Mathematics 342, Term 2, 2018. Instructor: Reichstein.

Problem Set 5. Due Tuesday, March 27

(1) Let $q = 7$ and C be a q-ary code of length 6 with parity check matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}.$$

Assuming that at most one error occurred in transmission, decode the following received words, if possible:

(a) $(1, 1, 1, 1, 1, 0)$,

(b) $(0, 0, 0, 0, 1, 1)$,

(c) $(3, 5, 0, 2, 4, 6)$.

Hint: Use a decoding scheme similar to the one in Example 7.12 in the book.

(2) Problem 7.11. Hint: Follow the solution outline in the back of the book.

(3) Find the minimum distance of the ternary linear code C with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{pmatrix}.$$

(4) Construct a generator matrix and a parity check matrix for the ternary Hamming code Ham(2, 3).

(5) Assume a codeword $\mathbf{x}$ from for the ternary Hamming code Ham(2, 3) was sent and the word $\mathbf{y}$ was received. Use the partiy check matrix you constructed in Problem 4 to decode $\mathbf{y}$ in each part using syndrome decoding:

(a) $\mathbf{y} = (1, 1, 1, 0)$,

(b) $\mathbf{y} = (2, 2, 2, 2)$,

(c) $\mathbf{y} = (1, 2, 1, 2)$.

(6) Find the minimum distance of the binary linear code C with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

Show that $C \neq C^{\perp}$ but that C is equivalent to $C^{\perp}$.

(7) Recall that a linear code C is self-dual if $C = C^{\perp}$. Show that the extended binary Hamming code Hâm(3, 2) is self-dual.

(8) Problem 8.10. Assume $q \geq 3$ is a prime.

Hint: Follow the solution outline in the back of the book.