Sol. Attached

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	201	All
Examination	**Date**	**Duration**
Alternate Midterm	29 October, 2016	1 h 30 min
Special Instructions:	Only approved calculators are allowed **Show all your work for full marks.**	

1. [12] (a) A line segment has the endpoints (-3,5) and (8,-5). Find the distance between the midpoint of the segment and the origin $(0,0)$.

(b) Find the equation of the line passing through the point $(6,0)$ and which is perpendicular to the line segment indicated in (a) above.

(c) Write the equation of the circle with the center at $(1,-1)$ and passing through the point $(4,3)$. (Hint: find first the radius of the circle.)

2. [9] Consider the quadratic function $f(x) = -2x^2 + 4x + 7$.

(a) Express $f(x)$ in standard form.

(b) Find its vertex and indicate is it the maximum or the minimum of f.

(c) Find the x- and y-intercepts.

3. [5] Consider the functions $f(x) = -2 + \sqrt{x^2+4}$ and $g(x) = \sqrt{x-3}$.

Find the domain and the range of f.

Find the function $f \circ g$ and determine its domain.

4. [12] Find the solutions of the following equations:

(a) $2^{2x} + 2^{x+2} = 3$

(b) $\log_3(2x+1) - \log_3(x-1) = 2$

(c) $\log_2(4x) + \log_2(x-3) = 4$

5. [6] Given the function $F(x) = \dfrac{2x^3 - 8x}{(x^2+3x+2)\sqrt{x^2+1}}$, find

(a) all horizontal asymptotes, and (b) all vertical asymptotes of $F(x)$.

6. [6] Consider the function $f(x) = \dfrac{2x+1}{4x+3}$.

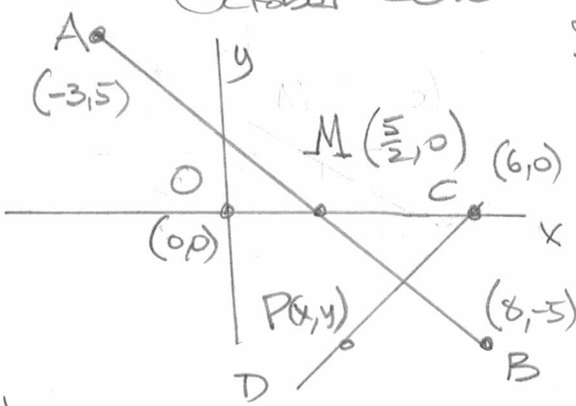
(a) Find the inverse function $f^{-1}(x)$.

(b) Find the domain and the range of f, and the domain and range off^{-1}.

Bonus. [3]: Let we know that some function $f(x)$ is invertible, the inverse $f^{-1}(x)$ has the domain $(-\infty,\infty)$ and the range $(-1,1)$. Can we say wether $F(x) = f^2(x)$ is invertible or not? Explain why yes or why no.

Math 201 Alternate Exam
October 2016

1 a)



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Step 1 Mid pt M_{AB} : $x = \frac{x_1+x_2}{2} = \frac{-3+8}{2} = \frac{5}{2}$

$y = \frac{y_1+y_2}{2} = \frac{5+(-5)}{2} = 0$

Step 2 distance OM:

$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(\frac{5}{2}-0)^2+(0-0)^2}$

$= \frac{5}{2}$

b) Step 1 ① Find slope AB:

$m_{AB} = \frac{y_2-y_1}{x_2-x} = \frac{-5-5}{8-(-3)} = \frac{-10}{11}$

② $\Rightarrow$ slope CD $= \frac{11}{10}$

4

Step 2 Equation CD. Let P(x,y) be any pt on line CD

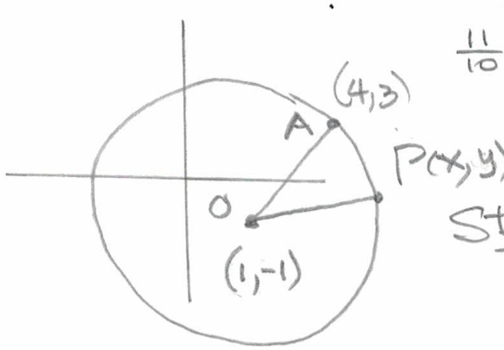
$m_{CP} = \frac{y-0}{x-6}$

$\frac{11}{10} = \frac{y}{x-6} \Rightarrow 10y = 11(x-6)$

$y = \frac{11}{10}(x-6)$

$y = \frac{11}{10}x - \frac{66}{10}$

c)



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Step 1 Radius OA $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-1)^2+(3-[-1])^2}$

$= \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

Step 2 Equation: Let P(x,y) be any pt on circle

$OP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$5 = \sqrt{(x-1)^2+(y-[-1])^2}$

$25 = x^2-2x+1+y^2+2y+1$

$x^2+y^2-2x+2y-23=0$

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2. a) $y = -2x^2+4x+7$

$y = -2(x^2-2x+\underline{1}) + 7 + \underline{(2)(1)}$

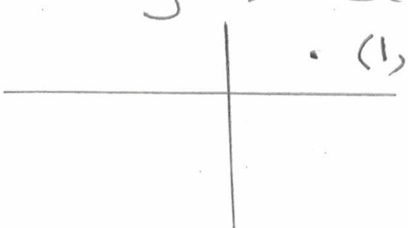
3

$y = -2(x-1)(x-1)+9$

$y = -2(x-1)^2+9$

OR $y-9 = -2(x-1)^2$

· (1,9) is vertex



3 b) opens down $\Rightarrow$ Vertex is MAX.

3 c)

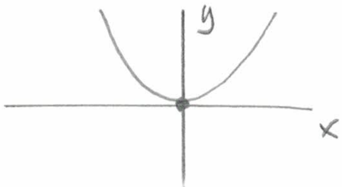
x int (y=0)	y int (x=0)
$-2x^2+4x+7=0$ $x = \frac{-(+4)\pm\sqrt{4^2-4(-2)(7)}}{2(-2)}$	$y=7$

$x = \frac{-4\pm\sqrt{16+56}}{-4}$

$x = \frac{-4\pm\sqrt{72}}{-4} = \frac{-4\pm6\sqrt{2}}{-4}$

$x = 1 \pm \frac{3\sqrt{2}}{4}$

3. (i) Domain $f(x)$: $x^2+4 \geq 0 \Rightarrow x^2 \geq -4 \Rightarrow$ No x Restriction



$\Rightarrow$ Domain $x \in \mathbb{R}$ or $(-\infty, \infty)$

$\Rightarrow$ Range $y \in \mathbb{R} \mid y \geq 0$ or $[0, \infty)$

(ii) $g(x) = \sqrt{x-3}$

5

$f(x) = -2 + \sqrt{x^2+4}$

$\Rightarrow f(g(x)) = -2 + \sqrt{(g(x))^2+4} = -2 + \sqrt{(\sqrt{x-3})^2+4}$

$= -2 + \sqrt{x-3+4}$

$= -2 + \sqrt{x+1}$

$\Rightarrow$ DOMAIN : $x+1 \geq 0 \Rightarrow x \geq -1$

$\Rightarrow$ DOMAIN $x \in \mathbb{R} \mid x \geq -1$

OR $[-1, \infty)$

4. a) $2^{2x} + 2^{x+2} = 3$

$(2^x)^2 + 2(2^x) - 3 = 0$

Same Form as Quadratic when $2^x = A$

$A^2 + 2A - 3 = 0$

$(A-1)(A+3) = 0$

$A-1=0$	$A+3=0$
$A=1$	$A=-3$

4

$\Rightarrow$

$2^x = 1$	$2^x = -3$
$2^x = 2^0$	No x
$x = 0$	

b) $\log_3(2x+1) - \log_3(x-1) = 2$

$\log_3 \frac{2x+1}{x-1} = 2$

$\frac{2x+1}{x-1} = 3^2$

4

$\frac{2x+1}{x-1} = \frac{9}{1}$

$9(x-1) = 1(2x+1)$

$9x - 9 = 2x + 1$

$9x - 2x = 1 + 9$

$7x = 10$

$x = \frac{10}{7}$

c) $\log_2 4x + \log_2 (x-3) = 4$

$\log_2 4x(x-3) = 4$

$4x(x-3) = 2^4$

4

$4x^2 - 12x - 16 = 0$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x+1=0$	$x-4=0$
$x=-1$	$x=4$
DISCARD	

Domain of log func > 0

5. VA : b) $(x^2+3x+2)\sqrt{x^2+1} = 0$

$x^2+3x+2=0$	$\sqrt{x^2+1}=0$
$(x+2)(x+1)=0$	No x

$x+2=0$	$x+1=0$
$x=-2$	$x=-1$
Since numerator also $=0$ when $x=-2$, $x=-2$ is Not a VA	$x=-1$ is the only VA

a) HA

$$\lim_{x\to\infty} \frac{2x^3-8x}{(x^2+3x+2)\sqrt{x^2+1}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3}-\frac{8x}{x^3}}{\left(\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{2}{x^2}\right)\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{2-\frac{8}{x^2}}{\left(1-\frac{3}{x}+\frac{2}{x^2}\right)\left(\sqrt{1+\frac{1}{x^2}}\right)} = \frac{2-0}{(1-0+0)\sqrt{1+0}}$$

$\Rightarrow y=2$ is HA.

6. a) $y = \dfrac{2x+1}{4x+3}$

(3)

$$\frac{x}{1} = \frac{2y+1}{4y+3}$$

$$1(2y+1) = x(4y+3)$$

$$2y+1 = 4xy+3x$$

$$2y-4xy = 3x-1$$

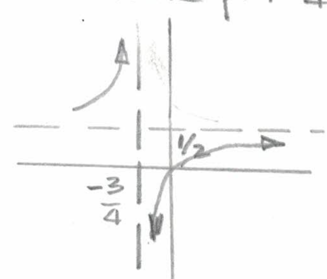
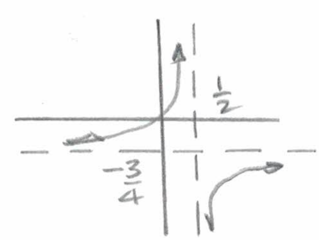
$$y(2-4x) = 3x-1$$

$$y = \frac{3x-1}{2-4x}$$

or $f^{-1}(x) = \dfrac{3x-1}{2-4x}$

b)

(3)

f	f^{-1}
Domain: $4x+3\neq 0$, $x\neq -\frac{3}{4}$	Domain: $2-4x\neq 0$, $-4x\neq -2$, $x\neq \frac{1}{2}$
Domain $x\in R \mid x\neq -\frac{3}{4}$	Domain $x\in R \mid x\neq \frac{1}{2}$
	
VA: $x=-\frac{3}{4}$	VA $x=\frac{1}{2}$
HA: $y=\frac{1}{2}$	HA $y=-\frac{3}{4}$
Range $y\in R \mid y\neq \frac{1}{2}$	Range $y\in R \mid y\neq -\frac{3}{4}$

Bonus

(3)

Range f^{-1} is Domain f

$\Rightarrow$ Domain $f=(-1,1)$

Domain f^{-1} is Range of f

$\Rightarrow$ Range f $(-\infty,\infty)$

$\Rightarrow$ Range of $f^2=[0,\infty) \Rightarrow$ Not 1-1 $\Rightarrow$ Not Surject