18/20 :)

Carleton University, School of Mathematics and Statistics

MATH 1119 A, fall 2017

Test 1, October 4, 2017

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Part A. True or false questions. Each question carries one mark

1. Any *m x n* matrix can be reduced to a matrix in an echelon form. T F

2. A linear system can have exactly three solutions T F

3. The number zero can be a pivot. T F

4. When the linear system corresponding to the reduced echelon form of the augmented matrix of a linear system has free variables, the solution set of the initial system is infinite. T F

5. If the reduced echelon form of the augmented matrix of a linear system has a row with zero in all entries (zero row), then the linear system has no solution. T F

Part B. Essay questions

6. Find the solution set of the following linear system by reducing its augmented matrix to the reduced echelon form

$$x_2 + 2x_3 = 6$$

$$x_1 + x_2 + x_3 = 4$$

$$-3x_1 + x_2 + x_3 = 8$$

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[5 marks]

$$\begin{bmatrix} 0 & 1 & 2 & 6 \\ 1 & 1 & 1 & 4 \\ -3 & 1 & 1 & 8 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ -3 & 1 & 1 & 8 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + 3R_1} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ 0 & 4 & 4 & 20 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 4R_2} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & -4 & -4 \end{bmatrix} \xrightarrow{R_3/-4} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_3} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_3}$$

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 &= -1 \\ x_2 &= 4 \\ x_3 &= 1 \end{aligned}\right\} \text{Solution set.}$$

coefficient

7. Consider the following matrices

$$A = \begin{bmatrix} 0 & 2 & 5 & 3 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

a. Is the matrix A in echelon form? Justify your answer. **[1 marks]** 1

$\boxed{\text{No,}}$ row the " all zero" should be below all non zero rows.

b. Is the matrix B in a reduced echelon form? Justify your answer. **[1 marks]** 1

$\boxed{\text{NO}}$ the pivot position in row 3 column 4 is not "1" as it should be in reduced echelon form.

c. Is the matrix C in echelon form? Justify your answer. **[1 marks]** 0

$\boxed{\text{yes,}}$ all entries below the pivot position are "0" X No

d. Assume that A is the augmented matrix of a linear system of four equations in three variables. What is the solution set of this system? **[1 marks]**

$\boxed{\text{There is no solution set,}}$ row 4 or $0 = 5$ is not valid. the system is inconsistent 1

8. Consider the following system

$$x_1 + 2x_2 = 6$$

$$2x_1 + hx_2 = k$$

where h and k are parameters. **Give separate answers for each of the following questions.** Find all the values of h and k such that the given system has

a. No solution such that $b \neq 0$ and $x_1, x_2 = 0$ **[2 marks]**

$$\begin{bmatrix} 1 & 2 & 6 \\ 2 & h & k \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 6 \\ 0 & h-4 & k-12 \end{bmatrix}$$

$\therefore$ $h - 4 = 0$

$\boxed{h = 4}$

$k - 12 \neq 0$

$\boxed{k \neq 12}$

$\therefore$ There is no solution when $h = 4$, and when k is equal to any number except for 12.

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b. A unique solution **[2 marks]**

$\begin{bmatrix} 1 & 2 & 6 \\ 0 & h-4 & k-12 \end{bmatrix}$ such that $h \neq 0$, and $k = \in \mathbb{R}$

$\therefore h-4 \neq 0$ $\boxed{h \neq 4}$, and $k-12 = \in \mathbb{R}$

$\boxed{\therefore h \neq 4 \text{, and } k = \text{any number}}$

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c. Infinitely many solutions. In this case, describe the solution set. **[2 marks]**

$\begin{bmatrix} 1 & 2 & 6 \\ 0 & h-4 & k-12 \end{bmatrix}$ such that $a_{11} = 0$, $a_2 = 0$ & $b = 0$

$\therefore h-4 = 0 \Rightarrow \boxed{h = 4}$, and $k-12 = 0 \Rightarrow$

$k = 12$ $\therefore$ there are infinitely many solution

when $\boxed{h = 4}$ and $\boxed{k = 12}$

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