Problem set 4. Due Tuesday, March 13

Mathematics 342, Term 2, 2018. Instructor: Reichstein.

There are 8 problems on this assignnment. Don't miss Problems 7 and 8 on page 2.

(1) Suppose C is a binary linear code of length 6 with generator matrix

$$G = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Find the minimal distance of C.

(2) Give an example of a linear code C which does not have a generator matrix in standard form.

(3) Show that if a linear code C has a generator matrix G in standard form, such a generator matrix is unique. In other words, if G' is a generator matrix in standard form for C then $G' = G$.

(4) Let $q = 5$ and consider the q-ary linear code C of length 4, consisting of all words (a_1, a_2, a_3, a_4) such that $a_1 + 2a_2 + 3a_3 + 4a_4 \equiv 0 \pmod 5$. Show that this is a linear code. Find a generator matrix in standard form for this code.

(5) Construct a standard array for the binary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

Use this array to decode 01010 and 11101.

(6) Construct a standard array for the ternary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Using this array to decode 1211 and 2102.

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(7) Let $q = 11$ and C be the q-ary linear code with parity check matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{pmatrix}.$$

(a) What is the minimal distance of this code?

(Recall that Problem 5 on Midterm 1 asked you to show that $d(C) \leq 3$. This question is asking for the exact value of $d(C)$.)

(b) Suppose a word $\mathbf{x}$, transmitted using this code, is received as

$$\mathbf{y} = (1, 1, 0, 0, 0, 0, 0, 0, 0, 0).$$

Assuming at most one error could occur in transmission, find $\mathbf{x}$. Explain your answer.

(8) Let C be the binary linear code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

How many cosets does C have? Find the coset leaders by going through words of small weight and calculating their syndromes until you exhaust all possible syndromes. Decode received words (a) 1010101, (b) 1110001, (c) 0111111.