# ASSIGNMENT #2

# Linear Programming

# Formulation, Graphical Method and Excel Solver

# ADM2302

# Section X

**Jasmin Manseau**

**Problem 1**

**a. Formulate algebraically the LP problem that provides the optimal production schedule and minimizes the total cost. (10 points)**

Decision Variables:

Xi: the regular production of January to April. i=1 to 4

Yi: the overtime production of January to April. i=1 to 4

Ii: the inventory of January to April, i=1 to 4

Objective Function:

Minmize the total cost Z=500*(X1+X2+X3+X4)+650*(Y1+Y2+Y3+Y4)+40*(I1+I2+I3+I4)

Constraints:

Regular production of January to April:

$X1 \leq 3000$

$X2 \leq 2000$

$X3 \leq 3000$

$X4 \leq 3500$

Overtime production of January to April:

$Y1 \leq 500$

$Y2 \leq 400$

$Y3 \leq 600$

$Y4 \leq 800$

Inventory of January to April:

$I1 \geq 100$

$I2 \geq 100$

$I3 \geq 300$

$I4 \geq 300$

Demand of January to April:

X1+Y1-I1=2800

I1+X2+Y2-I2=3000

I2+X3+Y3-I3=3500

I3+X4+Y4-I4=3000

**b. Find the optimal schedule that minimizes total cost using Solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results). (5 points)**

**a.**

| | X1(regular | X2 | X3 | X4 | Y1(overtime | Y2 | Y3 | Y4 | I1(inventory | I2 | I3 | I4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Solution | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | |
| | | | | | | | | | | | | | Minized cost | | |
| Cost | 500 | 500 | 500 | 500 | 650 | 650 | 650 | 650 | 40 | 40 | 40 | 40 | 0 | | |
| Constraints | | | | | | | | | | | | | LHS | | RHS |
| Rrgular production | 1 | | | | | | | | | | | | 0 | ≤ | 3000 |
| Rrgular production | | 1 | | | | | | | | | | | 0 | ≤ | 2000 |
| Rrgular production | | | 1 | | | | | | | | | | 0 | ≤ | 3000 |
| Rrgular production | | | | 1 | | | | | | | | | 0 | ≤ | 3500 |
| Overtime production | | | | | 1 | | | | | | | | 0 | ≤ | 500 |
| Overtime production | | | | | | 1 | | | | | | | 0 | ≤ | 400 |
| Overtime production | | | | | | | 1 | | | | | | 0 | ≤ | 600 |
| Overtime production | | | | | | | | 1 | | | | | 0 | ≤ | 800 |
| Inventory | | | | | | | | | 1 | | | | 0 | ≥ | 100 |
| Inventory | | | | | | | | | | 1 | | | 0 | ≥ | 100 |
| Inventory | | | | | | | | | | | 1 | | 0 | ≥ | 100 |
| Inventory | | | | | | | | | | | | 1 | 0 | ≥ | 300 |
| monthly demand | 1 | | | | 1 | | | | -1 | | | | 0 | ≥ | 2800 |
| monthly demand | | 1 | | | | 1 | | | 1 | -1 | | | 0 | ≥ | 3000 |
| monthly demand | | | 1 | | | | 1 | | | 1 | -1 | | 0 | ≥ | 3500 |
| monthly demand | | | | 1 | | | | 1 | | | 1 | -1 | 0 | ≥ | 3000 |

By looking at the solver, the minimum cost will be $0.

**Problem 2**

**b. Formulate algebraically an LP model that will enable the mill operator to satisfy the orders with minimum trim loss. (*Hint:*List the different ways the 20-foot pieces could be cut into the desired sizes.) (8 points)**

Variables：

$x_1$ = the 20-foot wood produces two 8-foot wood. (4-foot waste)

$x_2$ = the 20-foot wood produces one 8-foot wood and one 10-foot wood. (2-foot waste)

$x_3$ = the 20-foot wood produces two 10-foot wood. (no waste)

$x_4$ = the 20-foot wood produces one 8-foot wood and one 12-foot wood. (no waste)

Objective Function:

$\min_z = 4x_1 + 2x_2$

Constraints:

$x_1 + x_2 + x_3 + x_4 <= 350$ (total 20-foot wood on hand)

$2x_1 + x_2 + x_4 => 276$ (8-foot wood ordered)

$x_2 + 2x_3 => 100$ (10-foot wood ordered)

$x_4 = > 250$ (12-foot wood ordered)

$x_1, x_2, x_3, x_4 => 0$ (non-negativity)

**b. Using your notation from *a*, write an equation for**

**i. The amount of waste that would result given a solution. (1 point)**

The amount of waste would be:

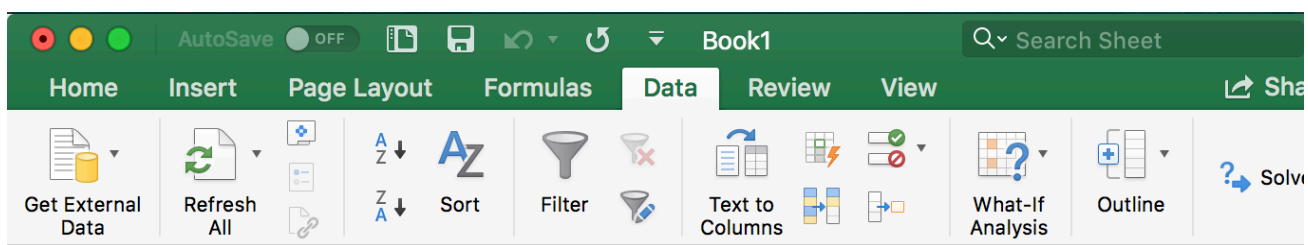
$4x_1 + 2x_2$

**ii. The number of pieces of each size (8-foot, 10-foot, and 12-foot). (1 point)**

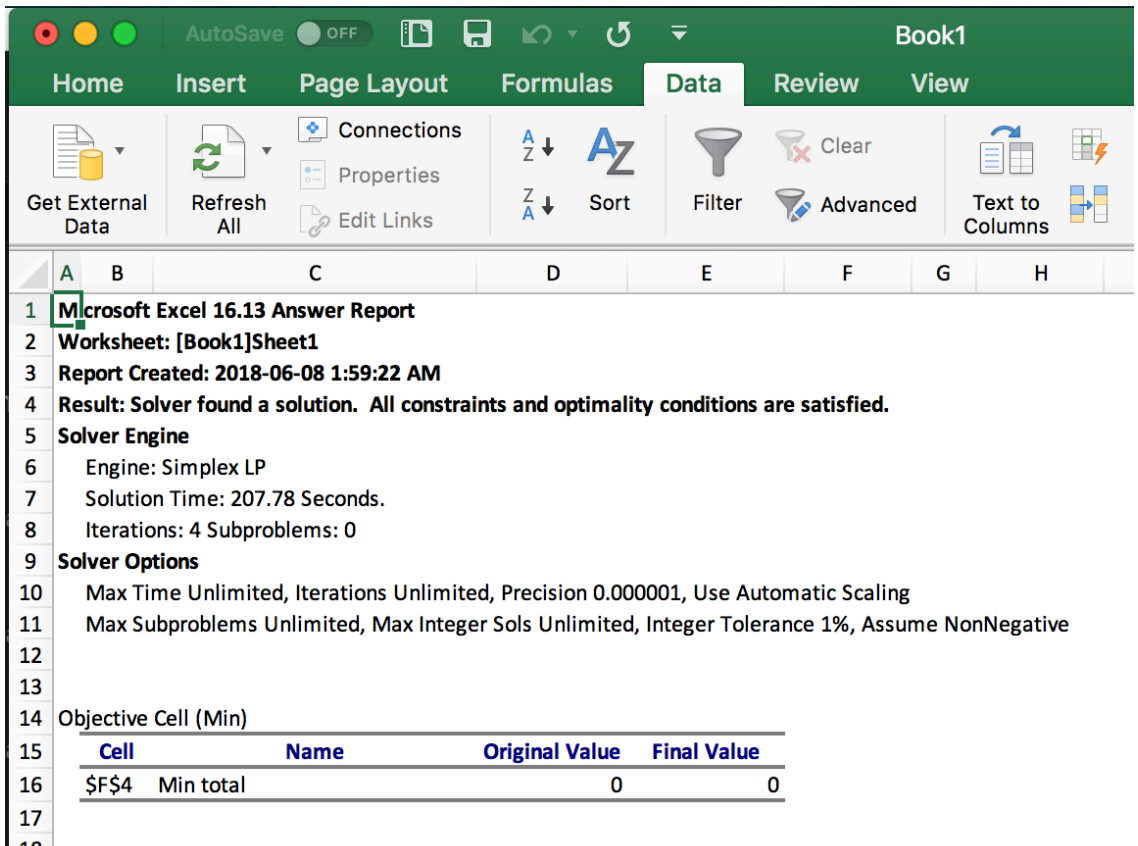
$2x_1 + x_2 + x_4$ = number of 8-foot wood

$x_2 + 2x_3$ = number of 10-foot wood
$x_4$ = number of 12-foot wood

**c. Formulate this same linear programming problem on a spreadsheet and SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Include "managerial statements" that communicate the results of the analyses (i.e. describe verbally the results). (5 points)**




| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | x1 | x2 | x3 | x4 | | | | |
| 2 | solution | 0 | 0 | 50 | 276 | | | | |
| 3 | | | | | | total | | | |
| 4 | Min | 4 | 2 | 0 | 0 | 0 | | | |
| 5 | | | | | | | | | |
| 6 | constrains | | | | | LHS | sign | RHS | |
| 7 | 8-foot wood | 2 | 1 | | 1 | 276 | >= | 276 | |
| 8 | 10-foot wood | | 1 | 2 | | 100 | >= | 100 | |
| 9 | 12-foot wood | | | | 1 | 276 | >= | 250 | |
| 10 | total 20-foot wood on hand | 1 | 1 | 1 | 1 | 326 | <= | 350 | |
| 11 | | | | | | | | | |



AutoSave OFF — Book1

Home | Insert | Page Layout | Formulas | Data | Review | View

Get External Data | Refresh All | Connections | Properties | Edit Links | Sort | Filter | Clear | Advanced | Text to Columns

**Microsoft Excel 16.13 Answer Report**
**Worksheet: [Book1]Sheet1**
**Report Created: 2018-06-08 1:59:22 AM**
**Result: Solver found a solution. All constraints and optimality conditions are satisfied.**
**Solver Engine**
Engine: Simplex LP
Solution Time: 207.78 Seconds.
Iterations: 4 Subproblems: 0
**Solver Options**
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $F$4 | Min total | 0 | 0 |

Variable Cells

| Cell | Name | Original Value | Final Value | Integer |
|---|---|---|---|---|
| $B$2 | solution x1 | 0 | 0 | Contin |
| $C$2 | solution x2 | 0 | 0 | Contin |
| $D$2 | solution x3 | 50 | 50 | Contin |
| $E$2 | solution x4 | 276 | 276 | Contin |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $F$10 | total 20-foot wood on hand LHS | 326 | $F$10<=$H$10 | Not Binding | 24 |
| $F$7 | 8-foot wood LHS | 276 | $F$7>=$H$7 | Binding | 0 |
| $F$8 | 10-foot wood LHS | 100 | $F$8>=$H$8 | Binding | 0 |
| $F$9 | 12-foot wood LHS | 276 | $F$9>=$H$9 | Not Binding | 26 |

The number of 8-foot wood order will be 276 pieces, the number of 10-foot wood order will be 100 pieces, and the number of 12-foot wood order will be 276 pieces. There will be 26 pieces of 12-foot wood extre (276-250) and can be save for future use.

**Header**

From: United Nations consultant

To: Igor Yeltsin

Date: June 10, 2018

Title: A operating plan for Yeltsin farm in order to maximiz the revenue

**Introduction**

Yeltsin farm, which is raising hogs as the mainly activity to operate, always run the farm with the instructions provides by Moscow's central planning agency. However, the owner of the farm Igor Yeltsin cannot follow the instructions to operate the farm anymore due to the new-market driven economy. In this report, we will provide the operating plan about how to run his farm efficiently, which include the optimal solution about how many hogs are supposed to feed, and the most profitable way to buy animal feed.

In order to show the process to find the optimal solution, we would analysis the sensitivity report and answer the questions that you asked, which include: The optimal solution to sell the hogs and the to purchase the corn, food scraps, and potatoes. The analysis about the reason to purchase in whole kilograms in hogs but fractional kilograms of corns and food scraps. The result of income if Igor sell another 5 hogs. The result if the cost of corn increase but the optimal solution would not change. The result if the amount of potatoes decrease from 600 kilograms to 500 kilograms. The different way to purchase of corn, food, or potatoes but keep the same optimal operating income.

**Recommendation**

To maximize the operating income in next month for Yeltsin farm, we recommend the farm to sell 100 hogs, and purchase 95.238 kg of corn, 742.857 kg of food scraped, and 600 kg of potatoes.

I suggest you to purchase 95 kilograms of corns and 174 kilograms of food scraped. However, by doing so, your income will decrease to 26755 rubles.

If you sell 5 more hogs, the operating income would increase to 28067.14.

The cost of corns can increase to 20.917, and at the same time, the optimal solution would not change.

The total income would decrease if the amount of potatoes decrease to 500. The profit would decrease 100*1.095=109.5

For the same optimal solution, there is only one plan to maximze the income.

Igor should start purchasing prepackaged dry hog food from Poland. He should purchase 1087.5 kilogram prepackaged dry hog food. He does not need to purchase any corn, scrap foods, and potatoes. By doing so, his profit will increase by 838.1 rubles, to 27600 rulbs.

**Analysis and Data**

1. From the appendix, the net income is 26761.906. Igor is supposed to sell 100 hogs, purchase 95.238 kg of corns, 742.857 kg of food scraped, and 600 kg of potatoes.
2. I suggest that you can use the method of round. By round down the number of corn purchased to 95 and the number of food scraped purchased to 743, all the constraints still remains in the feasible area. However, by doing so, the profit you will earn will reduce to 26755 rubles.
3. From the sensitivity report in appendix, the allowable increase is 5.405 and the shadow price is 261.048 of the constraints of pogs sold. So the net profit= 26761.905+5*261.048=28067.145
4. From the sensitivity report, the objective coefficient is -19, which means the price of cron is 19. The allowable increase is 1.917. So the corn can increase to 20.917.
5. From the sensitivity report in Appendix, if we change the amount of potatoes to 500, the shadow price would change to 1.095. So the total profit would decrease 100*1.095=109.5. The total profit is 26761.9-109.5=26652.4.
6. The solution from the solver report is the optimal solution, which is maximize the income for the farm. So there is no other optimal solution.
7. From the appendix for question 7, I have added a new variable, PD, which is the number of packed dry hog food purchased. After doing calculations, Igor only need to purchase prepacked dry hog food. He does not need to purchase any corn, food scraped, and potatoes. By doing so, his income will increase to 27600 rubles.

## Appendix

| | H | C | F | P | | | |
|---|---|---|---|---|---|---|---|
| | Hogs Sold | Corn purchased | Food scraped purchased | Potatoes purchased | | | |
| Decisions | 100.000 | 95.238 | 742.857 | 600.000 | 26,761.905 | | |
| Price | $450.00 | -$19.00 | -$10.00 | -$15.00 | | | |
| **Constraints:** | | | | | **LHS** | | **RHS** |
| Hogs supply limit | 1 | | | | 100.000 | <= | 100 |
| Food scraped supply limit | | | 1 | | 742.857 | <= | 800 |
| Potatoes supply limit | | | | 1 | 600.000 | <= | 600 |
| Crude Protein Net Excess Nutrients | -174 | 18 | 9 | 15 | 0.000 | >= | 0 |
| Kilo-Calories Net Excess Nutrients | -1400 | 30 | 120 | 80 | 0.000 | >= | 0 |

**Microsoft Excel 14.0 Sensitivity Report**
**Worksheet: [YelstinFarmsStudents.xls]Yelstin_Farms**
**Report Created: 10/02/2014 4:25:46 PM**

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$19 | Decisions Hogs Sold | 100 | 0 | 450 | 1E+30 | 261.048 |
| $C$19 | Decisions Corn purchased | 95.238 | 0 | -19 | 1.917 | 1 |
| $D$19 | Decisions Food scraped purchased | 742.857 | 0 | -10 | 0.5 | 24.694 |
| $E$19 | Decisions Potatoes purchased | 600 | 0 | -15 | 1E+30 | 1.095 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $F$22 | Hogs supply limit LHS | 100 | 261.048 | 100 | 5.405 | 21.739 |
| $F$23 | Food scraped supply limit LHS | 742.857 | 0 | 800 | 1E+30 | 57.143 |
| $F$24 | Potatoes supply limit LHS | 600 | 1.095 | 600 | 166.667 | 109.091 |
| $F$25 | Crude Protein Net Excess Nutrients LHS | 0.000 | -1.048 | 0 | 46800 | 1500 |
| $F$26 | Kilo-Calories Net Excess Nutrients LHS | 0.000 | -0.005 | 0 | 6000 | 78000 |

Objective Cell (Max)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $F$3 | Decisions | 26652.38095 | 26652.38095 |

Variable Cells

| Cell | Name | Original Value | Final Value | Integer |
|---|---|---|---|---|
| $B$3 | Decisions Hogs Sold | 100 | 100 | Contin |
| $C$3 | Decisions Com purchased | 152.3809524 | 152.3809524 | Contin |
| $D$3 | Decisions Food scraped purchased | 795.2380952 | 795.2380952 | Contin |
| $E$3 | Decisions Potatoed purchased | 500 | 500 | Contin |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $F$10 | Kilo-Calories Net Excess Nutrients LHS | -1.45519E-11 | $F$10>=$H$10 | Binding | 0 |
| $F$11 | Non-negative LHS | 100 | $F$11>=$H$11 | Not Binding | 100 |
| $F$12 | Non-negative LHS | 152.3809524 | $F$12>=$H$12 | Not Binding | 152.3809524 |
| $F$13 | Non-negative LHS | 795.2380952 | $F$13>=$H$13 | Not Binding | 795.2380952 |
| $F$14 | Non-negative LHS | 500 | $F$14>=$H$14 | Not Binding | 500 |
| $F$6 | Hogs supply limit LHS | 100 | $F$6<=$H$6 | Binding | 0 |
| $F$7 | Food scraped supply limit LHS | 795.2380952 | $F$7<=$H$7 | Not Binding | 4.761904762 |
| $F$8 | Potatoes supply limit LHS | 500 | $F$8<=$H$8 | Binding | 0 |
| $F$9 | Crude Protein Net Excess Nutrients LHS | -2.72848E-12 | $F$9>=$H$9 | Binding | 0 |

**Microsoft Excel 16.13 Sensitivity Report**
**Worksheet: [assignment 2 problem 3.xlsx]Sheet1**
**Report Created: 2018-06-10 3:57:00 PM**

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | Decisions Hogs Sold | 100 | 0 | 450 | 1E+30 | 261.047619 |
| $C$3 | Decisions Com purchased | 152.3809524 | 0 | -19 | 1.916666667 | 1 |
| $D$3 | Decisions Food scraped purchased | 795.2380952 | 0 | -10 | 0.5 | 24.69369369 |
| $E$3 | Decisions Potatoed purchased | 500 | 0 | -15 | 1E+30 | 1.095238095 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $F$10 | Kilo-Calories Net Excess Nutrients LHS | -1.45519E-11 | -0.004761905 | 0 | 500 | 83500 |
| $F$11 | Non-negative LHS | 100 | 0 | 0 | 100 | 1E+30 |
| $F$12 | Non-negative LHS | 152.3809524 | 0 | 0 | 152.3809524 | 1E+30 |
| $F$13 | Non-negative LHS | 795.2380952 | 0 | 0 | 795.2380952 | 1E+30 |
| $F$14 | Non-negative LHS | 500 | 0 | 0 | 500 | 1E+30 |
| $F$6 | Hogs supply limit LHS | 100 | 261.047619 | 100 | 0.45045045 | 34.7826087 |
| $F$7 | Food scraped supply limit LHS | 795.2380952 | 0 | 800 | 1E+30 | 4.761904762 |
| $F$8 | Potatoes supply limit LHS | 500 | 1.095238095 | 500 | 266.6666667 | 9.090909091 |
| $F$9 | Crude Protein Net Excess Nutrients LHS | -2.72848E-12 | -1.047619048 | 0 | 50100 | 300 |

**Question 7**

**Variables:**

H = Hogs sold

C = Corn Purchased

F = Food Scraped Purchased

P = Potatoes Purchased

PD = Packed dry hog food Purchased

**Objective Function:**

$Max_z = 450H - 19C - 10F - 15P - 16PD$

**Constraints:**

H <= 100

F <= 800

P <= 600

18C + 9F + 15P + 16PD => 174H

30C + 120F + 80P + 150PD => 1400H

H, C, F, P, PD, => 0

| | H | C | F | P | PD | | | |
|---|---|---|---|---|---|---|---|---|
| | Hogs Sold | Corn Purchased | Food Scraped Purchased | Potatoes Purchased | Packed dry hog food Purchased | | | |
| Decisions | 100 | 0 | 0 | 0 | 1087.5 | 27600 | | |
| Price | $450 | ($19) | ($10) | ($15) | ($16) | | | |
| Constrains: | | | | | | LHS | | RHS |
| Hogs supply limit | 1 | | | | | 100 | <= | 100 |
| Food scraped supply limit | | | 1 | | | 0 | <= | 800 |
| Potatoes supply limit | | | | 1 | | 0 | <= | 600 |
| Crude Protein Net Excess Nutrients | -174 | 18 | 9 | 15 | 16 | -1.819E-11 | >= | 0 |
| Kilo-Calories Net Excess Nutrients | -1400 | 30 | 120 | 80 | 150 | 23125 | >= | 0 |