MATHEMATICS 203 **Sample Midterm Test**

Total: 40 marks

Show all your work

Only approved calculators (**Sharp EL 531** or **Casio FX 300 MS**) permitted

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1. *(5 marks)* **(a)** Solve for x and explain how you got the solution (getting the answer with a calculator is not an acceptable solution).

(i) $\ln(x) = -1$ **(ii)** $2^{x-4} = 8$

(b) Let $f(x) = \sqrt{3-x}$, $g(x) = 4 - x^2$. Find the following functions **and their domains**:

(i) $f \circ g$ **(ii)** $g \circ f$

2. *(4 marks)* **(a)** Let $f(x) = x + \cos(\pi x)$ Find the average rate of change of $f(x)$ (i) over the interval $[0, 1/2]$; (ii) over the interval $[-1, 1]$;.

(b) Evaluate the following limits:

(i) $\displaystyle\lim_{t\to 3} \frac{3-t}{5-\sqrt{t^2+16}}$ **(ii)** $\displaystyle\lim_{x\to\infty} \frac{2x^{-2}+3x^{-4}}{5x^{-1}-x^{-3}}$

3. *(6 marks)* Find the following limits for the function $f(x) = \dfrac{1}{(x-1)(x+2)}$. **Explain your answers** in a sentence or two.

(a) $\displaystyle\lim_{x\to -2^-} f(x)$ **(b)** $\displaystyle\lim_{x\to 0} f(x)$

(c) $\displaystyle\lim_{x\to 1^+} f(x)$ **(d)** $\displaystyle\lim_{x\to\infty} f(x)$

(e) The equations of all asymptotes

4. *(5 marks)* Find the numbers a and b so that the function $f(x)$ is continuous.

$$f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x < 0 \\ ax+b & \text{if } 0 \le x < 1 \\ x & \text{if } 1 \le x \end{cases}$$

(it may help to sketch the graph).

5. *(12 marks)* For each of the following find $\dfrac{dy}{dx}$ (show all steps and do not simplify your answer):

(a) $y = \dfrac{x^2 + 2x^{3/2} - 5\sqrt{x}}{\sqrt{x}}$

(b) $y = e^{x^2}\,(x+7)^8$

(c) $y = \cos\left(x^2 - \tan x\right)$

(d) $y = \sqrt{1 + \sqrt{x + x^2}}$

6. *(8 marks)* Let $g(x) = 1 + \dfrac{1}{\sqrt{x}}$.

(a) Use the definition of the derivative :

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

to find $g'(4)$

(b) Check that your answer is correct by using the appropriate differentiation rule(s) to find $g'(x)$, and then substituting $x = 4$.

(c) Find the equation of the tangent line $T(x)$ to the graph of $g(x)$ at $(4, 3/2)$