Tutorial Session 6- Fall 2017

Mid Term Review

Review: Basic Formulas

- Simple Interest:
  - $FV_n = P + [n \times (P \times k)]$
    - n: number of years
    - P: principal
    - k: interest rate

Interest for n years

- Compounding:
  - $FV_n = PV_0 \times (1 + k)^n$

CVIF:
Compound Value
Interest Factor

- Discounting:
  - $PV_0 = FV_n \times [1/(1 + k)^n]$

PVIF:
Present Value Interest
Factor

Review: Basic Formulas

- Number of Years (calculated using the compounding formula):
- (Formulas not given in the final Equation List, but we learned how to get them using the basic formula so no need for memorizing!)

  - $n = Ln(FV_n / PV_0) / Ln(1 + k)$

- Interest Rates (calculated using the compounding formula):

  - $k = [FV_n / PV_0]^{(1/n)} - 1$

Review: Basic Formulas

- Equations related to Normal Annuity:

$$FV_n = PMT\left[\frac{(1+k)^n - 1}{k}\right]$$

$$PV_0 = PMT\left[\frac{1 - \frac{1}{(1+k)^n}}{k}\right]$$

PVAF:
Present Value
Annuity
Factor

Review: Basic Formulas

- Equations related to Annuity Due:(Formulas not given in the final Equation List)

$$FV_n = PMT\left[\frac{(1+k)^n - 1}{k}\right](1+k)$$

$$PV_0 = PMT\left[\frac{1 - \frac{1}{(1+k)^n}}{k}\right](1+k)$$

Review: Basic Formulas

- Perpetuity:

$$PV_0 = \frac{PMT}{k}$$

- Growing Perpetuity:

$$PV = \frac{C_1}{k - g}$$

- Growing Annuity:

$$PV = \frac{C_1}{k - g}\left[1 - \left(\frac{1+g}{1+k}\right)^n\right]$$

Review: Basic Formulas:

- Formula used to calculate the effective rate for a specific period given a compounding interval:
  - QR: Quoted Annual Rate
  - m: compounding frequency
  - f: frequency of payments per year

Effective rate $k = (1 + \frac{QR}{m})^{\frac{m}{f}} - 1$

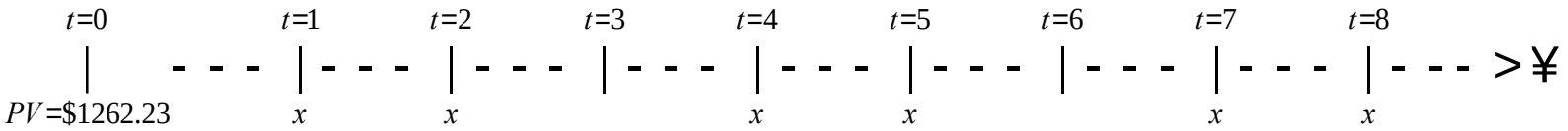
$$\frac{QR}{m} = effective\ rate$$

Example: $\frac{QR_{compounded\ monthly}}{12} = effective\ monthly\ rate$

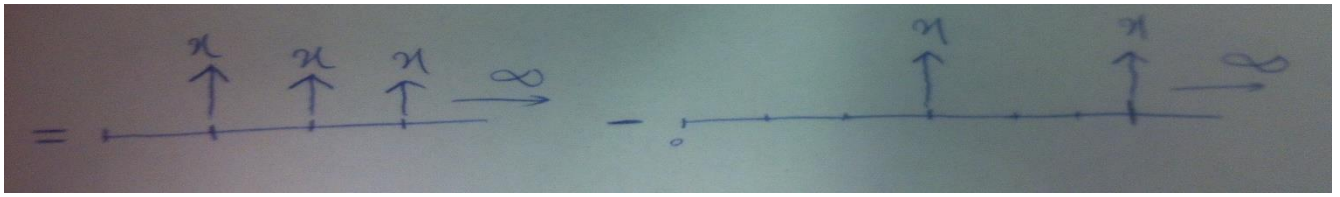
Example: $ESR = (1 + EMR)^{\frac{12}{2}} - 1$

(Summer 2 2014 – Q4)

The present value (t = 0) of the following cash flow stream is $1,262.23 when discounted at 12 percent annually. What is the value of the missing cash flows (x)? Assume the pattern of payments will continue in perpetuity (Every third year there is no payment).



A. $39.47
B. $151.47
C. $195.67
D. $215.26
E. $511.11



Effective 3 year rate $= (1.12)^3 - 1 = 0.404928$

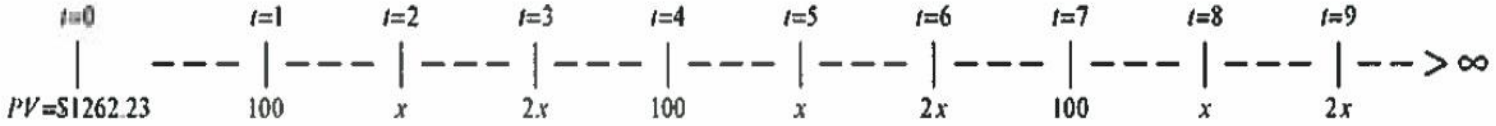
$PV_0 = 1262.23 = \frac{x}{0.12} - \frac{x}{0.404928} = x(8.26376)$

$\therefore x = \$215.26$

Answer: D

(Winter 2015 – Q5)

The present value (t = 0) of the following cash flow stream is $1,262.23 when discounted at 12 percent (EAR). What is the value of the missing cash flows (x)? Assume the pattern of payments continues in perpetuity.



A. $48.35

B. $100.00

C. $123.60

D. $300.00

E. None of the above

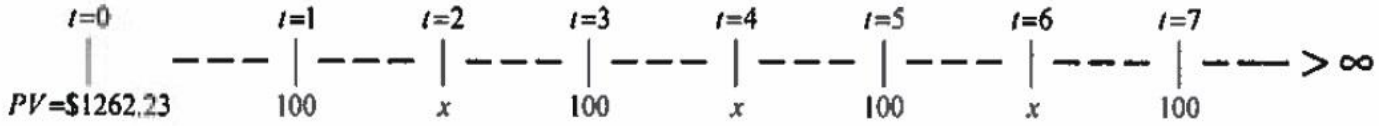
$\mathcal{E}_{3\text{year}} = (1.12)^3 - 1 = 40.492928\%$

$1262.23 = \frac{100}{0.404928} \times 1.12^2 + \frac{x}{0.404928} \times 1.12 + \frac{2x}{0.404928}$

$x = 123.60$

Summer1-2014/Q4

The present value (t = 0) of the following cash flow stream is $1,262.23 when discounted at 12 percent annually. What is the value of the missing cash flows (x)? Assume the alternating payment of $100 and $x continues in perpetuity.



A. $39.47

B. $190.94

C. $195.67

D. $209.11

E. $221.11

Effective 2 year rate: $k = 1.12^2 - 1$

$= 25.44\%$

$\frac{100}{0.2544} * (1.12) + \frac{x}{0.2544} = 1262.23$

$\therefore x = \$209.11$

Problem

A Canadian firm bought a new sewing machine for $100,000 which is in asset class 43 with a capital cost allowance (CCA) rate of 30 percent and useful life of five years. Calculate the amount of CCA associated with this asset in the fourth year.

A. $12,495

B. $10,290

C. $30,400

D. $41,650

Answer: A

UCC Beg year4 = 100,000(1-0.3/2)(1-0.3)^2 = 41,650

CCA year 4 = 41,650*30% = 12,495

Problem

Which of the following is the **most** correct?
_____ know their exposure is limited to the amount of capital they invest in the company.

A. Shareholders
B. Sole proprietors
C. General and limited partners
D. Limited partners and shareholders

Answer: D

Problem

Dana Inc. paid $4 million in dividends this year. Earnings are expected to grow by 15% each year for the next 10 years and 5% thereafter. Shareholders require a return of 10%. If there are 25 million shares outstanding, what is the estimated price per share?

A. $7.30

B. $8.91

C. $9.13

D. $9.78

Answer A

$$V_{equity} = \frac{4(1.15)}{(0.10-0.15)}\left[1-\frac{1.15^{10}}{1.10^{10}}\right]+\frac{4(1.15)^{10}(1.05)}{(.10-.05)(1.10)^{10}}$$

$$V_{equity} = \$51,495,863 + \$131,017,961 = 182,513,824$$

$$\text{Price per share} = \$182,513,824 \div 25,000,000$$

$$\text{Price} = \$7.30$$

(Summer 2 2014 – Q7)

Suppose someone offered you the choice of two equally risky annuities, each paying $10,000 per year for five years. One is an ordinary annuity, the other is an annuity due. Which of the following statements is most correct?

A. The present value of the ordinary annuity must be less than the present value of the annuity due, and, the future value of an ordinary annuity must be less than the future value of the annuity due.

B. The present value of the annuity due exceeds the present value of the ordinary annuity, while the future value of the annuity due is less than the future value of the ordinary annuity.

C. The present value of the annuity due exceeds the present value of the ordinary annuity, and the future value of the annuity due also exceeds the future value of the ordinary annuity.

D. If interest rates increase, the difference between the present value of the ordinary annuity and the present value of the annuity due remains the same.

E. Statements a and c are correct.

Answer: E

(Summer 2 2014 – Q26)

You are considering two perpetuities which are identical in every way except for the when the perpetuity payments will begin. Perpetuity A will begin making annual payments of a fixed amount, with the first payment being made two years from today. Perpetuity B pays the same fixed annual payment, but will make the first payment one year from today. Which of the following statements is most correct?

A. The PV of perpetuity A is greater than the PV of perpetuity B by the amount of the fixed payment.
B. The PV of perpetuity B is greater than the PV of perpetuity A by the amount of the fixed payment.
C. The PV of perpetuity A is equal to the PV of perpetuity B.
D. The PV of perpetuity A is greater than the PV of perpetuity B by the present value of the amount of the fixed payment.
E. The PV of perpetuity B is greater than the PV of perpetuity A by the present value of the first payment of perpetuity B.

Answer: E

(Fall 2012, Question 3)

"We pay you $1,000 a year for 10 years and thereafter you will pay us $1,000 a year forever!" –reads the Highlander (who lives forever) in an ad. What must be the rate of interest (EAR) in order for this to be a fair deal (i.e. the rate of interest that makes the present value of these two series of cash flows equal). Assume that all payments occur at the end of the year, so the Highlander receives the first payment at the end of year one, and he makes his first payment at the end of year 11.

A. No such interest rate exists.

B. 0%.

C. 6.82979%.

D. 7.1773%.

E. 10%.

- Ans: D

$$\frac{1000}{k}\left[(1+k)^{10}-1\right]=\frac{1000}{k}$$

$$\therefore (1+k)^{10}-1=1$$

$$\Rightarrow k=7.1773\%$$

(Summer2 2012, Question 21)

- A buyer can afford no more than $500 per month in payments. The most favorable loan available in the market is a 30-year loan at 10% (APR compounded semi-annually). What is the maximum affordable house with a 10% down payment? (Pick the closest number)

- A. $55,000
- B. $57,959
- C. $64,399
- D. $65,679
- E. 180,000

  - Ans: C

$$k = \left(1 + \frac{0.1}{2}\right)^{\frac{2}{12}} - 1 = 0.008165$$

$$PV = \frac{500}{0.008165} * \left[1 - \frac{1}{1.008165^{360}}\right] = 57959.73$$

$$0.9 \times x = 57959.73 \Rightarrow x = 64399.69$$

(Summer1 2012, Question 4)

- Jane Doe Inc. invested $735,000 at an 11.25% rate of return (effective annual). The company sold their investment for $1,067,425. How much more would Jane Doe's selling price have been if they had waited another 1.5 years to sell their investment?

- A) $120,085.31
- B) $185,102.87
- C) $277,654.31
- D) $1,252,527.87
- E) Insufficient information

  - Ans: B

$$(1+0.1125)^{1.5} \times 1,067,425 - 1,067,425 = 185,102.87$$

(Summer1 2012, Question 15)

You are going to pay $800 into an account at the beginning of each of 20 years (First payment at t=0). The account will then be left to compound for an additional 20 years. At the end of the 41st year you will begin receiving a perpetuity from the account. If the account pays 14%, how much each year will you receive from the perpetuity (round to nearest $1,000)?

A) $140,000

B) $150,000

C) $160,000

D) $170,000

E) None of the above

- Ans: C

$$FV_{20} = \frac{800}{0.14} \times \left(1.14^{20} - 1\right) \times 1.14 = \$83014.734$$

$$FV_{40} = \$83014.734 \times 1.14^{20} = \frac{PMT_{41}}{0.14}$$

$$PMT_{41} = \$83014.734 \times 1.14^{20} \times 0.14 = 159,727.70$$

Winter 2015- Q11

For a given constant effective annual rate, the quoted rate _____ as the compounding frequency increases.

A. Does not change.

B. Increases.

C. Decreases.

D. It depends on the effective rate.

E. It depends on frequency of payments.

(Summer1 2012, Question 13)

Which of the following investments would provide an investor the highest effective annual return?

A) An investment which has a 9 percent quoted rate with semi-annual compounding.

B) An investment which has a 9 percent quoted rate with monthly compounding.

C) An investment which has a 9.2 percent quoted rate with annual compounding.

D) An investment which has an 8.9 percent quoted rate with monthly compounding.

E) An investment which has an 8.9 percent quoted rate with quarterly compounding.

- Ans: B (Its between B&C, now: $EAR_B = (1 + EMR)^{\frac{12}{1}} - 1$
- EMR=0.09/12, So EAR_B= 9.38%

Fall 2014-Q5

Which one of the following is true concerning amortized loans?

A. A loan where **annual** payment includes a part of the principal plus a fraction of the interest due.

B. A loan where **annual** payment includes all the interest due plus a fraction of the principal.

C. A loan where **monthly** payment includes all the interest due plus a fraction of the principal.

D. Both B and C, but not A

E. A, B, and C are correct.

(Fall 2012, Question 27)

Your company is planning to borrow $2,500,000. It will repay this loan in ten equal annual instalments (first payment at the end of year 1). The quoted rate is 9 percent (EAR). What fraction of the payment made at the end of the third year will represent repayment of principal?

A. 46.04%.
B. B. 50.19%.
C. C. 54.70%.
D. D. 59.63%.
E. E. 64.99%.

$$PMT = 2,500,000 \times 0.09 \times \left[1 - \frac{1}{1.09^{10}}\right]^{-1} = 389,550.2248$$

$$OB_2 = \frac{389,550.2248}{0.09} \times \left[1 - \frac{1}{1.09^{8}}\right] = \$2,156,090.03$$

$$Interest_3 = 0.09 \times 2,156,090.03 = \$194,048.10 \Rightarrow Principal_3 = \$195,502.12$$

$$Principal_3 / PMT = 50.19\%$$

Summer1-2014-Q1

Your company is planning to borrow $500,000 on a 5-year, 7 percent, annual payment, fully amortized term loan. What fraction of the payment made at the end of the fifth year will represent repayment of principal?

A. 76.29%

B. 42.82%

C. 50.28%

D. 49.72%

E. None of the above

what fraction of the last payment is principal?

If x is the principal

$$x + 0.07x = PMT$$

$$x = \frac{PMT}{1.07} \Rightarrow \frac{x}{PMT} = \frac{1}{1.07} = 93.46\%$$

(Winter 2012, Question18)

- How many years (to the nearest half year) will it take for $X to triple with a stated interest rate of 9.6% (APR compounded monthly)?

- A) 11 years.
- B) 11.5 years.
- C) 12 years.
- D) 12.5 years.
- E) Not enough information to answer the question.

- Ans: B

$$EAR = \left(1 + \frac{0.096}{12}\right)^{12} - 1 = 10.0339\%$$

$$3x = x \times (1.100339)^{n} \Rightarrow 3 = (1.100339)^{n}$$

$$n = \frac{\ln(3)}{\ln(1.100339)} = 11.49\,years \approx 11.5\,years$$

(Winter 2012, Question1)

You are offered the choice between receiving two annuities. Each pays $200 per year for T years (assume both are equally risky). The first is an annuity due. The second is a regular annuity. If you are a rational investor, which would you chose (assume that interest rates are greater than zero)?

A) The ordinary annuity

B) The annuity due

C) Either one, because they have the same present value.

D) Without information on the interest rate, we do not have enough information to decide.

E) Without information on T, we do not have enough information to decide.

- Ans: B

Problem

You are trying to plan for retirement in 25 years and currently you have $92,000.00 in a savings account and $105,500.00 in stock. In addition you plan on adding to your savings by depositing $6,000.00 per year in your SAVINGS account at the end of each of the next 10 years and then $6,000.00 per year at the end of each year for the final 15 years until retirement. Assuming your savings account returns 7% compounded annually while your investment in stocks will return 9% compounded annually, how much will you have when you retire in 25 years?

Problem - Solution

- This must be worked in multiple steps.
- Find the FUTURE VALUE for the SAVINGS account after the first 10 years. N=10, I=7, PV=92000, PMT=6000, FV=? **ANSWER: $263,876.61**
- Find the FUTURE VALUE for the SAVINGS account after the next 15 years. N=15, I=7, PV=263876.612638, PMT=6000, FV=? **ANSWER: $878,818.03**
- Find the FUTURE VALUE for the STOCK account after 25 years. N=25, I=9, PV=105500, PMT=0, FV=? **ANSWER: $909,735.01**
- Add all of the future values together => $878,818.03 + $909,735.01 = **$1,788,553.04**

Summer II, 2013, Q28

- Q28. Your subscription to Jogger's World Monthly is about to run out and you have the choice of renewing it by sending in the $10 a year regular rate or of getting a lifetime subscription to the magazine by paying $100. Your cost of capital is 7 percent. How many years would you have to live to make the lifetime subscription the better buy? Payments for the regular subscription are made at the beginning of each year. (Round up if necessary to obtain a whole number of years.):

A. 7 years.
B. 8 years.
C. 10 Years.
D. 16 years.
E. 18 years.

$$\frac{10}{0.07} * \left[1 - \frac{1}{1.07^n}\right] * (1.07) = 100$$

$$1 - \frac{1}{1.07^n} = \frac{100 \times 0.07}{1.07 * 10}$$

$$1 - \frac{1}{1.07^n} = 0.6542 \rightarrow -\frac{1}{1.07^n} = 0.6542 - 1 \rightarrow -\frac{1}{1.07^n} = -0.3457$$

Solution: D

$$\frac{1}{1.07^n} = 0.3457 \rightarrow 1.07^{-n} = 0.3457 \rightarrow -n \ln 1.07 = \ln 0.3457 \rightarrow n = 15.69$$

Summer1-2015/Q5

You deposit $1,000 in an account today. You will deposit $600 at the end of each month for the next 12 months and $800 each month for the following 12 months. How much interest will you have earned in two years if the account pays 5.5% compounded monthly?

a. $795.42
b. $827.65
c. $849.42
d. $962.57
e. $979.00

$$FV = 1{,}000\left(1+\frac{0.055}{12}\right)^{24} + 600\left\{\frac{\left(1+\frac{0.055}{12}\right)^{12}-1}{0.055/12}\right\}\left(1+\frac{0.055}{12}\right)^{12} + 800\left\{\frac{\left(1+\frac{0.055}{12}\right)^{12}-1}{0.055/12}\right\}$$

$$= 1{,}115.9976 + 7{,}800.8344 + 9{,}845.7356$$

$$FV = 18{,}762.5676$$

$$Cap = 17{,}800.0000 = 1{,}000 + 600(12) + 800(12)$$

$$Int = \$962.5676$$

Summer1-2015/Q7

The Friendly Bank wants to earn an effective rate of 9% on its auto loans. If interest is compounded monthly, what APR must they charge?

a. 8.44%
b. 8.58%
(c.) 8.65%
d. 9.17%
e. 9.38%

$$0.09 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$r = 0.0865 \quad (8.65\%)$$

Summer1-2015/Q11

Investment A makes annual payments of $813.73 for each of the next 10 years, while investment B makes annual payments of $500 per year forever. At what interest rate would you be indifferent between the two investments?

a. 9%
b. 10%
c. 11%
d. 12%
e. 13%

$$813.73\left\{\frac{1-\frac{1}{(1+r)^{10}}}{r}\right\} = \frac{500}{r}$$

$$\sqrt[10]{(1+r)^{10}} = \sqrt[10]{2.593727\ldots}$$

$$r = 0.10\ (10\%)$$

Summer II, 2013, Q5

You are going to deposit $800 into an account at the beginning of each year for the next 20 years (First payment is today). Starting in year 21, you will begin receiving perpetuity from the account. First payment from the perpetuity will be at the beginning of year 21. If the account pays 14% (EAR), how much will you receive in each year from this perpetuity?

A. $10,194.79
B. $11,622.06
C. $13,249.15
D. $94,636.80
E. $592,962.39

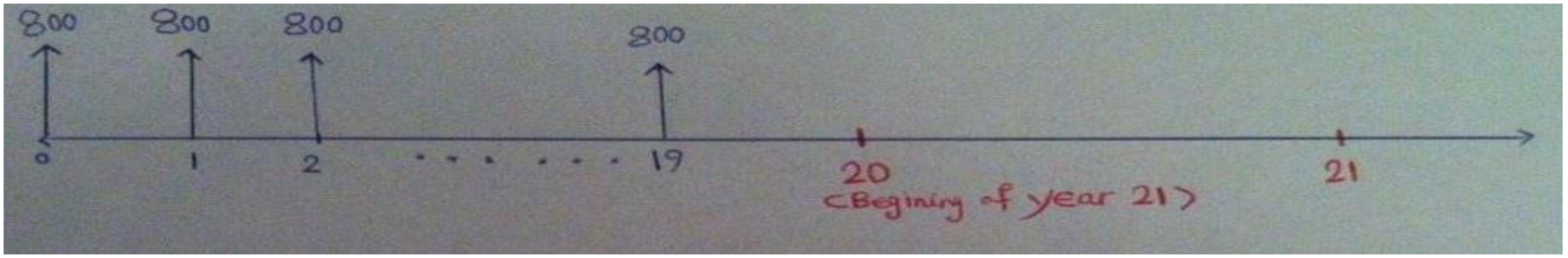
- Solution: A

PV0 of annuity due with 20 payments

$$FV_{19} = \frac{800}{0.14} * \left[1 - \frac{1}{1.14^{20}}\right] * (1.14) * (1.14)^{19} = \$72819.94$$

Let the payment from perpetuity be x

$$FV_{19} = \frac{x}{0.14} \Rightarrow x = 0.14 * 72819.94 = \$10194.79$$



Problem

A stock with a required rate of return of 10 percent sells for $30 per share. The stock's dividend is expected to grow at a constant rate of 7 percent per year. What is the expected year-end dividend, D_1, on the stock?

A. $0.87

B. $0.95

C. $1.02

D. $0.90

Answer D

30= D1/10%-7%

30= 33.33 D1

D1 = 0.90

Problem

Six years from now you will begin to receive cash flows of $200 per year. These cash flows will continue forever. If the appropriate discount rate is 5%, what is the present value of these cash flows? Choose the closest value.

A. $4,000

B. $3,134

C. $2,157

D. $1,567

PV(t=5) = 200/5%= $4000

PV(t=0) = 4000/(1+5%)^5

= $3,134.10

Problem

What is the price of a semi-annual corporate bond that has a 6% coupon rate (compounding is semi-annually), face value of $1000, 12 years to maturity and a discount rate of 7%.

A. $1083.84

B. $1677.42 N=24, FV=1000, 3.5=I/Y, 30 PMT

C. $919.71

D. $922.57

Answer C

Problem

The semi-annual 6 percent coupon paying bonds of “XYZ” company, have a quoted price (i.e., clean price) of $910. If the last coupon payment occurred on June 30th, 2016; the cash price of the bond on July 30th, 2016 is closest to which one of the following choices? Assume that face value of the bond is $1000.

A. $960

B. $955

C. $920

D. $915

$$cash\ price = quoted\ price + accrued\ interest$$

$$cash\ price = 910 + \frac{1}{6}(30) = \$915$$

Answer D

Problem

If investors require a 6% nominal return and the expected inflation rate is 2.5%, what is the expected real return?

A. 8.65%

B. 3.5%

C. 3.4%

D. 3.3%

=(1 + real rate) = (1 + RF) / (1 + expected inflation)

= 1.06/1.025= 3.4

Answer C

- Questions?