CHAPTER 2

✠ PROBABILITY

Probability is used in inference statistics as a tool to make statement for population from sample information.

- **Experiment** is a process for generating observations
- **Sample space** is all possible outcomes of an experiment.
- **Event** is a collection of one or some outcomes from sample space, usually denoted by a capital letter.
  - **Simple Event:** The event that cannot decomposed.
- **Venn Diagram** is used to show the result of an experiment, for this reason all simple event show in a box by a point.
- **Tree Diagram** is used when the experiment generated in several steps.

⧫ Some Relations Between Events

- **Union:** The union of events A and B, denoted by $A \cup B$ is the event that contains all outcomes that are *either in* A *or* B *or both*.
- **Intersection:** The intersection of events A and B, denoted by $A \cap B$ is the event that contains all outcomes that are in *both* A *and* B.
- **Complement:** The complement of an event A, denoted by A' is the event that contains all outcomes *in sample space* S *but not in* A.

■ Two events are **mutually exclusive** or **disjoint**, if they don't have any common outcome, or when one event occurs, the other cannot, and vice versa.

⧫ Calculating Probability: $P(A)$ is a measure of the chance that A will occur.

- **Calculating Probability by Using Relative Frequency:**

$$P(A) = \lim_{n \to \infty} \text{relative frequency} = \lim_{n \to \infty} \frac{\text{frequency}}{n}$$

  Frequency is the number of times that event A occurred.
  n is number of times that experiment repeat.

- **Calculating Probability by Using Sample Space:**
  First assign same probability to each simple event such that each probability be a number between 0 and 1 also the sum of all probabilities be 1 , then the probability of event A is equal to the sum of probabilities of simple events contained in A.

Axioms of Probability

1. For any event A, $P(A) \geqslant 0$.
2. $P(S) = 1$.
3. If $A_1, A_2, \cdots$ is an infinite collection of disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i)$$

Properties of Probability

- $P(A) = 1 - P(A') \quad \forall A.$
- $P(A \cap B) = 0 \quad \forall A$ and B mutually exclusive events.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \forall A$ and B.
- For any three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Example: Consider the following able

	Used eyeglasses for reading	
Judge to need eyeglasses	Yes	No
Yes	0.44	0.14
No	0.02	0.40

If a person is selected from this large group, find the probability of each event:
a. The adult is judged to need eyeglasses.
b. The adult needs eyeglasses for reading but does not use them.
c. The adult uses eyeglasses for reading whether he or she needs them or not.

Counting Techniques

One of the method for computing probability is using simple events and

$$P(A) = \frac{n(A)}{n}$$

n : number of simple events in sample space.
$n(A)$: number of simple events contained in A.

There is some rule for counting n and $n(A)$ which are needed for calculate $P(A)$.

The mn rule: If an experiment is performed in two stages, with **m** ways to accomplish the first stage and **n** ways to accomplish the second stage, then there are **mn** ways to accomplish the experiment.
If experiment has **k** stages, then $\mathbf{n} = \mathbf{n_1} \times \mathbf{n_2} \times \cdots \times \mathbf{n_k}$ such that $\mathbf{n_1}$ is number of ways for first stage, ...

Permutation: There are **n** distinct objects and want to choose **k** objects **in order**, then there are

$$P_{k,n} = \frac{n!}{(n-k)!} \quad \text{ways.}$$

Combination: There are **n** distinct objects and want to take **r** objects **at a time**, then there are

$$C_{r,n} = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \text{ways.}$$

Example: A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers?

♦ Conditional Probability

The conditional probability of A given that B has occurred, is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{if } P(B) \neq 0$$

Example: A new magazine publishes three columns entitled "Art" (A),"Book" (B), and "Cinema" (C). Reading habits of a randomly selected reader with respect to these columns are

Read regularly	A	B	C	$A \cap B$	$A \cap C$	$B \cap C$	$A \cap B \cap C$
Probability	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Find $P(A|B),\ P(A|B \cup C),\ P(A \cup B|C)$.

Multiplication Rule

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

Law of Total Probability

If $A_1, A_2, \cdots, A_k$ be mutually exclusive and exhaustive events,for an event B,

$$P(B) = P(B|A_1)P(A_1) + \cdots + P(B|A_k)P(A_k) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

Bayes' Rule

Let $A_1, A_2, \cdots, A_k$ be mutually exclusive and exhaustive events,if an event B occurs, then

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)} \quad j = 1, \cdots k$$

Example: Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur 99% of the time, whereas an individual without the disease will show a positive test result only 2% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?

Independence

Two events A and B are independent if the

$$P(A \cap B) = P(A)P(B)$$

$A,\ B$ and C are mutually independent if

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

Example: Two cards are drown from a deck of 52 cards. calculate the probability that the draw includes an ace and a ten.

Suggested Exercises for Chapter 2: 3, 11, 13, 17, 21, 23, 25, 45, 47, 49, 51, 63, 71, 73, 77,79, 80, 83, 87, 91