Chapter 1.

Experiment as used in this text which is not an experimental design: a set of conditions under which some variable is observed, that is, a trial, a series of events or observations.

Outcome of an experiment – or **Element**: the result of the observation.

Sample Space, ***S***: collection of all possible outcomes of an experiment.

(Def. 1.1 text)

Examples:

$S = \{H,T\}$ is the sample space for a coin tossing experiment.

$S = \{x |\ x$ is a city with a population > 1 million$\}$ is the sample space for an observational experiment about city size.

Event: a collection of sample points, or a subset of a sample space.

(Def 1.2 text)

Examples:

$\emptyset$– Null Set

S - the whole sample set.

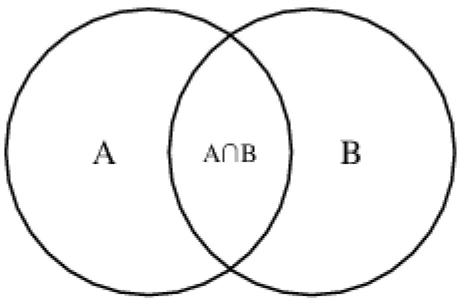
A = { H } - a head on a toss of a coin.

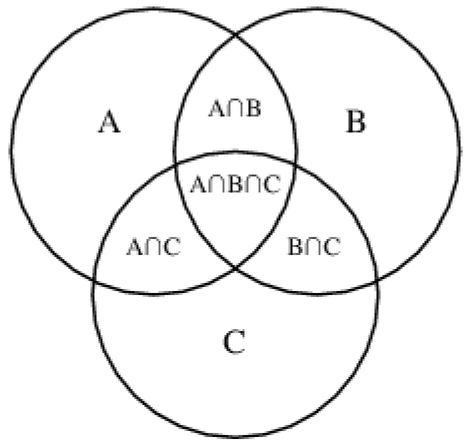
B = {Spade} - a spade in a deck of cards.

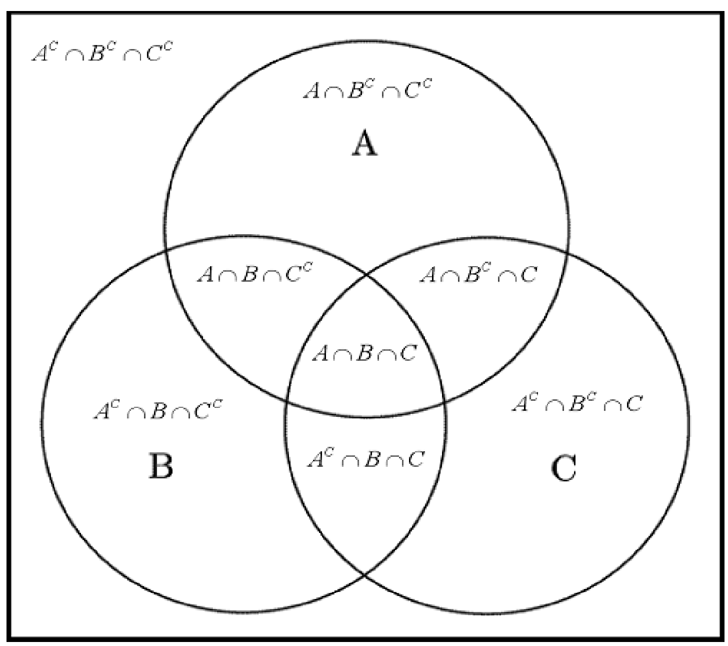
Venn Diagram:

A Venn Diagram is used in statistics to depict collections of sets and represent their relationships.

Examples:





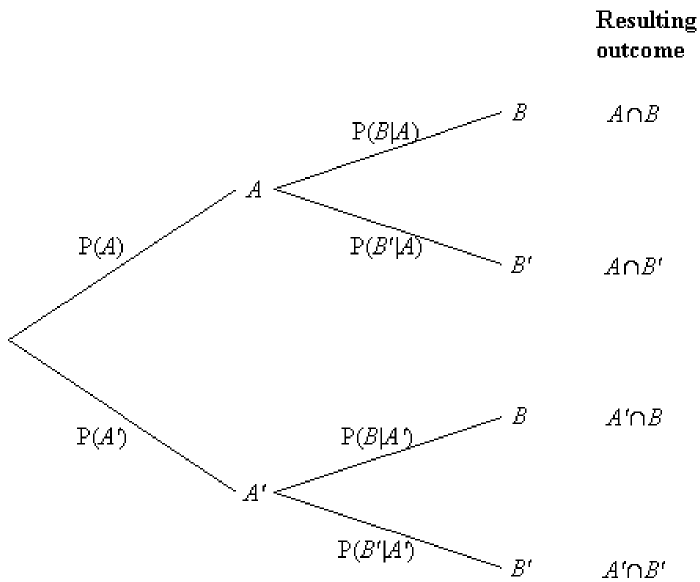


As you can see above there are many ways to correctly label (and thus to calculate) the intersections.

Tree Diagram:

Tree diagrams allow us to see all the possible outcomes of an event and calculate each of their probabilities. Each branch in a tree diagram represents a possible outcome from the experiment. The sum of the probabilities for any set of branches is always 1. Each event can be labeled as either prior to the future events on the tree or conditional on the previous events in the tree. You can also note that in a tree diagram to find a probability of an outcome we multiply along the branches and add vertically.

Example :



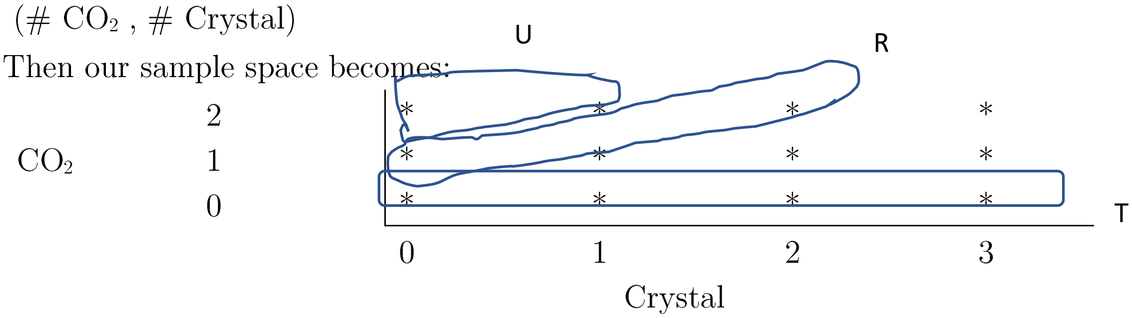
Source: http://paulgoddard.co.uk/tuition/resources/mathematics/statistics/probability.html

Grid:

A grid is another way to visualize a sample space.

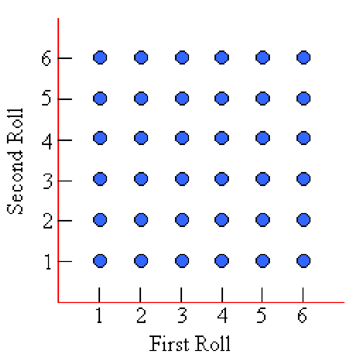
Example:

1) A technician needs to check 3 solid crystal lasers and 2 CO_2 lasers to see if they are useful for a particular task. If we are interested in the number of lasers of each type that are useful, then we can define our outcome as:
   (# CO_2 , # Crystal)
   Then our sample space becomes:



   Then we can define several events.
   Let R be the event with an equal number of crystal and carbon dioxide that are useful. Then
   R = {(0,0), (1,1), (2,2)}.
   Let U be the event where there are fewer crystal than carbon dioxide that are useful. Then
   U = {(1,0),(2,1), (2,0)}
   Let T be the event that no carbon dioxide is suitable. Then
   T = {(0,0), (0,1), (0,2), (0,3)}

2) Rolling two six-sided dice, the sample space would be as follows:



Source: https://www.mathsteacher.com.au/year10/ch05_probability/03_representation/repres.htm

Complement:

(Def. 1.3 text)

The complement of an event A is the set of all objects not in A and it is denoted A’ (though in some texts it is denoted as A^C)

Intersection:

(Def 1.4 text)

The intersection of 2 events A and B, denoted by the symbol A∩B, is the event containing all elements that are in common to A **and** B.

Union:

(Def 1.6 text)

The union of two events A and B, denoted by A∪B, is the event containing all the elements that belong to both A and B.

Mutually Exclusive:

Two events are mutually exclusive OR disjoint, if the intersection A∩B = ∅, that is, if A and B have no elements in common (Ai∩Aj = ∅, for all i ≠ j). If $A_{1,}$…,A_k are mutually exclusive and exhaustive then the collection of events A is called a partition of *S*. (i.e. $A_i \cap A_j = \emptyset$; if i ≠ j and $A_1 \cup \ldots \cup$ Ak = S)

Exhaustiveness - union of all events is sample space (A1∪A2∪...∪An = *S*). If events are mutually exclusive and exhaustive then they are disjoint and span the space *S*.

Permutation:

A permutation is an arrangement of all or part of a set of objects. In a premutation we care about the order of the elements (so really you have permutation locks not combination locks!).

$${}_nP_r = \frac{n!}{(n-r)!}$$

where n is the total number of objects and r is the number we are placing in a row.

Permutations with repetition is:

n^r

Combination:

$_nC_r = \frac{n!}{(n-r)!r!} = \binom{n}{r}$

where n is the total number of objects and r is the number we are choosing

Combinations with repetition is:

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!\,(n-1)!}$$

The binomial distribution uses the combination of r objects selected from n to count the ways in which outcomes can arise in n repeated Bernoulli trials.

Probability:

Probability is the likelihood of a certain event occurring out of a total possible number of events. The probability of an event is the proportion of times the event would occur in a long run of repeated experiments.

Examples:

P(A) = # of ways in which A can happen / total number of possible events that could occur.

P(H with a fair coin) = ½

P(spade) = 13/52

Equally Likely Outcomes: If an experiment can result in any one of N different, but equally likely outcomes, and if exactly n of those outcomes correspond to event A, then the probability of event A is:

P(A) = n/N

Examples:

P(Heads with a fair coin) = ½

Properties/axioms of probability/general additivity rules

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- $P(\emptyset) = 0$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$ if A and B are mutually exclusive

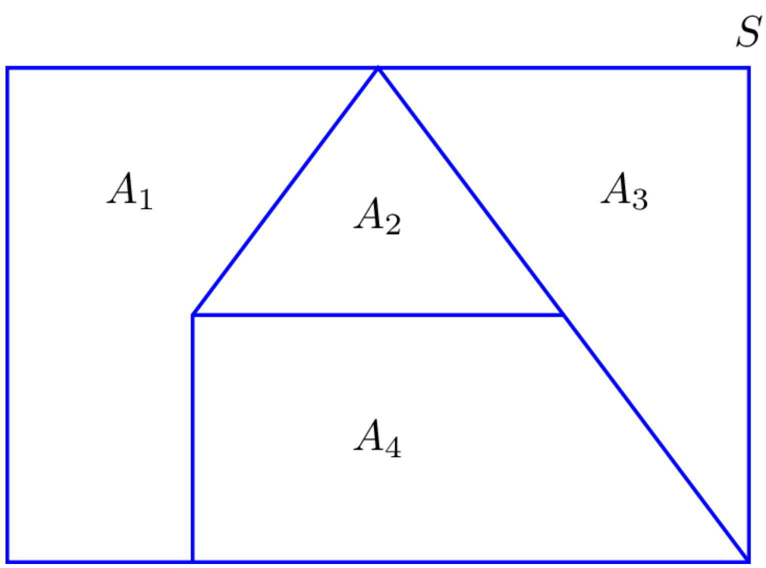
If A and A" are complementary events, then

$P(A) + P(A') = 1$

If $A_1, \ldots, A_k$ are mutually exclusive and exhaustive then the collection of events A is partition of *S*. Then:

$P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) + P(A_2) + \ldots + P(A_k) = P(S) = 1$

Example:



Here we have partitioned S into 4 collections of events A_1 to A_4.

The probability of $P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(A_1) + P(A_2) + P(A_3) + P(A_4) = P(S) = 1$

Total probability rule:

If $A_1, \dots, A_k$ are mutually exclusive and exhaustive (i.e. $A_i \cap A_j = \emptyset$; if $i \neq j$ and $A_1 \cup \dots \cup Ak = S$), then for any event B

$P(B) = P(B|A_1)P(A_1) + \dots + P(B|A_k)P(A_k)$

Conditional probability:

The probability that an event A occurs when we know that event B has occurred.

Definition:

$P(A \mid B) = P(A \cap B) / P(B)$, provided $P(B) > 0$

We can see that, $P(A \cap B)$ can also be obtained from above as

$P(A \cap B) = P(B)P(A|B) = P(A)\ P(B|A)$

Independence:

Two events A and B are independent if A and B have no influence on the probability of each other. A and B are independent if:

$P(A|B) = P(A)$, or equivalently if

$P(B|A) = P(B)$, or if

$P(A \cap B) = P(A)\ P(B)$.

Bayes Rule:

Bayes' theorem is a formula that describes how to update the probabilities of hypotheses when given evidence for an experiment. It follows simply from the axioms of conditional probability.

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$