Introduction to Semantics and Pragmatics

Roman Numerals and their Interpretation

Solutions Problem Set 1

Let RN be the set of Roman numerals which can be expressed in terms of the symbols in the set N, that is, the set $\{I, V, X, L, C, D, M\}$, and which include the expressions *IIII*, *VIIII*, *XXXX*, *LXXXX*, *CCCC* and *DCCCC*, but exclude the expressions *IV*, *IX*, *XL*, *XC*, *CD* and *CM*. (Indeed, the replacement of the former expressions by the latter was a late innovation in the Roman numeral notation.)

1. (2 points)
None of the symbols *V*, *L* or *D* may occur more than once. None of the symbols *I*, *X*, *C* or *M* may occur more than four times. And given the following ranking of the symbols, no symbol more lowly ranked may occur to the left of any symbol more highly ranked. Here is the ranking: $M > D > C > L > X > V > I$, that is, *M* outranks *D*, which outranks *L*, which outranks *X*, which outranks *V*, which outranks *I*.

2. (2 points)
The purported recursive definition of RN:

(1.1) If $x \in$ N, then $x \in$ RN;
(1.2) If $y \in$ RN and $z \in$ RN, then $yz \in$ RN;
(1.3) Nothing else is a member of RN.

includes in RN such expressions as *VX* and *IIIII*, which are not Roman numerals.

3. (2 points)
Let the symbols in N be ranked as follows: $M > D > C > L > X > V > I$, that is, *M* outranks *D*, which outranks *L*, which outranks *X*, which outranks

V, which outranks I.

(2.1) If $x \in$ N, then $x \in$ RN;
(2.2) If $y \in$ RN and $z \in$ RN, no symbol in z outranks any symbol in y and, in y and z taken together, V, L and D occur at most once and I, X, C and M occur at most four times,
then $yz \in$ RN;
(2.3) Nothing else is a member of RN.

4. (2 points)
We define the function v as follows:

(3.1) $v(I) = 1, v(V) = 5, v(X) = 10, v(L) = 50, v(C) = 100, v(D) = 500$ and $v(M) = 1000$;
(3.2) If (1) $y \in$ RN and $z \in$ RN, (2) no symbol in z outranks any symbol in y and (3) in y and z taken together, V, L and D occur at most once and I, X, C and M occur at most four times,
then $v(yz) = v(y) + v(z)$.

5. (2 points)
RN has a numeral, namely *MMMMDCCCCLXXXXVIIII*, whose value is greater than that of any other numeral in RN, relative to the valuation rule.
Its value is 4999.