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CVG 2140 A01: Mechanics of Materials I

Report for Bridge Project:
Modified Warren Truss Bridge

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I. ABSTRACT

The purpose of this project is to apply theory seen in class to design and build a truss bridge. For this project, the bridge model chosen was an adapted version of a warren bridge design.

The design of the bridge was first drawn, and a complete truss analysis was performed using hand calculations and software. Methods of connecting the bridge members were researched and a final design was made. The truss was then constructed using four different dimension types of balsa wood and by following all the design requirements. The pieces were joined together using wood glue and clamps.

To test the structure of the bridge, a concentrated load was applied until failure. According to the theoretical truss analysis conducted, the maximum load that the truss bridge can sustain before failing is 902 Newtons. The calculated load was found to be higher than the experimental one which was 822 N. This difference can be explained by the fact that in actual conditions, the joints could have experienced moments. Moreover, the load was not applied at a single point as considered in the analysis, so the actual distribution varied. As calculations predicted, certain members were found to be in compression and others in tension during the application of the load.

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1.0 INTRODUCTION AND OBJECTIVES

The objective of this project is to design and build a truss bridge by applying theory learned in class. The project allows students to be exposed to the full engineering design process, which includes designing their truss bridge, analyzing the bridge using methods seen in class, and then building and testing the bridge. For this project, the truss bridge chosen was an adapted version of the Warren bridge.

The design process of the construction of this bridge included research into truss bridges, choosing a design and adapting it to the objectives of the project. The theoretical design was then analysed to determine the forces acting on it, and a final design was determined using these calculations to construct the bridge.

The objective of the bridge design is to support the highest load to self-weight ratio, while respecting established constraints. The bridge must have a maximum height of 250 mm, a width between 150mm and 250mm, and an unsupported span of 750 mm between its simple supports. The materials provided for the construction of this truss bridge include four different thickness sizes of balsa wood. These pieces of wood are joined using a wood glue, and held together using gusset plates.

To test the structure and design of the bridge, a bearing plate of dimensions 250mm by 250mm will be placed on the chords of the bridge. For the load testing, a Galdabini universal testing machine will apply a concentrated load at the mid-span of the truss bridge. The load will be applied until the truss fails.

2.0 THEORETICAL BACKGROUND

A truss is an engineering design concept in which straight members are connected in a triangular form. This design creates a rigid structure because it distributes the load over a larger area instead of at a single point. A rigid truss is one that does not deform under a small load. A non-rigid truss will deform. The triangular truss design is an example of a rigid and stable truss. Although often used for bridges, a truss design can be found in many other applications: roofs, buildings and gantry cranes. A truss bridge is one whose main component is the truss design [2].

There are many different designs of truss bridges, including: Howe, Pratt, Fink, Warren, bowstring, and the three-hinged arch. The bridge chosen for this project is an adapted version of the Warren bridge. A truss bridge is created by making a triangular rigid truss. Each time two members are attached, another triangle is formed, creating a larger compound truss. In a simple statically determinate truss, the equation $m = 2n - 3$ represents the number of members (m) and the number of joints (n) [3].

A truss is a two dimensional structure that is designed to carry applied loads on the same plane. Because the truss members are often slender and cannot support lateral loads, the loads are applied at the joints. In general, joints that cannot transfer any moments are preferred.

In a truss, each member experiences either two tension or compression forces, that are equal and opposite in direction. These members are referred to as two-force axial members. There are two methods of calculating the forces in a statically determinate truss: the method

of sections and the method of joints. The method of joints will be demonstrated later in this report [3].

Furthermore, joints are a very important aspect of a truss bridge because it is at these points that the load is applied. During the design of the modified Warren bridge, several different methods for joining the wood together were considered: simple overlapping joints, double overlapping joints, butt joints, and joints with gusset plates. Ultimately, the gusset plate method was chosen for this design bridge [3].

Moreover, balsa wood was chosen as the material to design the modified Warren bridge. Historically, wood was a common material used to design truss bridges because of its low cost, ease of building, simplicity of analysis, and its span capability. Before the industrial revolution, bridges were mainly built with stone, but since then, wood and iron were found to resist tension and compression forces much better [2]. Wood with similar properties to Balsa wood, is a common modelling wood. This wood is grown in South American rainforest and can be kiln-dried to maximize its strength to weight ratio. The modulus of elasticity of balsa wood is between 3500 and 3710 MPa [1], and its ultimate tensile strength is 23.5 MPa [3].

The stress strain relationship of Balsa Wood is strictly linear as it only experiences elastic strain. Stress and strain values are calculated using the following equations:

$$(1) \quad \sigma = F/A$$

Where F is the force in Newtons, and A is the area in mm², and

$$(2) \quad \varepsilon = \Delta L/L_0$$

Where ΔL is the change in length in mm, and L_0 is the original length in mm.

The maximum compressive load that can be applied before buckling is found using Euler's

formula:

$$(3) \quad P_{cr} = \pi^2 EI / L_e^2$$

Where E is the elastic modulus in MPa, I is the minimum moment of inertia of the cross section in mm⁴, and L_e is the effective length of the member in mm [3].

3.0 PROPOSED DESIGN

When determining the design of the truss bridge, the main objective was to create the most efficient truss system possible, that would hold as much weight without being too heavy. In order to achieve this goal, various truss designs were examined, and an adaptation of the Warren bridge truss design was chosen based on its overall symmetry and redundancy.

The final design chosen for the Warren bridge has a height of 200 mm, a width of 150 mm, and an unsupported span of 750 mm between its simple supports. A triangular pattern was used for a stable geometry. Gusset plates were added at all joints in order to stabilize the structure.

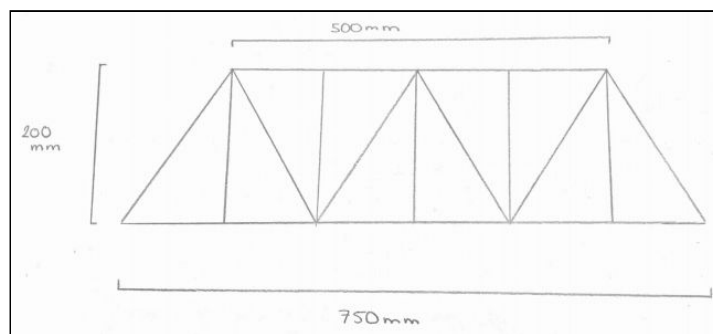


Figure 1. Front View of Truss Design

To reduce buckling, some pieces of wood (gusset plate type) were used in the top and bottom chords of the truss bridge. As seen in Figure 2, the wood was placed in a way to

increase the moment of inertia and the resistance to the rotation. The reasoning behind this orientation was for I_z to be large, and for the cross section to spread away from the z-axis.

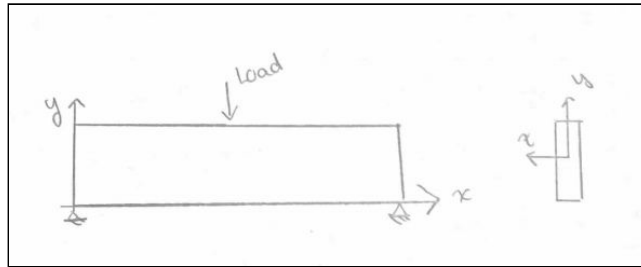


Figure 2. Orientation of Wood for Bottom and Top Chords

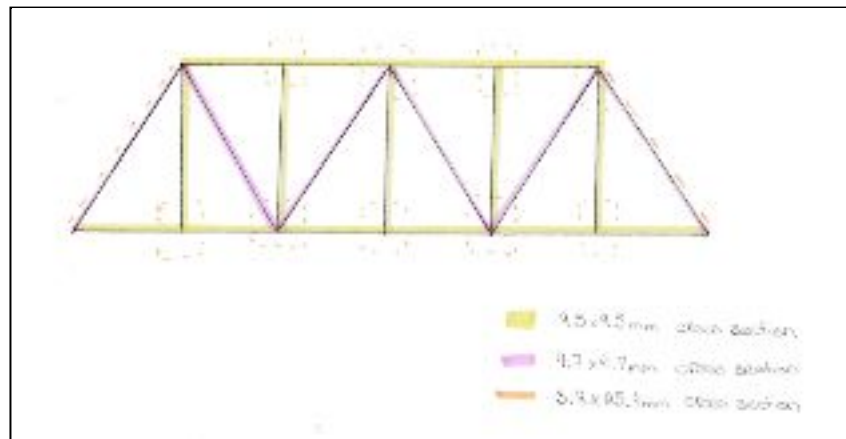


Figure 3. Cross Sections of Wood Pieces Chosen for Design

In addition, lateral bracings were placed along the top and bottom of the truss, as seen in Figure 4. This bracing joined each section of the frame. Because these pieces were used mainly for stability, the wood with the smallest cross-sectional area was used. This decision was also done to reduce the total weight of the truss bridge.



Figure 4. Top View of Bridge Design

4.0 MATERIALS AND EQUIPMENT

The material and equipment used in the construction of the modified Warren bridge are:

- Balsa Wood (four types of pieces)
 - 5 sticks of balsa wood (9.5mm x 9.5mm x 914mm)
 - 5 sticks of balsa wood (4.7mm x 4.7mm x 914mm)
 - 5 sticks of balsa wood (3.2mm x 3.2mm x 914mm)
 - 5 sticks of balsa wood (3.2mm x 25.4mm x 914mm)
- X-Acto Knife
- Saw
- Measuring Tape
- Gorilla Extra Strength Wood Glue
- Clamps
- Universal Testing Machine (Galdabini)
- Sandpaper
- Scale

5.0 ANALYSIS

The first step in analysing a truss is to know whether the proposed design is statically determinate or indeterminate. The following equations can be used to determine the state of a truss: :

$$(4) \quad m + r = 2j \quad \text{statically determinate}$$

$$(5) \quad m + r > 2j \quad \text{statically indeterminate}$$

Where m is the number of internal forces, r is the number of external support reactions and j is the number of joints.

As shown in Figure 5, the modified truss bridge has 21 internal forces, three external support reactions, and 12 joints. Using the equations provided above, the truss design is proven to be statically determinate:

$$21 + 3 = 2(12).$$

There are two methods for analyzing statically determinate trusses: the method of joints and the method of sections. During the design of this truss bridge, the method of joints was used to calculate the forces acting on each member.

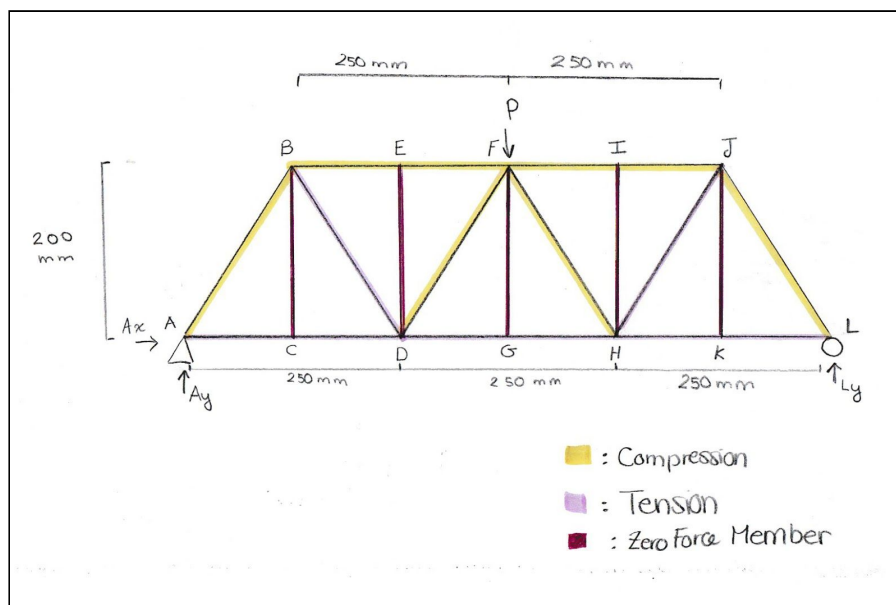


Figure 5. Truss Analysis

Assumptions

Due to symmetry, all members right of the line of symmetry formed by member FG, will be equal to their corresponding members on the left side. Moreover, members BC, ED, FG, IH, and JK form zero-force members.

When analyzing the truss, the external forces were calculated first. Table 1 shows the external forces acting on the truss.

Table 1. External Forces of the Truss

External Forces (N)
$A_x = 0$
$A_y = 0.5 P$
$L_y = 0.5 P$

Sample Calculations for the External Forces

$$(6) \quad \Sigma F_x = A_x = 0$$

$$(7) \quad \Sigma M_A = L_y(750m) - P(375m) = 0$$

$$L_y = 0.5PN$$

$$(8) \quad \Sigma F_y = A_y + L_y - P = 0$$

$$A_y = 0.5PN$$

After evaluating all external forces, the internal forces of each member of the truss were calculated using the Method of Joints. The results obtained are shown in Table 2.

Table 2. Internal Forces obtained by the Joint Method

Member Forces	Value of Force in Member (N)
$F_{BC}, F_{ED}, F_{FG}, F_{IH}, F_{JK}$	0 (zero-force member)
F_{AB}, F_{LJ}	0.589P (C)
F_{AC-CD}, F_{LK-KH}	0.312P (T)
F_{BD}, F_{JH}	0.589P (T)
F_{BE-EF}, F_{JI-IF}	0.624P (C)
F_{DF}, F_{FH}	0.589P (C)
F_{DG-GH}	0.939P (T)

Sample Calculations for the Internal Force

$$(9) \quad \Sigma F_y = 0.5PN - F_{AB} \sin(58^\circ) = 0$$

$$F_{AB} = 0.589PN \text{ (C)}$$

$$(10) \quad \Sigma F_x = F_{AC} - F_{AB} \cos(58^\circ) = 0$$

$$F_{AC} = 0.312PN \text{ (T)}$$

6.0 DESIGN

After the initial analysis and preliminary design concept had been completed, the cross sectional area of each member in the truss had to be determined.. These values were calculated using a form of the stress equation $A = \sigma P$, and the properties of balsa wood. Three different cross-sectional areas were used to build the truss bridge. Table 3 shows the maximum force a member with a specific cross sectional area can withstand before it fails.

Table 3. Tension Member Capacity (N)

Cross Sectional Area (in ²)	$\frac{3}{8}'' \times \frac{3}{8}''$	$\frac{3}{16}'' \times \frac{3}{16}''$	$\frac{1}{8}'' \times \frac{1}{8}''$
P_{max} (N)	2127	531.6	236.2

Table 4 shows the chosen pieces of wood for each member that will be in tension. This decision was made by matching the piece of wood that could withstand a certain amount of force to a member than had a similar force. In general, members in tension fail due to the lack of strength against the internal stress they experience as a result of the applied load. In order to avoid tensile failure, the members in tension need to have sufficient cross-sectional area to ensure that the stress is less than their tensile strength.

Table 4. Design of Members in Tension

Member	Length (mm)	Force in Member (N)	Chosen Cross Sectional Area of member (in ²)	Corresponding Truss Load P based on yielding (N)	Corresponding Truss Load of Entire Bridge (N)
AC-CD, LK-KH	125 (2x)	0.312P (T)	$\frac{3}{8}'' \times \frac{3}{8}''$	6817	13 634
BD, JH	236	0.589P (T)	$3/16'' \times 3/16''$	903	1806
DG-GH	125 (2x)	0.939P (T)	$\frac{3}{8}'' \times \frac{3}{8}''$	2265	4530

Sample Calculations for Tension for Piece BD

$$P = F/P_{\max}, \quad P_{\max} = 0.589, \quad F = 531.6 \text{ N for } 3/16'' \times 3/16'' \dots P = 531.6/0.589 = 903 \text{ N}$$

Table 5 shows how the pieces of wood were picked for each member that will be in compression. This step was done by matching the piece of wood that could withstand a certain amount of force to the member than had a similar force. Members in compression fail due to a crushing or buckling. The longer the member, and the smaller its cross section, the higher its chance of buckling. Hence, members in compression should be kept short unless they are thick and heavy.

Table 5. Design of Members in Compression

Member	Force in Member (N)	Chosen Cross Sectional Area of Member (in ²)	Length (mm)	Corresponding Load P Based on Buckling (N)	Corresponding Load P for Entire Bridge (N)
AB, LJ	0.589P (C)	2 x 3/16" x 3/16" (& Gusset plates)	236	450.94	901.89
BE-EF-FI-I J	0.624P (C)	3/8" x 3/8"	125 (x4)	3214.8	6429.6
DF, FH	0.589P (C)	2 x 3/16" x 3/16"	236	450.94	901.89

Sample Calculations for Buckling (Compression)

$$(11) \quad P_{cr} = \pi^2 EI / KL_e^2$$

→ Piece BE:

$$E_{Balsa} = 3.71 \text{ GPa} [1] = 3710 \text{ MPa}, L = 125 \text{ mm}$$

K=0.5 single piece → fixed

$$I = b * h^3 / 12 = (9.525 \text{ mm})^4 / 12 = 685.92 \text{ mm}^4$$

$$P_{cr} = \pi^2 * 3710 * 685.92 / (0.5 * 125^2) = 3214.8 \text{ N}$$

→ Piece DF:

$$I = b * h^3 / 12 = 4.7625 (9.525)^3 / 12 = 342.96 \text{ mm}^4, L=236 \text{ mm},$$

K=0.5 → fixed due to glue

$$P_{cr} = \pi^2 * 3710 * 342.96 / (236^2 * 0.5) = 450.94 \text{ N}$$

As the cross sectional area increases and length decreases, the load that the member can sustain increases. Thus, this trend was incorporated into the design. Furthermore, looking at all the load constraints of the modified truss bridge, the predicted maximum load sustained during the load testing should be approximately 902 N.

Safety Factor

Another important measurement to consider in the design is the safety factor, which is used to calculate the load bearing capacity of the structure. This step is done using a safety factor value, in order to ensure safety when the bridge is put into use. The higher the safety factor value, the lower the event of structural failure [4]. For the case of a bridge, safety is a very important aspect, as failure could result in severe damage and human injuries.

$$(12) \quad N = \frac{F_{BD}}{F_{BA}}$$

where $F_{BD} = 902\text{N}$ is the yielding force on the member that failed (BD), and

$F_{BA} = 450.94\text{N}$ is the limiting load of the buckling force on member BA

$$N = \frac{902}{450.94} = 2.00$$

The final calculated safety factor is 2.00. According to theory seen in class, the higher the safety factor, the better the overall safety of the bridge. Although a factor of 2.00 is low, it is still considered acceptable [4].

7. CONSTRUCTION PROCESS

The materials were first gathered and counted to ensure that all the necessary items were collected. All pieces were inspected to ensure that they were defect-free. After determining the design, a measuring tape was used to measure the pieces of wood of the required lengths based on the dimensions of the bridge design shown in Figure 1. An X-Acto knife and a saw were used to cut each piece according to the desired dimensions. The measurements were used during the cutting process to keep track of how many of each kind of wood were needed, and to avoid exceeding the amount of wood provided. Each piece of wood cut was sanded with sandpaper to ensure a smooth surface. Each piece was labelled for easy reference when gluing them together at a later time.

Before construction all the pieces were laid out in their proper positions and there was a discussion about how to form the connections so that the proper angle for each triangle was maintained. Figure 6 shows the pieces organized in the design of the truss.



Figure 6. Members Organized in the Truss Design

After all the pieces were cut, they were joined together using wood glue. This step was done for every member twice to create the two frames for the bridge. The pieces were joined together using clamps to allow the glue to dry for 30 minutes. Clamps were used during the drying process to ensure that the glue dried properly at the joints.

After joining the main two frames separately, the gusset plates were cut to dimensions determined experimentally. For each joint, two gusset plates were glued to both sides. The total number of gusset plates used was 46.

To join the main two frames together, the 3.2mm x 25.4mm balsa wood pieces were attached across the zero force members. Lateral bracings were also included at the top of the bridge. Wood pieces were added at the side of bridge ensure that the bridge had strong supports at its sides. These pieces were attached just below the gusset plates holding the frame together. Shown in Figure 6, a frame was then built in a grid formation, at the top of the bridge, in order to form a plate for the load to be applied.

After the bridge was completed, excess glue was removed with sandpaper to reduce the total weight.

During the building of the truss, a few difficulties were encountered. First, all the pieces of wood used had to be sanded so that they were all the exact length, to 1 mm of exactness. Moreover, the pieces of wood were quite fragile so they could not be clamped too tightly, which resulted with less rigidness of the bridge. The limited number of wood pieces also influenced the overall design of the project, since some pieces were very fragile.

8. TESTING AND PERFORMANCE

First, the supports of the bridge were placed on the support of the machine. The load was applied as a concentrated load by the universal testing machine. The load was increased until the truss bridge failed. The truss was able to withstand 822 N before it failed. The bridge failed at joints H and J. As seen in Figure. 7, the bridge failed due to the glue unsticking from these joints, and slight fracture occurred along the top and bottom chords. This behaviour can be explained by the small cross section areas and low strength in members HJ and BD. Moreover, the bridge had less lateral bracing on the side extremities of the top and bottom chords, which provided less support and resistance to the load applied, ultimately causing upwards bending on one side of the structure. Moreover, in actual conditions, the joints could have experienced moments.

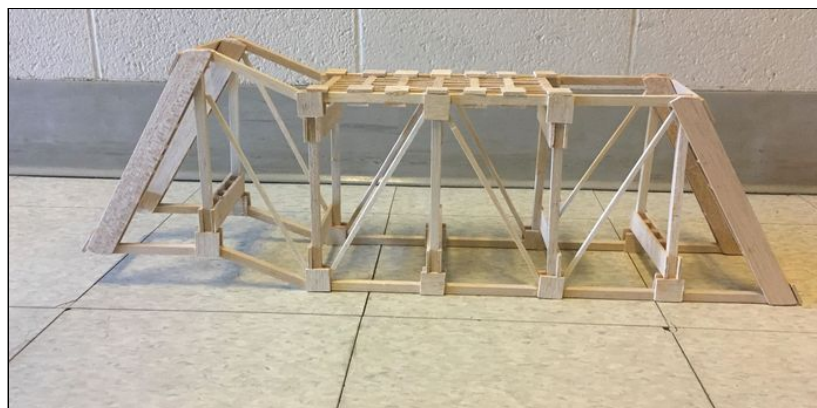


Figure 7. Bridge Failure

This failure could have been avoided by increasing the cross sectional area of members BD and JH, and adding additional bracing at the side extremities of the top and bottom chords. Unfortunately, due to constraints related to the amount of material, the bridge was less effective in supporting the applied load.

9. CONCLUSION

In conclusion, a truss bridge was designed and built based on the theory learned in class. The expected load the bridge could take was 902 Newtons, and the experimental load sustained was 822 Newtons. This difference can be explained by the fact that in actual conditions, the joints could have experienced moments. Moreover, the load was not applied at a single point as considered in the analysis, so the actual distribution varied, and theoretical zero-force members actually provided support. There was also no way to factor in the strength the glue adds to balsa wood. The safety factor was calculated using the theoretical limiting load and the load sustained by the failing member. The safety factor was calculated to be 2.00. This value is acceptable, as anything lower than 2 would be considered unsafe, however a higher safety value in real application would be preferred [4].

The final bridge dimensions were 750mm x 200mm x 150mm. The bridge weighed 283g and the load to weight ratio was 2.9.

The size and material constraints were followed in the construction of the bridge. A full analysis of the forces acting on the bridge was conducted before the construction process began, in order to optimize the design.

To improve the results obtained in the bridge testing, more lateral bracing should be added to the top and bottom chords, and a better method of connecting the joints should be considered.

To summarize, this project was successful in learning all aspects of the engineering design process. This project allowed students to be exposed to the full engineering design

process, which included designing their truss bridge, analyzing the bridge using methods seen in class, and then building and testing the bridge.

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