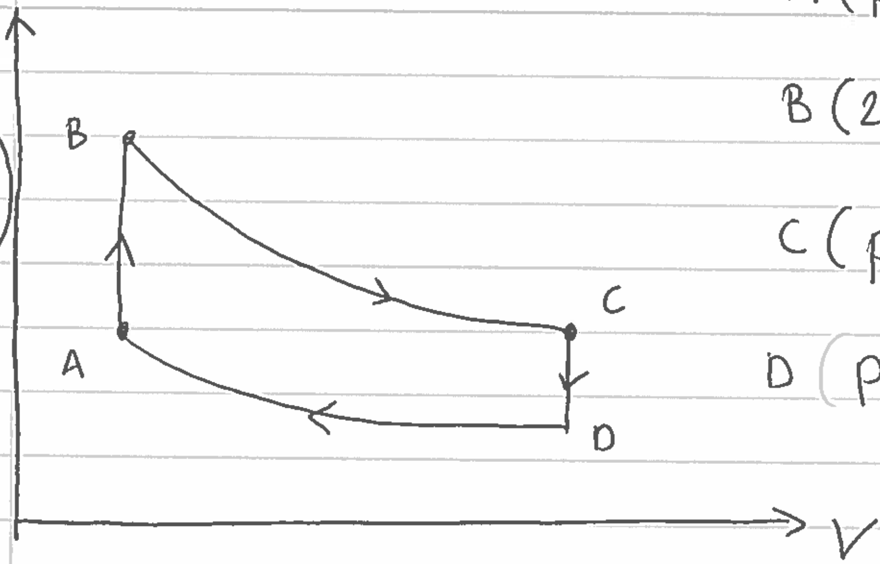
ANS B $\qquad \Delta l = \alpha l_i \, \Delta T = 17\times10^{-6}\cdot(0.9)\cdot(47.9-750)$

$$\Delta l = -0.0107\,m \simeq -0.01\,m$$

ANS B : Change of length → shortening by 1cm

Problem #3

ANS A



$A(p_i, V_i)$

$B(2p_i, V_i)$

$C(p_C, 3V_i)$

$D(p_D, 3V_i)$

A $\begin{cases} P_A = 101300 \text{ Pa} \\ V_A = 0.020 \text{ m}^3 \\ T_A = 243 \text{ K} \end{cases}$ this allows us to find n

$P_A V_A = nRT_A \rightarrow n = 1.003 = 1 \text{ mole}$

B $\begin{cases} P_B = 2P_A = 202600 \text{ Pa} \\ V_B = 0.020 \text{ m}^3 \\ T_B = 487 \text{ K} \end{cases}$

C $\begin{cases} P_C = ? \\ V_C = 3V_A = 0.06 \text{ m}^3 \\ T_C = ? \end{cases}$

$P_B V_B^{\gamma} = p_C V_C^{\gamma}$

$p_C = \left(\frac{V_B}{V_C}\right)^{\gamma} P_B$

$p_C = (2p_i) \cdot \left(\frac{1}{3}\right)^{5/3}$

ANS B $\longrightarrow$ $P_C = 32466.6 \text{ Pa}$

ANS C $P_C V_C = nRT_C \longleftrightarrow T_C = 234.4 \text{ K}$

ANS D: (Freebe) $T_f = T_i = 243 \text{ K}$ (cycle!)

ANS E $\quad Q_h = Q_{AB} \leftarrow \left.\begin{matrix} V = \text{CONST} \\ P \text{ increases} \end{matrix}\right\} \Rightarrow T \nearrow$

$$Q_{AB} = n C_v \, \Delta T$$

$$Q_{AB} = n C_v \frac{V_i \Delta p_{AB}}{nR} = \frac{C_v}{R} \Delta p_{AB} V_i = \frac{3}{2} p_i V_i$$

$$Q_{AB} = \frac{3}{2} \, 101\,300 \times 0.02 \text{ J} = \underline{3039 \text{ J}}$$

ANS : E

$$\underline{Q_h = 3039 \text{ J}}$$

ANG $\quad W_{BC} = \frac{1}{\gamma - 1} \left(p_C V_C - p_B V_B \right)$

$$W_{DA} = \frac{1}{\gamma - 1} \left(p_A V_A - p_D V_D \right)$$

$$W = W_{BC} + W_{DA} = \frac{1}{\gamma - 1} \left[p_C V_C + p_A V_A - p_B V_B - p_D V_D \right]$$

$pV = nRT$

$$W = \frac{nR}{\gamma - 1} \left(T_C + T_A - T_B - T_D \right]$$

$$W = \frac{8.31}{\left(\frac{2}{3}\right)} \left[234.4 + 243 - 487 - 117 \right]$$

$$W = \underline{-1{,}583 \text{ J}}$$ ← engine does 1583 J of work

$$e = \frac{|W|}{|Q_h|} = \frac{1583}{3039} = 0.52 \rightarrow \underline{\underline{52\%}}$$

Q4 $$P(v)\,dv = 4\pi\left[\frac{1}{2\pi}\frac{m}{kT}\right]^{3/2} v^2 e^{-\frac{mv^2}{2kT}}\,dv$$

But it is not necessary! $\left\{\right.$ It is useful to express the above using molar mass M and R rather than m, k_B

$$P(v)\,dv = 4\pi\left[\frac{1}{2\pi}\frac{M}{RT}\right]^{3/2} v^2 e^{-\frac{Mv^2}{2RT}}\,dv$$

$$P(v)\,dv = \frac{N_v}{N} \quad ; \quad 1\text{ mole} \rightarrow N = N_A$$

$$N_v = \underset{\uparrow\ 6.02\times10^{23}}{N_A} \cdot 4\pi\left[\frac{1}{2\pi}\frac{0.028}{8.31\cdot300}\right]^{3/2} e^{-\frac{(0.028)(321)}{2(8.31)\cdot300}} \times \times \underbrace{(321)^2}_{v^2}\ \underbrace{1\,m}_{dv}$$

point A.

$$N_{321} = 6.02\times10^{23}\cdot(4\pi)\left[\frac{1}{2\pi}\frac{0.028}{8.31\ \ 300}\right]^{3/2}\cdot(321)^2 e^{-\frac{(0.028)\,321}{2(8.31)\,300}}\cdot(1)$$

units (not mandatory but interesting point of this

$$N_v = \underbrace{N_A}_{\text{dimensionless}}\ 4\pi \underbrace{\left[\quad\right]^{3/2}}_{\left[\frac{m}{s^2}\right]^{-3}} \overset{(m/s^2)^2}{v^2}\ \underbrace{e^{-\frac{Mv^2}{2RT}}}_{\text{dimensionless}}\ \overset{m/s^2}{dv}$$

$$[N_v] = \left[\frac{m}{s^2}\right]^{-3}\cdot\left[\frac{m}{s^2}\right]^{3} = 1.$$

4 b

$$v_{MP} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2 \cdot 8.31 \cdot 300}{0.028}}$$

$$v_{MP} = 421.985 \simeq 422 \ m/s$$

4 c)

$$v_{AVG} = \sqrt{\frac{8kT}{\pi m}} = 422 \ m/s$$

$$\frac{8RT}{\pi M} = 422^2$$

$$T = \pi (0.028) \cdot 422^2 \cdot \frac{1}{8 \cdot 8.31}$$

$$T = 235.6 \ K$$

ANS : average v will be $422 \ m/s$ in $T = 235.6K$.

4 a) v_{MP} - most probable v corresponds to maximum of $P(v)$ function

$$\frac{dP}{dv} = 0 \Rightarrow \frac{d}{dv} (\text{const})\, v^2 e^{-\frac{mv^2}{2kT}} = 0$$

$$\text{const} \frac{d}{dv} \left(v^2 e^{-\frac{mv^2}{2kT}} \right) = 0$$

$$2v e^{-\frac{mv^2}{2kT}} + v^2 \left(-\frac{m(2v)}{2kT}\right) e^{-\frac{mv^2}{2kT}} = 0$$

$$\underbrace{e^{-\frac{mv^2}{2kT}}}_{\text{never } 0} \underbrace{\left(2v - \frac{mv^3}{kT} \right)}_{} = 0$$

$$2v - \frac{m}{kT} v^3 = 0$$

$$v\left(2 - \frac{m}{kT} v^2\right) = 0$$

$$v = 0 \text{ or } \quad 2 - \frac{m}{kT} v^2 = 0$$

$$v^2 = \frac{2kT}{m}$$

$$v = \sqrt{\frac{2kT}{m}}$$

4d) $v_{rms} = \sqrt{\overline{v^2}}$ root mean square

$$\overline{v^2} = \int_0^\infty v^2 P_v \, dv = \int_0^\infty v^2 \left(4\pi \left[\frac{1}{2\pi} \frac{m}{kT}\right]^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv\right.$$

$$\overline{v^2} = 4\pi \left(\frac{1}{2\pi} \frac{m}{kT}\right)^{3/2} \underbrace{\int_0^\infty v^4 e^{-\frac{mv^2}{2kT}} dv}_{\text{Gaussian Integral of the type: } \int_0^\infty x^4 e^{-\frac{mx^2}{2kT}} dx}$$

$$\overline{v^2} = 4\pi \left(\frac{1}{2\pi} \frac{m}{kT}\right)^{3/2} \frac{3}{8} \sqrt{\frac{\pi}{\left(\frac{m}{2kT}\right)^5}} = \frac{4\pi}{(2\pi)^{3/2}} \cdot \sqrt{\pi} \cdot \frac{3}{8} \cdot \left(\frac{m}{kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{5/2}$$

$$\overline{v^2} = \frac{4 \cdot 3 \cdot 2^{5/2}}{2^{3/2} \cdot 8} \left(\frac{kT}{m}\right) = \frac{4 \cdot 3 \cdot 2}{8} \left(\frac{kT}{m}\right) = \frac{3kT}{m}$$

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad (\text{or}) \quad \sqrt{\frac{3RT}{M}}$$

4d) v_{Avg} - average v

$$\overline{v} = \int_0^{+\infty} v P_v \, dv = \int_0^{+\infty} v \, 4\pi \left[\frac{1}{2\pi} \frac{m}{kT}\right]^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

$$\overline{v} = 4\pi \frac{1}{(2\pi)^{3/2}} \left(\frac{m}{kT}\right)^{3/2} \cdot \underbrace{\int_0^{+\infty} v^3 e^{-\frac{mv^2}{2kT}} dv}_{\text{Gaussian Integral of the type } \int_0^\infty x^3 e^{-\frac{mx^2}{2kT}} dx}$$

$$\overline{v} = \frac{4\pi}{2^{3/2} \pi^{3/2}} \left(\frac{m}{kT}\right)^{3/2} \frac{1}{2\left(\frac{m}{2kT}\right)^2} = \frac{4\pi \, 4}{\pi^{3/2} \cdot 2^{3/2} \, 2} \cdot \left(\frac{kT}{m}\right)^{1/2}$$

$$\overline{v} = \frac{8}{\sqrt{\pi} \, 2^{3/2}} \left(\frac{kT}{m}\right)^{1/2} = \frac{\sqrt{8}}{\sqrt{\pi}} \sqrt{\frac{kT}{m}} = \sqrt{\frac{8kT}{\pi m}} \; //$$

Q5

part A $\Delta E_{int} = Q + W$

i) I Law of Thermodynamics applied to isobaric transformation gives:

$$n C_v \Delta T = n C_p \Delta T - p \Delta V$$

$$n C_v \Delta T = n C_p \Delta T - n R \Delta T$$

$$C_v = C_p - R$$

$$C_p = C_v + R$$

ii) I Law of Thermodynamics applied to adiabatic transformation gives:

$$W = \Delta E_{int}$$

$$\frac{1}{\gamma-1}(p_f V_f - p_i V_i) = n C_v \Delta T$$

$$\frac{1}{\gamma-1}(nRT_f - nRT_i) = n C_v \Delta T$$

$$\frac{nR}{\gamma-1}(T_f - T_i) = n C_v \Delta T$$

$$\frac{\cancel{n}R}{\gamma-1} \cancel{\Delta T} = \cancel{n} C_v \cancel{\Delta T}$$

$$\frac{R}{\gamma-1} = C_v$$

$$\frac{R}{C_v} = \gamma - 1$$

$$\frac{C_p - C_v}{C_v} = \gamma - 1$$

$$\frac{C_p}{C_v} = \gamma$$

3

$$pV = nRT$$

$$d(pV) = d(nRT)$$

$$p\,dV + V\,dp = nR\,dT$$

we could combine it with First Law of Thermodynamics for adiabatic transformation

$$dE_{int} = dW \qquad (dQ = 0$$

$$nC_V\,dT = -p\,dV$$

$$dT = -\frac{p\,dV}{nC_V}$$

$$p\,dV + V\,dp = nR\left(-\frac{p\,dV}{nC_V}\right)$$

$$p\,dV + V\,dp = -\frac{R}{C_V}\,p\,dV$$

$$p\left(1 + \frac{R}{C_V}\right)dV + V\,dp = 0$$

$$p\left(\frac{C_V + R}{C_V}\right)dV + V\,dp = 0$$

$$p\left(\frac{C_p}{C_V}\right)dV + V\,dp = 0 \qquad \frac{C_p}{C_V} = \gamma$$

$$p\gamma\,dV + V\,dp = 0 \;/ : pV$$

$$\gamma\frac{dV}{V} + \frac{dp}{p} = 0$$

$$\gamma \frac{dV}{V} = -\frac{dp}{p}$$

$$\int_i^f \gamma \frac{dV}{V} = \int_i^f - \frac{dp}{p}$$

$$\gamma \ln V \Big|_i^f = - \ln p \Big|_i^f$$

$$\gamma \left(\ln V_f - \ln V_i\right) = -\left(\ln_f - \ln_i\right)$$

$$\gamma \ln \frac{V_f}{V_i} = - \ln \frac{f}{i}$$

$$\ln \left(\frac{V_f}{V_i}\right)^\gamma = \ln \left(\frac{p_f}{p_i}\right)^{-1}$$

$$\ln \left(\frac{V_f}{V_i}\right)^\gamma = \ln \left(\frac{p_i}{p_f}\right)$$

$$\left(\frac{V_f}{V_i}\right)^\gamma = \frac{p_i}{p_f}$$

$$V_f^\gamma p_f = V_i^\gamma p_i$$

$$\Downarrow$$

$$V^\gamma p = \text{CONST.}$$

Alternative way

$$-p\,dV = n\,C_v\,dT$$

$$-\frac{p\,dV}{T} = \frac{n\,C_v\,dT}{T}$$

$$\frac{p}{T} = \frac{nR}{V} \longrightarrow -\frac{nR\,dV}{V} = n\,C_v\,\frac{dT}{T}$$

$$-R\,\frac{dV}{V} = C_v\,\frac{dT}{T}$$

$$-\frac{dV}{V} = \frac{C_v}{R}\,\frac{dT}{T}$$

$$-\frac{R}{C_v}\int\frac{dV}{V} = \int\frac{dT}{T}$$

$$\mathrm{CONST}_2 \quad -\frac{C_p-C_v}{C_v}\ln V = \ln T \quad + \mathrm{CONST}_1$$

$$\underbrace{\mathrm{CONST}_2 - \mathrm{CONST}_1}\ = \ln T + \left(\frac{C_p}{C_v} - 1\right)\ln V$$

$$\mathrm{CONST.} = \ln T + (\gamma - 1)\ln V$$

$$\mathrm{CONST.} = \ln T + \ln V^{(\gamma-1)}$$

$$\mathrm{CONST.} = \ln T V^{\gamma-1}$$

$$T V^{\gamma-1} = e^{\mathrm{CONST}}$$

Almost what we want!

$$\frac{T V^{\gamma}}{V} = e^{\mathrm{CONST}}$$

$$\frac{p}{nR} V^{\gamma} = e^{\mathrm{CONST}}$$

$$pV^{\gamma} = \underbrace{nR\,e^{\mathrm{CONST}}} = \mathrm{CONST}'$$

5B (ii)

$$P_v\, dv \longrightarrow P_E\, dE$$

$$P_v dv = 4\pi \left[\frac{1}{2\pi}\frac{m}{KT}\right]^{3/2} v^2 e^{-\frac{mv^2}{2KT}} dv$$

$$E = \frac{mv^2}{2}$$

$$P_E dE = 4\pi \left[\frac{1}{2\pi}\frac{m}{KT}\right]^{3/2} \frac{2}{m}\frac{mv^2}{2} e^{-\frac{mv^2}{2KT}} \frac{dE}{\sqrt{2m}\sqrt{E}} \quad (*)$$

$$P_E dE = 4\pi \left[\frac{1}{2\pi}\frac{m}{KT}\right]^{3/2} \frac{2}{m}\frac{1}{\sqrt{2m}} E \frac{1}{\sqrt{E}} e^{-\frac{E}{KT}} dE$$

$$P_E dE = 4\pi \left[\frac{1}{2\pi}\right]^{3/2} \frac{m^{3/2}}{(KT)^{3/2}} \frac{\sqrt{2}}{m^{3/2}} \sqrt{E}\, e^{-\frac{E}{KT}} dE$$

$$P_E dE = \frac{{}^{2}\cancel{4}\pi\sqrt{2}}{2\sqrt{2}\,\pi\sqrt{\pi}} \frac{1}{(KT)^{3/2}} \sqrt{E}\, e^{-E/KT} dE = \frac{2}{\sqrt{\pi}}\frac{1}{(KT)^{3/2}}\sqrt{E}\, e^{-E/KT} dE$$

$$P_E = \frac{2}{\sqrt{\pi}}\frac{1}{(KT)^{3/2}}\sqrt{E}\, e^{-E/KT} dE$$

$(*)$

$$E = \frac{mv^2}{2} \rightarrow dE = \frac{m}{2} 2v\, dv = mv\, dv$$

$$\frac{dE}{mv} = dv$$

$$dv = \frac{dE}{\sqrt{m}\sqrt{m}\, v} \cdot \frac{dE}{\sqrt{m}\sqrt{2}\left(\frac{\sqrt{m}v}{\sqrt{2}}\right)} = \frac{dE}{\sqrt{2m}\sqrt{E}} \quad \therefore$$