MATH 1119A Test #1 May 28th 2018

(This is a 50-minute test. Programmable calculators are not permitted.)

NAME: ________ STUDENT #: ________

This test paper has two parts. Total of 40 marks. Part I has 5 multiple choice questions. Part II has 2 long answer questions.

PART I: Multiple Choice Questions. Circle the correct answer. Three marks each. No part marks.

1) Consider the following augmented matrix of a system of linear equations:

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \quad R_3 \to (-1)R_2 + R_3 \quad \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_2 + R_1 \quad \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system has

a) infinitely many solutions with one free variable
b) infinitely many solutions with two free variables
c) unique solution
d) no solutions

$x_1 = 2$ ✓
$x_2 + x_3 = 1$
$x_3 = 1 - x_2$

2) Let $A = \begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & k^2 - 9 & k+3 \end{bmatrix}$

$3^2 - 9$ $3+3$
$9 - 9 =$
$0, 0, 0$, 6 inconsistent

be the augmented matrix of a given system. For which value of k will the system has no solutions?

a) $k = -3$ b) $k = 3$ c) $k = 9$ d) $k = -9$.

3) Which of the following statements is TRUE?

a. Row echelon form of a matrix is unique.
b. The number of vectors in Span{u, v} is 2.
c. The matrix equation AX = 0 is always consistent. ✓
d. None of the above are true.

4) Let $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ $\begin{matrix} R_1 \to R_3 + R_2 \\ R_2 \to (-1)R_3 + R_2 \end{matrix}$ $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

Which one of the following is the reduced echelon form of A

a) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, b) $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, c) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$, d) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

5) Let $\mathbf{b} = \begin{bmatrix} h \\ -3 \\ 2h-5 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$,

For what value of h is **b** in the span of **u** and **v**?

a) $h = 2$, b) $h = 5/2$, c) $h = 4$, d) $h = -2$.

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PART II: Long answer questions. Show all your work.

1. Given a linear equation system

$$\text{①}\; 3x_1 + 9x_2 - 6x_3 = 3$$
$$\text{②}\; 4x_1 + 10x_2 + 2x_3 = 0$$
$$\text{③}\; 3x_1 + 2x_2 + 41x_3 = 1$$

   a. Use Gauss Elimination method to get echelon form of the augmented matrix of the system. Then determine whether the system is consistent or not, why? (8 marks)

   b. Use Gauss-Jordan Elimination method to solve the system (hint: continue the step a) to get reduced echelon form, then solve the system). (8 marks)

$$\begin{bmatrix} 3 & 9 & -6 & 3 \\ 4 & 10 & 2 & 0 \\ 3 & 2 & 41 & 1 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{3}R_1} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 4 & 10 & 2 & 0 \\ 3 & 2 & 41 & 1 \end{bmatrix} \begin{matrix} R_2 \to (-4)R_1 + R_2 \\ R_3 \to (-3)R_1 + R_3 \end{matrix} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -2 & 10 & -4 \\ 0 & -7 & 47 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -2 & 10 & -4 \\ 0 & -7 & 47 & -2 \end{bmatrix} \xrightarrow{R_2 \to (-\frac{1}{2})R_2} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & -7 & 47 & -2 \end{bmatrix} \xrightarrow{R_3 \to (7)R_2 + R_3} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 12 & 12 \end{bmatrix} \xrightarrow{R_3 \to \frac{1}{12}R_3} \checkmark \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & -5 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} R_1 \to (2)R_3 + R_1 \\ R_2 \to (5)R_3 + R_2 \end{matrix} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 \to (-3)R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 & -18 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \boxed{\begin{matrix} x_1 = -18 \\ x_2 = 7 \\ x_3 = 1 \end{matrix}} \checkmark$$

b. ① $3(-18) + 9(7) - 6(1) = 3$

$-54 + 63 - 6 = 3$

$3 = 3$

② $4(-18) + 10(7) + 2(1) = 0$

$-72 + 70 + 2 = 0$

$0 = 0$

③ $3(-18) + 2(7) + 41(1) = 1$

$-54 + 14 + 41 = 1$

$1 = 1$

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a. The system is consistent ✓ because $[0, 0, 0, \#]$ does not exist in the Therom. It has a unique solution.

2. Let vector $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

   a) Find $x_1\mathbf{u} + x_2\mathbf{v}$. where x_1 and x_2 are two variables (4 marks)

   b) Solve x_1 and x_2 from the equation $x_1\mathbf{u} + x_2\mathbf{v} = \mathbf{w}$. (5 marks)

$$\begin{bmatrix} 1 & -4 & 0 \\ 2 & -6 & 2 \end{bmatrix} \xrightarrow{R_2 \to (-2)R_1 + R_2} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 2 & 2 \end{bmatrix} \xrightarrow{R_2 \to \frac{1}{2}R_2} \begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1 \to (4)R_2 + R_1}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{b.} \begin{bmatrix} x_1 = 4 \\ x_2 = 1 \end{bmatrix} \checkmark$$

$$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ -6 \end{bmatrix} \checkmark = \begin{bmatrix} x_1 - 4x_2 \\ 2x_1 - 6x_2 \end{bmatrix}$$

-0.5

a. $1x_1 - 4x_2 = 0$

$2x_1 - 6x_2 = 2$

b. $1(4) - 4(1) = 0$

$4 - 4 = 0$

$0 = 0$

$2(4) - 6(1) = 2$

$8 - 6 = 2$

$2 = 2$

24.5

$3^2-9 \quad 3+3$

$9-9 \quad 6$

$0 \quad 6$

$-3^2-9 \quad -3+3$

$9-9 \quad 0$

$0 \quad 0$

$9^2-9 \quad 9+3$

$81-9$

$72 \quad 12$

$-9^2-9 \quad 9+3$

$81-9 \quad 12$

5. $\begin{bmatrix} 1 & 2 & h \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix} R_3 \to (-3)R_2 + R_3 \begin{bmatrix} 1 & 2 & h \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{bmatrix} R_1 \to (-2)R_2 + R_1 \begin{bmatrix} 1 & 0 & 6h \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{bmatrix}$

$2h+4=0$

$\frac{2h}{2} = \frac{4}{2}$

$\boxed{h=2}$