

University of Ottawa
Department of Mathematics and Statistics

MAT 1320F-Winter 2018-Feb 7th - Midterm 1

Professor: Xinhou Hua

Family Name _____ First Name _____

Student # _____ DGD # _____

- Time: 17:30–18:45.
- **NO Calculators. NO Notes. NO Books.**
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- **Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.**

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Question	1	2	3	4	5	6	7	Total
Maximum	1	1	1	4	3	4	6	20
Answer				X	X	X	X	X
Mark								

Part I: Three Multiple Choice Questions, no partial marks.

1. [1 point] Let $f(x) = \frac{x-1}{2x-3}$. Find $f^{-1}(2/5)$.

- (A) 0 (B) 1 (C) -1 (D) $\frac{3}{2}$ (E) 3 (F) None of them.

Solution: (C). Since $f(-1) = 2/5$.

2. [1 point] Solve for x : $\ln(4x - 3) = 6$

- (A) $\frac{1}{4} \ln(9)$ (B) $\frac{1}{4} e^3$ (C) $\frac{1}{4} e^6 + \frac{3}{4}$ (D) $\frac{1}{4} e^6 - \frac{3}{4}$ (E) $\frac{1}{4} \ln(6) + \frac{1}{4} \ln(3)$ (F) $\frac{9}{4}$

Solution: (C). $4x - 3 = e^6$, $4x = e^6 + 3$.

3. [1 point] Which of the following is a solution of $2 \sin(2x - \frac{\pi}{2}) = 1$ with $0 \leq x \leq \pi$?

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{3}$ (E) $\frac{5\pi}{6}$ (F) $\frac{5\pi}{8}$

Solution: (B). $2x - \frac{\pi}{2} = \frac{\pi}{6}$, $2x = \frac{4\pi}{6}$, $x = \frac{\pi}{3}$.

Part II: Long Answer Questions, you have to show your work clearly.

4. [4=2+2 points] Calculate the following limits. (You can not use l'Hopital's rule).

(a) $\lim_{x \rightarrow 1} \frac{5x^2 - x - 4}{2x^2 + x - 3}$.

Solution: $\lim_{x \rightarrow 1} \frac{5x^2 - x - 4}{2x^2 + x - 3} = \lim_{x \rightarrow 1} \frac{(x-1)(5x+4)}{(x-1)(2x+3)} = \lim_{x \rightarrow 1} \frac{5x+4}{2x+3} = \frac{9}{5}$.

(b) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 5} - x)$.

Solution:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 5} - x) = \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 4x + 5} - x)(\sqrt{x^2 + 4x + 5} + x)}{(\sqrt{x^2 + 4x + 5} + x)}$$

$$= \lim_{x \rightarrow \infty} \frac{4x + 5}{(\sqrt{x^2 + 4x + 5} + x)} = \lim_{x \rightarrow \infty} \frac{4 + (5/x)}{(\sqrt{1 + (4/x) + (5/x^2)} + 1)} = \frac{4}{2} = 2.$$

5. [3 points] Find the values of a and b such that the function $f(x)$ is continuous in \mathbb{R} .

$$f(x) = \begin{cases} 2x & \text{if } x < 1 \\ ax^2 + b & \text{if } 1 \leq x \leq 2 \\ 4x & \text{if } x > 2 \end{cases}$$

Solution: Note that

$$\lim_{x \rightarrow 1^-} f(x) = 2 = \lim_{x \rightarrow 1^+} f(x) = a + b = f(1)$$

and

$$\lim_{x \rightarrow 2^-} f(x) = 4a + b = f(2) = \lim_{x \rightarrow 2^+} f(x) = 8.$$

Thus $a = 2$ et $b = 0$.

6. [4=1+3 points]

a) Write down the definition of the derivative of the function $f(x)$ at the point $x = a$.

Solution: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

b) Let $f(x) = \frac{x+1}{x+2}$. Use the definition of the derivative to find the $f'(1)$.

Solution:

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2+h}{1+h+2} - \frac{2}{1+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(2+h) - 2(3+h)}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{h}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{1}{3(3+h)}$$

$$= \frac{1}{9}.$$

7. [6=2+2+2 points] Calculate the derivative of the following three functions:

(a) $f(x) = e^{3x^2} + 2^x$

Solution:

$$f'(x) = e^{3x^2}(6x) + 2^x \ln 2.$$

(b) $g(x) = \frac{\tan(x)}{x^2 + 1}$

Solution: By the quotient rule,

$$g'(x) = \frac{\tan'(x)(x^2 + 1) - (x^2 + 1)' \tan(x)}{(x^2 + 1)^2} = \frac{\sec^2(x)(x^2 + 1) - 2x \tan(x)}{(x^2 + 1)^2}.$$

(c) $h(x) = \cos(x^3 + x + 11)$

Solution: By the chain rule,

$$h'(x) = \cos'(x^3 + x + 11)(x^3 + x + 11)' = -(3x^2 + 1) \sin(x^3 + x + 11).$$