HW 1

Due: 23:59, Thursday, May 24, 2018

CS ID 1: _____

CS ID 2: _____

Instructions:

1. Do not change the problem statements we are giving you. Simply add your solutions within the space that we've allotted by editing this latex document.
2. If you need more space, add a page between the existing pages using the `\newpage` command.
3. Export the completed assignment as a PDF file for upload to gradescope.
4. On gradescope, upload only **one** copy per partnership. (Instructions for uploading to gradescope will be posted on the assignments page of the course website.)

Academic Conduct: I certify that my assignment follows the academic conduct rules for of CPSC 121 as outlined on the course website. As part of those rules, when collaborating with anyone outside my group, (1) I and my collaborators took no record but names away, and (2) after a suitable break, my group created the assignment I am submitting without help from anyone other than the course staff.

1. (12 marks) One way to better understand a computational system is to look at the minimum set of primitives (simple operations) that are sufficient to express all the tasks performed by the system. In this question, you will prove that every truth table can be implemented by a circuit that uses only binary (2-input) `NOR` gates. To do that you need to show the following steps:

   (a) (3 marks) Show that $\neg$ can be simulated using a `NOR` gate. That is, design a circuit of `NOR` gates, that takes as input a signal x, and whose output is $\neg x$.

   (b) (3 marks) Show that $\wedge$ can be simulated using `NOR` gates. That is, design a circuit whose only gates are `NOR` gates, that takes as inputs two signals x and y, and whose output is $x \wedge y$. (Hint: take advantage of what you learned in the previous part!)

(c) (3 marks) Show that $\vee$ can be simulated using `NOR` gates. That is, design a circuit whose only gates are `NOR` gates, that takes as inputs two signals x and y, and whose output is $x \vee y$. (Hint: take advantage of what you learned in the previous parts!)

(d) (3 marks) Now argue that any logic function that is represented by a truth table over k propositional variables can be implemented with a circuit that uses only binary `NOR` gates.

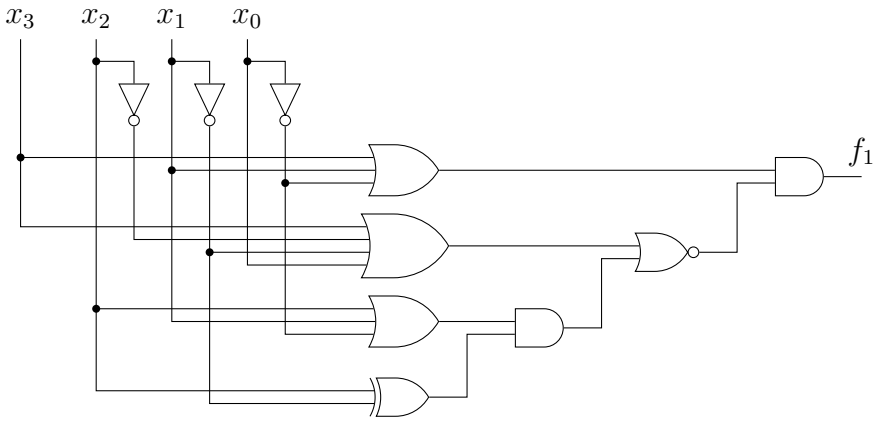
2. (8 marks) The truth table below defines the truth value of s for each combination of truth values of p, q and r.

p	q	r	s
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	F
T	T	F	T
T	T	T	F

(a) Find a logic formula for s that uses each variable at most twice. Then verify the correctness of your formula by drawing a truth table corresponding to this formula, including the truth values of all relevant sub-formulas. Enter your response in the table below. We have provided at least as many columns as you'll need. Be sure to fill in the column headings!

p	q	r					
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

(b) Modify the image below to create a circuit with inputs p, q and r, whose output is the value s described by the truth table. Your marks will depend on the number of gates you use, and fewer is better!



3. Consider the following logical expressions and circuits:

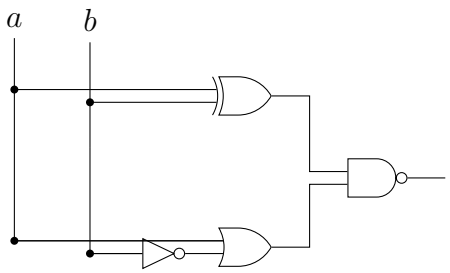
   i) $(c \vee a) \rightarrow (c \vee b)$

   ii) $\neg a \vee b$

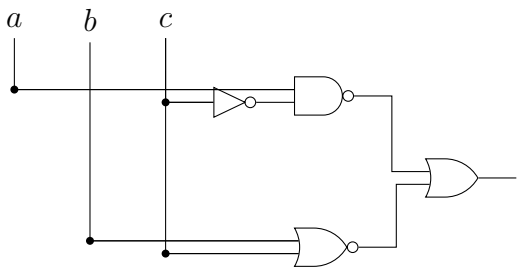
   iii) $c \vee (a \rightarrow b)$

   iv) $(a \wedge (b \vee c)) \rightarrow (a \wedge c)$

   v)



   vi)



Each proposition or circuit is logically equivalent to exactly one other proposition or circuit. In each of the three sections below, write down an equivalence and write a proof in the space provided. Each proof must use a sequence of the logical equivalences listed in the "Logical Equivalence Laws" and "Implication" sections of the 121 "official" formula list shown at http://www.ugrad.cs.ubc.ca/~cs121/2018S/handouts/formulasheet/formulasheet.pdf, **not a truth table**. (You can assume that $x \rightarrow y \equiv \neg x \vee y$). You may also use the abbreviations for the rule names which are shown on this handout.

(a) _____≡_____

expression 1		expression 2	reason
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		

(b) _____≡_____

expression 1		expression 2	reason
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		

(c) _____≡_____

expression 1		expression 2	reason
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
	≡		
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	≡		
	≡		
	≡		
	≡		
	≡		

4. Design a circuit that takes three inputs a, b, and c and returns the value that the majority of inputs have (this is called a majority vote).

   (a) Show the boolean formula for this problem and explain how you found it.

   (b) Draw the circuit.

5. This puzzle is adapted from Shakespeare's *The Merchant of Venice*. Portia wants to find a very intelligent husband. She does not care about his fortune, but she does care about his brains, so she asks him to play the following game with her: on the table she places a gold box, a silver box, and a lead box. Inside one of the boxes is an engagement ring. She asks her boyfriend to choose one box. If he chooses the one with the ring, she will marry him. Otherwise, they will break up. The three boxes have the following labels:

   - GOLD: The ring is not in this box
   - SILVER: The ring is in this box
   - LEAD: The ring is not in the silver box

   Portia explains to her boyfriend that at most one of the three statements is true. Which box should her boyfriend choose if he wants to marry Portia? Prove that your choice is correct by arguing that each of the other choices will lead to an impossibility.