uOttawa

Faculté de génie
Faculty of Engineering

GNG 1105 H – Engineering Mechanics

Midterm Exam

Length of Examination: 1h 10m

Professor: Christian Viau

Date: Monday, October 16th 2017

Last Name: Viau

First Name: Christian

Student Number: ______________________

Signature: Midterm Solution

Number of Booklets Submitted: ________

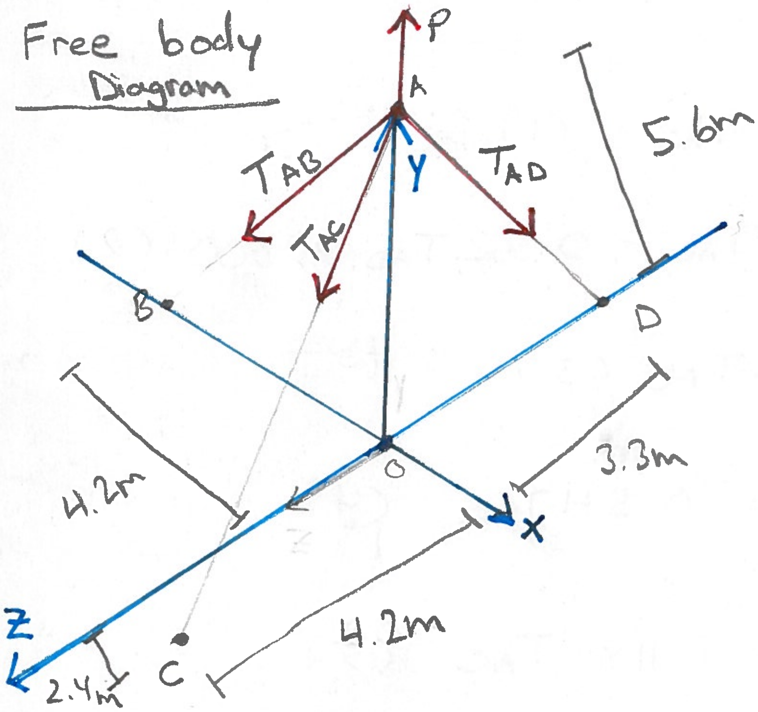
Instructions:

- This is a closed book exam. No notes or textbooks are permitted.
- The examination question paper **MAY NOT** be taken from the examination room. Please leave it in your booklet(s) upon submission.
- Only work in the examination booklet(s) will be graded. Anything written on this examination question paper will be disregarded.
- Please verify that your exam contains 7 pages.
- If you do not understand a question, clearly state an assumption and proceed.

Question 1 (40 points)

Three cables are used to tether a balloon as shown in Figure 1.

Knowing that the balloon exerts an 800-N vertical force at A, **determine the tension in each cable.**



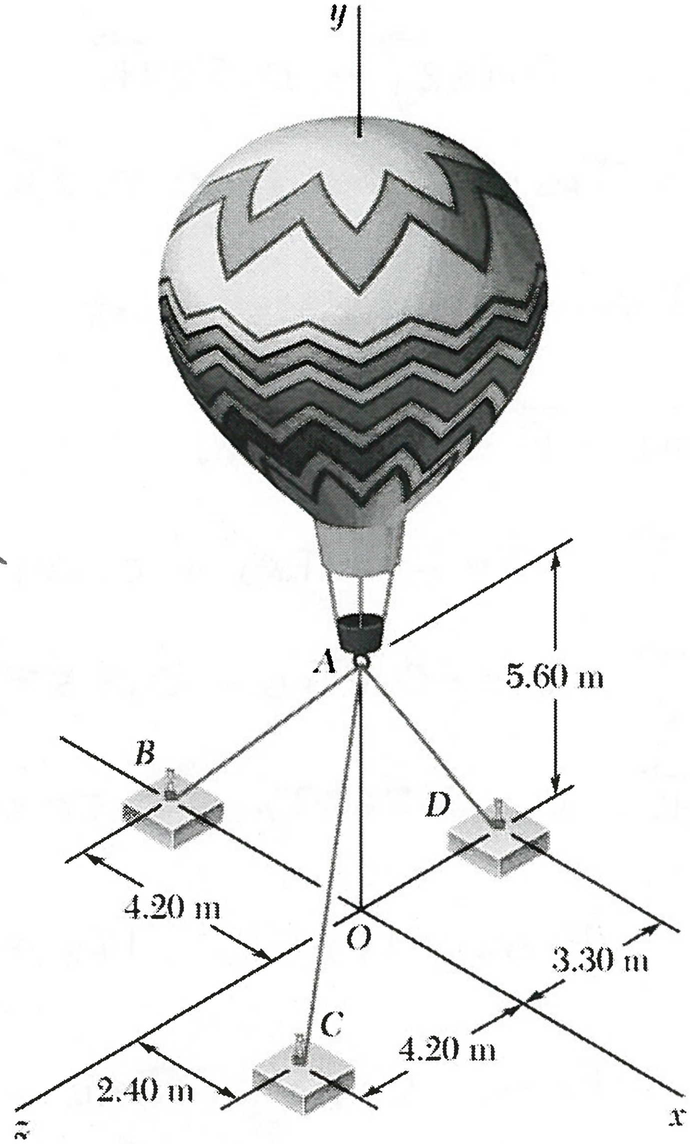


Figure 1

- $\vec{AB} = -4.2\vec{i} - 5.6\vec{j}$, $|AB| = \sqrt{4.2^2 + 5.6^2} = 7m$

$\vec{T_{AB}} = T_{AB} \cdot \lambda_{AB}$ where $\lambda_{AB} = \dfrac{-4.2\vec{i} - 5.6\vec{j}}{7}$

$= -0.6\vec{i} - 0.8\vec{j}$

$\vec{T_{AB}} = T_{AB}(-0.6\vec{i} - 0.8\vec{j})$

- $\vec{AC} = 2.4\vec{i} - 5.6\vec{j} + 4.2\vec{k}$, $|AC| = 7.4m$

$\lambda_{AC} = 0.324\vec{i} - 0.757\vec{j} + 0.568\vec{k}$

$\vec{T_{AC}} = T_{AC} \cdot (0.324\vec{i} - 0.757\vec{j} + 0.568\vec{k})$

$\vec{AD} = -5.6\vec{j} - 3.3\vec{k}$, $|AD| = 6.5m$

$\lambda_{AD} = -0.862\vec{j} - 0.508\vec{k}$

$\vec{T}_{AD} = T_{AD} \cdot (-0.862\vec{j} - 0.508\vec{k})$

Equilibrium condition at "A" : $\Sigma\vec{F} = 0$: $\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{P} = 0$

where $\vec{P} = 800\vec{j}$ N

on $\vec{i}$: $0 = -0.6 \cdot T_{AB} + 0.324 \cdot T_{AC}$ (1)

$\vec{j}$: $0 = -0.8 T_{AB} - 0.757 \cdot T_{AC} - 0.862 \cdot T_{AD} + 800$ (2)

$\vec{k}$: $0 = 0.568 \cdot T_{AC} - 0.508 \cdot T_{AD}$ (3)

From (1): $T_{AB} = 0.54 \cdot T_{AC}$ (4)

From (3): $T_{AD} = 1.118 \cdot T_{AC}$ (5)

Putting (4) + (5) into (2), we find:

$2.153 T_{AC} = 800 \rightarrow \boxed{T_{AC} = 372 N}$

Finally, from (4) + (5):

$T_{AB} = 0.54 \cdot 371.57 = \boxed{201 N}$

$T_{AD} = 1.118 \cdot 371.57 = \boxed{415 N}$

Question 2 (25 points)

a) Knowing that the tension in wire BD is 1300 N, **determine the reactions at the fixed support C** of the frame shown in Figure 2.

b) If wire BD were to fail suddenly, **determine the reactions at support C.**

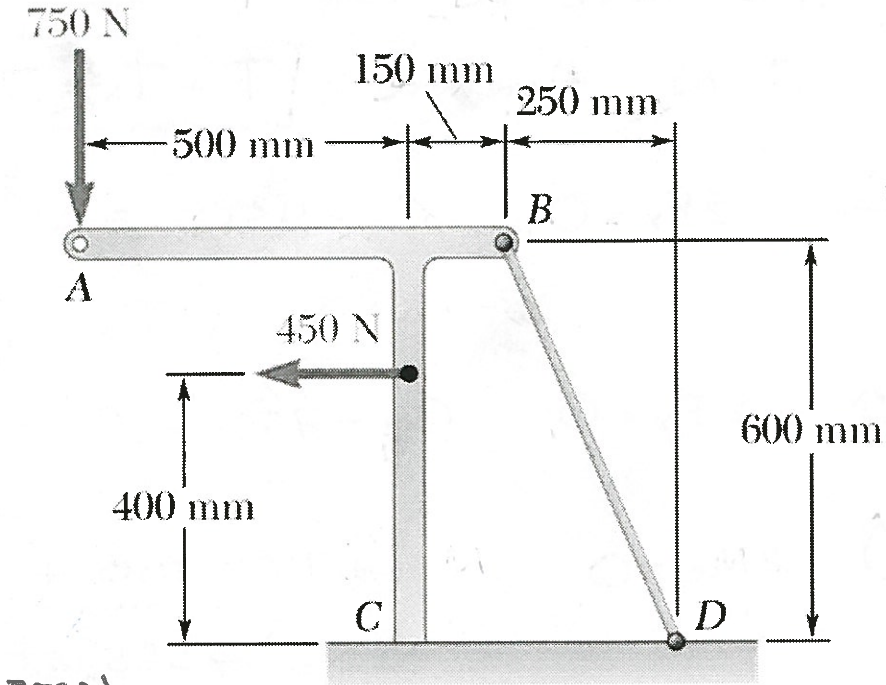
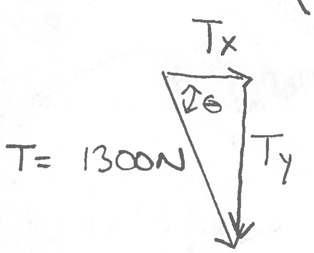
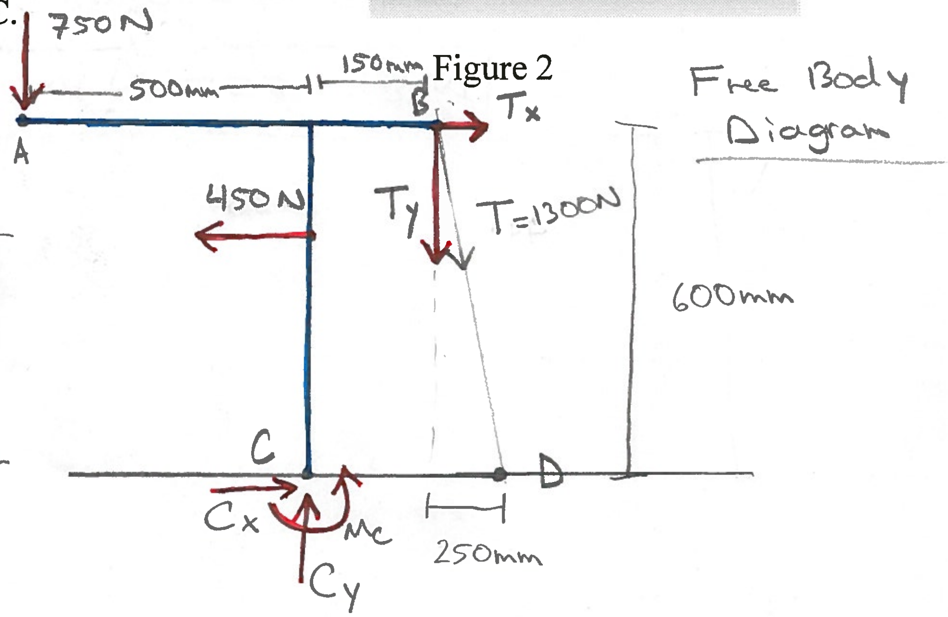


Figure 2

a)

$\theta = \tan^{-1}\left(\frac{600}{250}\right) = 67.4^\circ$





$T_x = 1300 \cos 67.4 = 500\ N$

$T_y = 1300 \sin 67.4 = 1200\ N$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \ C_x - 450 + 500 = 0$

$\rightarrow C_x = -50N$

$\boxed{C_x = 50\ N \leftarrow}$

$+\uparrow \ \Sigma F_y = 0: \ C_y - 750 - 1200 = 0$

$\rightarrow C_y = 1950\ N$

$\boxed{C_y = 1950\ N \uparrow}$

$+\circlearrowleft \ \Sigma M_C = 0: 0 = M_C + 750 \cdot 500 + 450 \cdot 400 - 1200 \cdot 150 - 500 \cdot 600$

$M_C = -75{,}000\ N \cdot mm \rightarrow \boxed{M_C = 75\ N \cdot m \circlearrowright}$

b) If wire BD were to fail, T would become 0 N, therefore $\boxed{T = T_x = T_y = 0 N}$

$\xrightarrow{+}$ $\Sigma F_x = 0$: $C_x - 450 = 0 \rightarrow C_x = 450 N$

$\boxed{C_x = 450 N \rightarrow}$

$+\uparrow$ $\Sigma F_y = 0$: $C_y - 750 = 0 \rightarrow \boxed{C_y = 750 N \uparrow}$

$\circlearrowleft +$ $\Sigma M_c = 0$: $M_c + 750 \cdot 500 + 450 \cdot 400 = 0$

$M_c = -555,000 N \cdot mm$

$\boxed{M_c = 555 N \cdot m \circlearrowright}$

Question 3 (35 points)

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle BE that is supported by bearings at C and D, as shown in Figure 3.

If a 720-N vertical load is applied at A when the lever is horizontal, **determine the reactions at C and D when T = 1200 N**. Assume that the bearing at D does not exert any axial thrust.

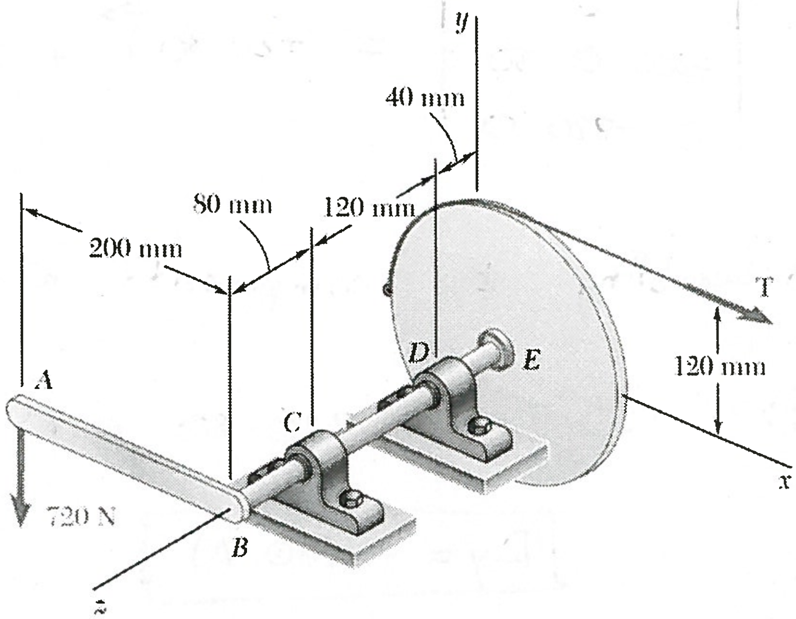
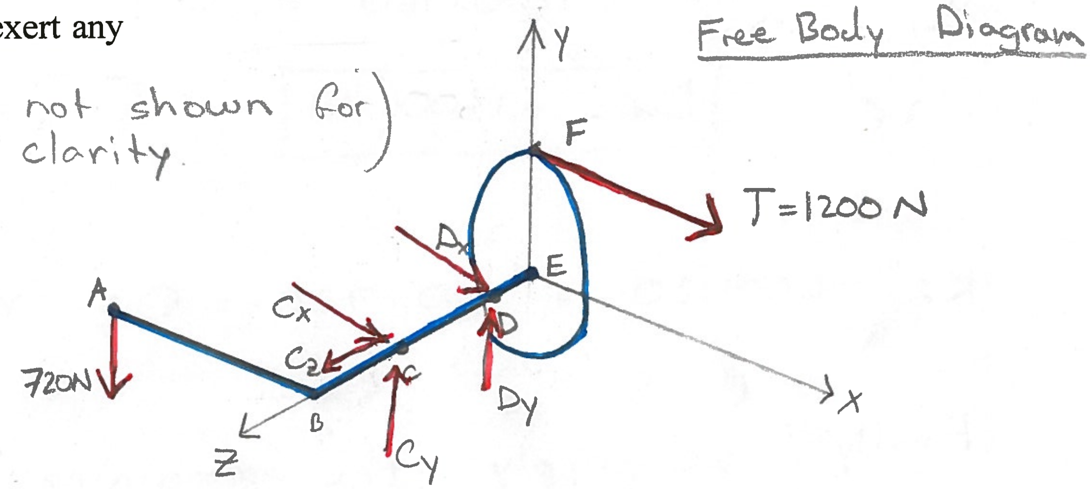


Figure 3

(Dims. not shown for clarity.)



$$\Sigma M_C = 0: \quad \underbrace{\vec{r}_{CD} \times \vec{D}}_{1} + \underbrace{\vec{r}_{CF} \times \vec{T}}_{2} + \underbrace{\vec{r}_{CA} \times (-720\vec{j})}_{3} = 0$$

1:

$$\begin{vmatrix} i & j & k \\ 0 & 0 & -120 \\ D_x & D_y & 0 \end{vmatrix} = 120 D_y \vec{i} + 120 D_x \vec{j}$$

2:

$$\begin{vmatrix} i & j & k \\ 0 & 120 & -160 \\ 1200 & 0 & 0 \end{vmatrix} = 1200 \cdot 160 \vec{j} - 1200 \cdot 120 \vec{k}$$

3:

$$\begin{vmatrix} i & j & k \\ -200 & 0 & 80 \\ 0 & -720 & 0 \end{vmatrix} = 720 \cdot 80 \vec{i} + 200 \cdot 720 \vec{k}$$

Assembling by components and equating to zero:

i: $120 \cdot D_y + 720 \cdot 80 = 0$

$$\boxed{D_y = -480 \text{ N}}$$

j: $120 \cdot D_x + 1200 \cdot 160 = 0$

$$\boxed{D_x = -1600 \text{ N}}$$

k: $-1200 \cdot 120 + 200 \cdot 720 = 0$ ✓ OK

Finally, we apply the remaining equilibrium equations:

$\Sigma F_x = 0$: $C_x + D_x + 1200 = 0$

$$\boxed{C_x = 400 \text{ N}}$$

$\Sigma F_y = 0$: $C_y + D_y - 720 = 0$

$$\boxed{C_y = 1200 \text{ N}}$$

$\Sigma F_z = 0$: $\boxed{C_z = 0}$

Optional - Question 4 (5 points)

If the frame from Question 2 was actually pinned at C (see Figure 4), **would the frame remain in equilibrium if wire BD were to fail suddenly?**

Explain and show your reasoning using what we learned in class.

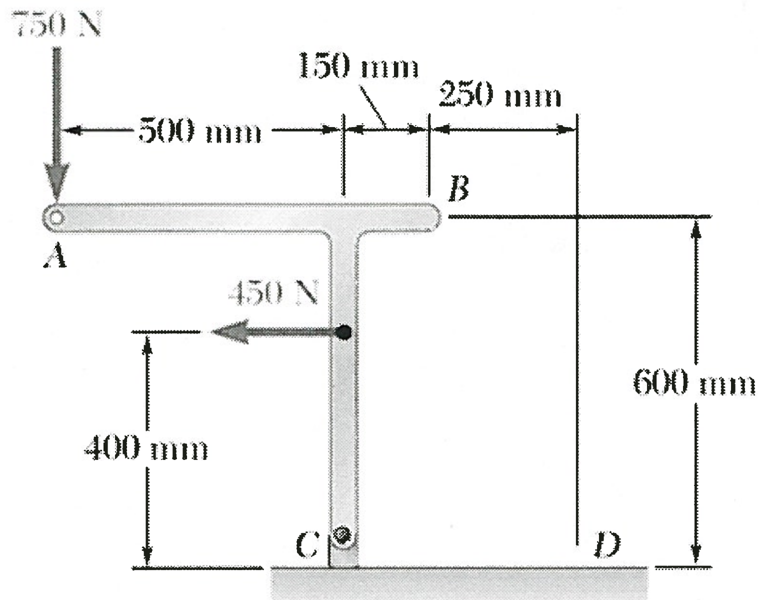


Figure 4

Intuitively, we can conclude that without either wire at B or fixity at C, there is nothing preventing the frame from falling over.

This can be proven simply applying ΣM_c and seeing if it equals zero:

$$\Sigma M_c \stackrel{?}{=} 0 : \quad 750 \cdot 500 + 450 \cdot 400 \stackrel{?}{=} 0$$

$$\boxed{555,000 \neq 0}$$

∴ The frame would <u>not</u> be in equilibrium.