

1. (3 points)

$$\text{Let } f(x) = \frac{x^3(-2x - 4)}{(x^2 + 1)(x^2 + 2x)}.$$

List all vertical asymptotes and holes in the graph of $f(x)$, and compute $\lim_{x \rightarrow \infty} f(x)$. On what intervals is $f(x)$ continuous?

2. Compute the following limits, if they exist. If not, explain why not.

(a) (3 points)

$$\lim_{x \rightarrow 1} \frac{x(\ln(x))}{x^2 - 2x - 3}$$

(b) (3 points)

$$\lim_{x \rightarrow 0} \frac{1}{x}$$

(c) (3 points)

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 7x + 12}$$

3. Let $f(x) = |x|$.

(a) (1 point) Write and simplify the difference quotient for $f(x)$ at $x = 0$.

(b) (2 points) Compute

$$\lim_{h \rightarrow 0} \frac{|h|}{h},$$

if it exists, by computing the left and right limits. (Hint: write $|h|$ as a piecewise function of h .)

(c) (1 point) Interpret your work above in terms of the derivative of $f(x) = |x|$.

4. (a) (3 points) Find the derivative of $f(x) = \sqrt{x+1}$ using the limit definition of the derivative.
- (b) (3 points) Find the equation of the tangent line to $f(x)$ at the point $(3, 2)$.

5. Find the derivatives of the following functions using any technique you want. Clearly separate your work for each problem.

(a) (3 points)

$$(x^2 + 2x + 3)^4(x^3 - 1)^5 + \ln\left(\frac{x}{x-2}\right)$$

(b) (3 points)

$$\frac{e^{2x+3}}{9-14x}$$

6. (6 points) Two bugs, a ladybug and a gentlemanbug, land on your notebook while you are about to graph some functions for your math homework. The ladybug lands at the origin, and the gentlemanbug lands at the point $(4, 0)$. The ladybug immediately begins crawling upward along the y -axis at a rate of one centimeter per second, while the gentlemanbug begins crawling toward the origin at a rate of two centimeters per second. At what rate is the distance between the bugs changing at the instant that the gentlemanbug has traveled two centimeters?

7. (6 points) The function $H(t) = (t - 1)^3(t - 3)^2$, for $0 \leq t \leq 4$, models the temperature in degrees centigrade in your backyard over the course of 4 hours. When was this temperature at its very highest and its very lowest? Also list any local maxima and minima. List both the x -values at which the extrema occur and the extreme y -values themselves. Indicate which are local extrema and which are global.

8. (3 points) The marginal revenue at time t (in **months**) for a local taco stand is given by the formula $g(t) = 2t(3t^2 - 12)^3$. Assuming the revenue after one year is \$12,000, find a formula describing the revenue of the taco stand.

9. (5 points) Compute the area under the curve $f(x) = -(x - 5)(x + 3)$ on the interval $[-3, 5]$. What is the average value of $f(x)$ on this interval?

10. Compute the following definite and indefinite integrals. For definite integrals, simplify completely. Clearly separate your answers.

(a) (3 points)

$$\int x e^x dx$$

(b) (3 points)

$$\int_1^{e^3} \frac{4}{x} dx$$

(c) (3 points)

$$\int \left(x^5 - e^{-2x} + \sqrt[5]{x} - \frac{1}{x^3} \right) dx$$

(d) (3 points)

$$\int_{-2}^1 (2x + 5)^{100} dx$$