***** SOLUTIONS *****

1. Solve the following linear equations:

   NOTE: There is often more than one way to solve a linear equation. You will not lose marks if you choose a path that's different from what I show in these solutions. As long as you follow the rules of algebra, you will receive credit for a correct answer.

   You also do not have to write out the details in words as I have done in these solutions. They are there only to help you see what I am doing at each step.

   (a) $3a - 2 = 4a - 1$

$$\begin{aligned} 3a - 2 &= 4a - 1 & \\ 3a - 2 + 2 &= 4a - 1 + 2 & \text{Add 2 to both sides} \\ 3a &= 4a + 1 & \text{Simplify} \\ 3a - 4a &= 4a + 1 - 4a & \text{Subtract } 4a \text{ from both sides} \\ -a &= 1 & \text{Simplify} \\ -1(-a) &= -1(1) & \text{Multiply both sides by } -1 \\ a &= -1 & \text{Simplify} \end{aligned}$$

Check your solution:

$$\begin{aligned} 3(-1) - 2 &= 4(-1) - 1 \\ -3 - 2 &= -4 - 1 \\ -5 &= -5 \end{aligned}$$

(b) $-2x+4=2-x$

$$\begin{aligned}
-2x+4 &= 2-x \\
-2x+4+2x &= 2-x+2x && \text{Add } 2x \text{ to both sides} \\
4 &= 2+x && \text{Simplify} \\
4-2 &= 2+x-2 && \text{Subtract 2 from both sides} \\
2 &= x && \text{Simplify} \\
x &= 2 && \text{Switch the two sides of the equation}
\end{aligned}$$

Check your solution:

$$\begin{aligned}
-2(2)+4 &= 2-(2) \\
-4+4 &= 2-2 \\
0 &= 0
\end{aligned}$$

(c) $m + 3(m - 5) = 5(2m + 3)$

$$
\begin{aligned}
m + 3(m - 5) &= 5(2m + 3) && \\
m + 3(m) + 3(-5) &= 5(2m) + 5(3) && \text{Distribute the multiplication} \\
m + 3m - 15 &= 10m + 15 && \text{Simplify} \\
4m - 15 &= 10m + 15 && \text{Collect like terms} \\
4m - 15 - 4m &= 10m + 15 - 4m && \text{Subtract } 4m \text{ from both sides} \\
-15 &= 6m + 15 && \text{Simplify} \\
-15 - 15 &= 6m + 15 - 15 && \text{Subtract 15 from both sides} \\
-30 &= 6m && \text{Simplify} \\
\frac{-30}{6} &= \frac{6m}{6} && \text{Divide both sides by 6} \\
-5 &= m && \text{Simplify} \\
m &= -5 && \text{Switch the two sides of the equation}
\end{aligned}
$$

Check your solution:

$$
\begin{aligned}
(-5) + 3((-5) - 5) &= 5(2(-5) + 3) \\
-5 + 3(-10) &= 5(-10 + 3) \\
-5 - 30 &= 5(-7) \\
-35 &= -35
\end{aligned}
$$

(d) $9k + 3(k + 6) = 15k + 21$

$$\begin{aligned}
9k + 3(k+6) &= 15k + 21 \\
9k + 3k + 18 &= 15k + 21 && \text{Distribute the multiplication} \\
12k + 18 &= 15k + 21 && \text{Collect like terms} \\
12k + 18 - 12k &= 15k + 21 - 12k && \text{Subtract } 12k \text{ from both sides} \\
18 &= 3k + 21 && \text{Simplify} \\
18 - 21 &= 3k + 21 - 21 && \text{Subtract 21 from both sides} \\
-3 &= 3k && \text{Simplify} \\
\frac{-3}{3} &= \frac{3k}{3} && \text{Divide both sides by 3} \\
-1 &= k && \text{Simplify} \\
k &= -1 && \text{Switch the two sides of the equation}
\end{aligned}$$

Check your solution:

$$\begin{aligned}
9(-1) + 3((-1) + 6) &= 15(-1) + 21 \\
9(-1) + 3(5) &= 15(-1) + 21 \\
-9 + 15 &= -15 + 21 \\
6 &= 6
\end{aligned}$$

(e) $6(1-3x)+1 = 2x - [3(x-7)-20]$

$$
\begin{aligned}
6(1-3x)+1 &= 2x-[3(x-7)-20] && \text{Distribute the multiplications}\\
6-18x+1 &= 2x-[3x-21-20] && \text{Combine the constant terms}\\
7-18x &= 2x-[3x-41] && \text{Distribute the subtract on the right hand side}\\
7-18x &= 2x-3x+41 && \\
7-18x &= -x+41 && \text{Combine like terms}\\
7-18x+18x &= -x+41+18x && \text{Add } 18x \text{ to both sides}\\
7 &= 17x+41 && \text{Simplify}\\
7-41 &= 17x+41-41 && \text{Subtract 41 from both sides}\\
-34 &= 17x && \text{Simplify}\\
\frac{-34}{17} &= \frac{17x}{17} && \text{Divide both sides by 17}\\
-2 &= x && \text{Simplify}\\
x &= -2 && \text{Switch the two sides of the equation}
\end{aligned}
$$

Check your solution:

$$
\begin{aligned}
6(1-3(-2))+1 &= 2(-2)-[3((-2)-7)-20]\\
6(1+6)+1 &= 2(-2)-[3(-9)-20]\\
6(7)+1 &= 2(-2)-[-27-20]\\
6(7)+1 &= 2(-2)-[-47]\\
42+1 &= -4+47\\
43 &= 43
\end{aligned}
$$

(f) $\frac{1}{3}(t+1) = \frac{1}{3}t + 1$

$$\begin{aligned}
\tfrac{1}{3}(t+1) &= \tfrac{1}{3}t+1 && \\
3\left(\tfrac{1}{3}(t+1)\right) &= 3\left(\tfrac{1}{3}t+1\right) && \text{Multiply both sides by 3} \\
3\left(\tfrac{1}{3}(t+1)\right) &= 3\left(\tfrac{1}{3}t\right)+3(1) && \text{Distribute the multiplication on the right} \\
1(t+1) &= 1t+3 && \text{Simplify} \\
t+1 &= t+3 && \\
t+1-t &= t+3-t && \text{Subtract } t \text{ from both sides} \\
1 &= 3 && \text{Simplify}
\end{aligned}$$

This is a contradiction, so the original equation has no solution.

(g) $\dfrac{x-2}{6} = \dfrac{x+1}{15}$

I will first find the least common multiple (LCM) of 6 and 15:

$6 = 2 \times 3 \qquad 15 = 3 \times 5 \qquad \text{LCM}(6, 15) = 2 \times 3 \times 5 = 30$

Now I can multiply both sides of the original equation by 30:

$$
\begin{aligned}
(30)\left(\frac{x-2}{6}\right) &= (30)\left(\frac{x+1}{15}\right) \\
\frac{30}{6}\cdot(x-2) &= \frac{30}{15}\cdot(x+1) && \text{Rewrite the multiplication} \\
5(x-2) &= 2(x+1) && \text{Simplify} \\
5x-10 &= 2x+2 && \text{Distribute the multplication and simplify} \\
5x-2x &= 2+10 && \text{Subtract } 2x \text{ from, and add 10 to both sides} \\
3x &= 12 && \text{Simplify} \\
x &= 4 && \text{Divide both sides by 3 and simplify}
\end{aligned}
$$

Check your solution:

$$
\begin{aligned}
\frac{(4)-2}{6} &= \frac{(4)+1}{15} \\
\frac{2}{6} &= \frac{5}{15} \\
\frac{1}{3} &= \frac{1}{3}
\end{aligned}
$$

(h) $\dfrac{3p+3}{2} = 1 - p$

$$
\begin{aligned}
\frac{3p+3}{2} &= 1 - p \\
2\left(\frac{3p+3}{2}\right) &= 2(1-p) && \text{Multiply both sides by 2} \\
\not{2}\left(\frac{3p+3}{\not{2}}\right) &= 2(1-p) && \text{Cancel} \\
3p+3 &= (2)(1-p) && \text{Simplify} \\
3p+3 &= 2-2p && \text{Distibute the multiplications} \\
3p+3+2p &= 2-2p+2p && \text{Add } 2p \text{ to both sides} \\
5p+3 &= 2 && \text{Simplify} \\
5p+3-3 &= 2-3 && \text{Subtract 3 from both sides} \\
5p &= -1 && \text{Simplify} \\
\frac{5p}{5} &= \frac{-1}{5} && \text{Divide both sides by 5} \\
p &= -\tfrac{1}{5} && \text{Simplify}
\end{aligned}
$$

Check your solution:

$$
\begin{aligned}
\frac{3(-\frac{1}{5})+3}{2} &= 1-(-\frac{1}{5}) \\
\frac{-\frac{3}{5}+\frac{15}{5}}{2} &= \frac{5}{5}+\frac{1}{5} \\
\frac{\frac{12}{5}}{2} &= \frac{6}{5} \\
\frac{12}{5}\cdot\left(\frac{1}{2}\right) &= \frac{6}{5} \\
\frac{6}{5} &= \frac{6}{5}
\end{aligned}
$$

(i) $\frac{x}{2}+\frac{x}{5}+\frac{x}{10}=1$

Let's find the LCM of 2, 5, and 10:

$2=2 \qquad 5=5 \qquad 10=2\times 5 \qquad \text{LCM}(2,5,10)=2\times 5=10$

Now I can multiply both sides of the original equation by 10:

$$\begin{aligned}
10\left(\frac{x}{2}+\frac{x}{5}+\frac{x}{10}\right) &= (10)(1) & \\
10\left(\frac{x}{2}\right)+10\left(\frac{x}{5}\right)+10\left(\frac{x}{10}\right) &= 10 & \text{Distribute the multiplication}\\
\frac{10}{2}x+\frac{10}{5}x+\frac{10}{10}x &= 10 & \text{Rewrite the multiplication}\\
5x+2x+x &= 10 & \text{Simplify}\\
8x &= 10 & \text{Collect like terms}\\
x &= \frac{10}{8} & \text{Divide both sides by 8}\\
x &= \frac{5}{4} & \text{Reduce the fraction}
\end{aligned}$$

Check your solution:

$$\begin{aligned}
\frac{\left(\frac{5}{4}\right)}{2}+\frac{\left(\frac{5}{4}\right)}{5}+\frac{\left(\frac{5}{4}\right)}{10} &= 1\\
\frac{5}{4}\cdot\frac{1}{2}+\frac{5}{4}\cdot\frac{1}{5}+\frac{5}{4}\cdot\frac{1}{10} &= 1\\
\frac{5}{8}+\frac{5}{20}+\frac{5}{40} &= 1\\
\frac{25}{40}+\frac{10}{40}+\frac{5}{40} &= 1\\
\frac{25+10+5}{40} &= 1\\
\frac{40}{40} &= 1
\end{aligned}$$

(j) $\dfrac{2x+1}{x-2} = 2$

$$\begin{aligned}
\frac{2x+1}{x-2} &= 2 \\
(x-2)\left(\frac{2x+1}{x-2}\right) &= (x-2)(2) && \text{Multiply both sides by } x-2 \\
\frac{(x-2)}{(x-2)} \cdot (2x+1) &= (x-2)(2) && \text{Rewrite the multiplication} \\
2x+1 &= 2x-4 && \text{Distribute the multiplication on the right} \\
2x+1-2x &= 2x-4-2x && \text{Subtract } 2x \text{ from both sides} \\
1 &= -4 && \text{Simplify}
\end{aligned}$$

This is a contradiction, so the original equation has no solution.

2. What is the solution to the following equation: $-\frac{1}{2}(4x+5) = -3(x-1)$

   (i) $x = -\frac{1}{2}$ (ii) $x = \frac{11}{2}$ (iii) $x = 4$ (iv) No solution

   One way to answer a multiple choice question like this is to simply substitute each of the given values into the equation. If any of these values make the equation true, then that's the solution!

   But, since you'll be asked to solve equations on your final exam (and they won't be multiple choice!) we should go ahead and actually solve this equation.

$$-\frac{1}{2}(4x+5) = -3(x-1)$$

$$-\frac{1}{2}(4x) - \frac{1}{2}(5) = -3(x) - 3(-1)$$

$$-2x - \frac{5}{2} = -3x + 3$$

$$-2x - \frac{5}{2} + 3x = -3x + 3 + 3x$$

$$x - \frac{5}{2} = 3$$

$$x - \frac{5}{2} + \frac{5}{2} = 3 + \frac{5}{2}$$

$$x = \frac{11}{2}$$

   So the correct answer is option (ii).

3. What is the solution to the following equation: $\dfrac{-3}{x-1} = \dfrac{5}{x+1}$

   (i) $x = \dfrac{1}{8}$ (ii) $x = -\dfrac{1}{8}$ (iii) $x = \dfrac{1}{4}$ (iv) No solution

   We begin by "getting rid of" the fractions. The horizontal bars represent division, so we can cancel division by $x-1$ on the left hand side by multiplying both sides by $x-1$:

$$(x-1) \cdot \frac{-3}{x-1} = \frac{5}{x+1} \cdot (x-1)$$

$$\cancel{(x-1)} \cdot \frac{-3}{\cancel{x-1}} = \frac{5}{x+1} \cdot (x-1)$$

$$-3 = \frac{5(x-1)}{x+1}$$

   Now we can eliminate the fraction on the right hand side by multiplying both sides by $x+1$:

$$(x+1)(-3) = \frac{5(x-1)}{x+1} \cdot (x+1)$$

$$-3(x+1) = \frac{5(x-1)}{\cancel{x+1}} \cdot \cancel{(x+1)}$$

$$-3(x+1) = 5(x-1)$$

   You have likely heard of the process called "cross-multiplication". That is exactly what I just did, only I did it in two steps instead of one. We now have an equation that can be solved as easily as the ones in problem 1 (a) and (b).

$$-3x - 3 = 5x - 5$$

$$-3 + 5 = 5x + 3x$$

$$2 = 8x$$

$$x = \frac{2}{8}$$

$$x = \frac{1}{4}$$

   The correct answer is option (iii).

4. Translate each of the following sentences into a linear equation, and then solve the equation:

   (a) The sum of twice a number and fifteen is eleven.

   The equation is: $2x + 15 = 11$

   The solution is:

$$\begin{aligned} 2x + 15 &= 11 \\ 2x &= 11 - 15 \\ 2x &= -4 \\ x &= \frac{-4}{2} \\ x &= -2 \end{aligned}$$

   (b) When four is subtracted from ten times a number, the result is sixty-six.

   The equation is: $10x - 4 = 66$

   The solution is:

$$\begin{aligned} 10x - 4 &= 66 \\ 1x &= 66 + 4 \\ 10x &= 70 \\ x &= \frac{70}{10} \\ x &= 7 \end{aligned}$$

(c) Two more than twice a number is the same as ten less than half of the number.

The equation is: $2x + 2 = \frac{1}{2}(x) - 10$

The solution is:

$$\begin{aligned}
2x + 2 &= \tfrac{1}{2}x - 10 \\
2x &= \tfrac{1}{2}x - 10 - 2 \\
2x &= \tfrac{1}{2}x - 12 \\
2x - \tfrac{1}{2}x &= -12 \\
\tfrac{4}{2}x - \tfrac{1}{2}x &= -12 \\
\tfrac{3}{2}x &= -12 \\
2\left(\tfrac{3}{2}x\right) &= 2(-12) \\
3x &= -24 \\
x &= -8
\end{aligned}$$

(d) Ten divided by two more than a number is negative 5.

The equation is: $\dfrac{10}{x+2} = -5$

The solution is:

$$\begin{aligned}
\frac{10}{x+2} &= -5 \\
(x+2)\left(\frac{10}{x+2}\right) &= -5(x+2) \\
\cancel{(x+2)}\left(\frac{10}{\cancel{x+2}}\right) &= -5(x+2) \\
10 &= -5x - 10 \\
20 &= -5x \\
-4 &= x \\
x &= -4
\end{aligned}$$

(e) When twenty-seven is divided by three less than twice some number, the result is three.

The equation is: $\dfrac{27}{2x-3} = 3$

The solution is:

$$\begin{aligned}
\frac{27}{2x-3} &= 3 \\
(2x-3)\left(\frac{27}{2x-3}\right) &= 3(2x-3) \\
\cancel{(2x-3)}\left(\frac{27}{\cancel{2x-3}}\right) &= 3(2x-3) \\
27 &= 6x-9 \\
36 &= 6x \\
6 &= x \\
x &= 6
\end{aligned}$$

5. Thomas spent 1.5 times as much on his groceries this week as Dylan. Together they spent \$171.25. How much did they each spend on their groceries?

Let D represent the amount of money that Dylan spent on groceries. Since Thomas spent 1.5 times as much as Dylan, Thomas spent: $1.5D$. Together they spent \$171.25. This gives us the equation:

$$\begin{aligned}
D + 1.5D &= 171.25 \\
2.5D &= 171.25 \\
\frac{2.5D}{2.5} &= \frac{171.25}{2.5} \\
D &= 68.5
\end{aligned}$$

Dylan spent \$68.50 on groceries, while Thomas spent $1.5 \times 68.50 = \$102.75$. We can also check that: $68.50 + 102.75 = 171.25$.

6. The Oilers and Blackhawks combined to score 12 goals in a hockey game. The Oilers scored 3 times as many goals as the Blackhawks. How many goals did each team score?

   Let g represent the number of goals that the Blackhawks scored. Since the Oilers scored three times more goals, they scored: $3g$ goals. Together they scored 12 goals. This gives us the equation:

$$\begin{aligned} g + 3g &= 12 \\ 4g &= 12 \\ \frac{4g}{4} &= \frac{12}{4} \\ g &= 3 \end{aligned}$$

   This means that the Blackhawks scored 3 goals, and the Oilers scored $3 \times 3 = 9$ goals. We can also check that: $3 + 9 = 12$.

7. When four consecutive odd numbers are added, their sum is 96. What are the four numbers?

   Let's call the first number: x. Then the next three odd numbers are: $x + 2$, $x + 4$, and $x + 6$. Remember that if x is odd, then $x + 1$ is even.

   The sum of these four numbers must be 96. This gives us the equation:

$$\begin{aligned} x + (x + 2) + (x + 4) + (x + 6) &= 96 \\ x + x + 2 + x + 4 + x + 6 &= 96 \\ 4x + 12 &= 96 \\ 4x &= 96 - 12 \\ 4x &= 84 \\ \frac{4x}{4} &= \frac{84}{4} \\ x &= 21 \end{aligned}$$

   The four numbers are: 21, 23, 25, and 27. We can also check that: $21+23+25+27 = 96$.

8. Your car was recently repaired for a total cost of \$264. The bill indicated \$89 for parts, and 5 hours of labour. What is the hourly cost of the labour?

   Since we are being asked to find the hourly cost of the labour, let's use the variable c to represent this cost. This is the cost for one hour of labour. The bill indicates that there were 5 hours of labour, so the total cost of the labour is: $5c$. This amount must be added to \$89 to get the total amount of the bill. This gives us the equation:

$$\begin{aligned} 89 + 5c &= 264 \\ 5c &= 264 - 89 \\ \frac{5c}{5} &= \frac{175}{5} \\ c &= 35 \end{aligned}$$

   The labour costs \$35 per hour. We can also check that: $89 + 5(35) = 264$.

9. This month's cell phone bill was \$74.75. You know what your plan charges you \$42.50 each month, as well as \$0.50 for each minute of airtime. How many minutes of airtime did you use this month?

   If we let m represent the number of minutes, then the cost of those minutes is: $0.50m$. This amount must be added to \$42.50 to get the total amount of the bill. This gives us the equation:

$$\begin{aligned} 42.50 + 0.50m &= 74.75 \\ 0.50m &= 74.75 - 42.50 \\ \frac{0.50m}{0.50} &= \frac{32.35}{0.50} \\ m &= 64.5 \end{aligned}$$

   You used 64.5 minutes of airtime. You can also check that: $42.50 + 0.50(64.5) = 74.75$.

10. Find the mistake(s) that were made in solving the following equations:

Problem i:

$$9(x-1) = 99 \tag{1}$$

$$9(x-1) - 9 = 99 - 9 \tag{2}$$

$$x - 1 = 90 \tag{3}$$

$$x = 89 \tag{4}$$

On line (2), 9 is subtracted from both sides of the equation, but this would not cancel the 9 at the front of the expression, because that 9 is multiplying $(x-1)$. In order to cancel that 9, you should divide both sides of the equation by 9.

Also from line (3) to line (4), 1 is added to the left hand side to isolate x, but 1 is subtracted from the right hand side. The right hand side should be 91, not 89.

Problem ii:

$$2x + 3 = 3x + 4 \tag{5}$$

$$5x + 3 = 4 \tag{6}$$

$$5x = 7 \tag{7}$$

$$x = \frac{7}{5} \tag{8}$$

From line (5) to line (6), $2x$ became $5x$, which means $3x$ was added to the left side. But $3x$ needs to be subtracted from the right side, which means it also needs to be subtracted from the left side. The $5x$ should be $-x$.

From line (6) to line (7), 4 became 7, which means 3 was added to the right side. But 3 needs to be subtracted from the left side, which means it also needs to be subtracted from the right side. The 4 should be a 1.

Problem iii:

$$-(8-b) = -6 \tag{9}$$

$$-8 + b = -6 \tag{10}$$

$$b = \frac{-6}{-8} \tag{11}$$

$$b = \frac{3}{4} \tag{12}$$

Everything from line (9) to line (10) is correct. But from line (10) to line (11), the right hand side was divided by -8, which implies that the left hand side was also divided by -8. But on line (10), the number -8 and the variable b are being **added**. In order to undo that, -8 should be subtracted from the left hand side. This is the same thing as adding 8. So the correct thing to do would be to add 8 to both sides.

11. Calvin and Travis are arguing again, this time about the solution to the equation:

$$3(x+3) - 2x = 9 + 2x$$

Here is how they each solved this equation:

Calvin

$$\begin{aligned} 3x + 9 - 2x &= 9 + 2x \\ x + 9 &= 9 + 2x \\ x + 9 - 9 &= 9 + 2x - 9 \\ x &= 2x \\ x - 2x &= 0 \\ -x &= 0 \\ x &= 0 \end{aligned}$$

Travis

$$\begin{aligned} 3x + 9 - 2x &= 9 + 2x \\ x + 9 &= 9 + 2x \\ x + 9 - 9 &= 9 + 2x - 9 \\ x &= 2x \\ \frac{x}{x} &= \frac{2x}{x} \\ 1 &= 2 \end{aligned}$$

No solution

Who is right? Explain why.

Once again Calvin is right. Travis' mistake was on the second-to-last line when he divided by x. You should never divide by an expression that contains a variable because there may be a value for that variable that makes the expression equal to 0. And division by 0 is not defined.

In fact, in this particular example, notice that the solution to the equation *is* 0. So when Travis divided by x, he essentially divided by 0.