*** SOLUTIONS ***

1. Evaluate the following expressions, if possible. If it is not possible, explain why:

   (a) $5(3-7) - 2(12-2)$
   $= 5(-4) - 2(10)$
   $= -20 - 20$
   $= -40$

   (b) $-2(6-8) + 3(8-11)$
   $= -2(-2) + 3(-3)$
   $= 4 - 9$
   $= -5$

   (c) $\dfrac{a^2-1}{b^3+1}$ when $a = -1$, $b = 1$
   $= \dfrac{(-1)^2 - 1}{(1)^3 + 1}$
   $= \dfrac{1-1}{1+1}$
   $= \dfrac{0}{2}$
   $= 0$

   (d) $\dfrac{a^2-1}{b^3+1}$ when $a = 1$, $b = -1$
   $= \dfrac{(1)^2 - 1}{(-1)^3 + 1}$
   $= \dfrac{1-1}{-1+1}$
   $= \dfrac{0}{0}$
   Division by 0 is not defined, so we cannot evaluate this expression.

(e) $\dfrac{3x-4z}{\sqrt{y}}$ when $x=-1$, $y=25$, $z=-7$

$= \dfrac{3(-1)-4(-7)}{\sqrt{25}}$

$= \dfrac{-3+28}{5}$

$= \dfrac{25}{5}$

$= 5$

2. Which of the following are considered algebraic expressions?

   (a) $mx+b$: Yes

   (b) $\dfrac{9+x}{\sqrt{-x}}$: Yes

   (c) $\dfrac{11x}{3+}$: No

   (d) $\sqrt{\sqrt{9b}}$: Yes

   (e) $9 \div (-3)$: Yes

   (f) $9 - (\div 3)$: No

3. Distribute the following, and then combine like terms and simplify:

   (a) $-3(4a-b)+2(-a+b)$

   $= -3(4a)-3(-b)+2(-a)+2(b)$

   $= -12a+3b-2a+2b$

   $= -12a-2a+3b+2b$

   $= -14a+5b$

   (b) $(3m-2n)(3m+2n)$

   $= 3m(3m)+3m(2n)-2n(3m)-2n(2n)$

   $= 9m^2+6mn-6mn-4n^2$

   $= 9m^2-4n^2$

4. Perform the following operations, and then simplify by collecting like terms:

   (a) $(1 + 3x)(2x) - 6x$
   $= 1(2x) + 3x(2x) - 6x$
   $= 2x + 6x^2 - 6x$
   $= 6x^2 - 4x$

   (b) $2(t^2 + 5) - 3(t^2 + 5) + 5(t^2 + 5)$
   $= 2(t^2) + 2(5) - 3(t^2) - 3(5) + 5(t^2) + 5(5)$
   $= 2t^2 + 10 - 3t^2 - 15 + 5t^2 + 25$
   $= 4t^2 + 20$

5. What is the result when $x - 1$ is subtracted from $x + 1$?

   $(x + 1) - (x - 1)$

   $= x + 1 - x - (-1)$

   $= x + 1 - x + 1$

   $= 2$

6. Add the following algebraic expressions:

   (a) $(3b + 2a^2) + (a^2 - b) + (2b - 3a)$
   $= 3b + 2a^2 + a^2 - b + 2b - 3a$
   $= 2a^2 + a^2 - 3a + 3b - b + 2b$
   $= 3a^2 - 3a + 4b$

   (b) $(x^2 + 3x) + (3x - 2x^2) + (4x - 6 + 5x^2)$
   $= x^2 + 3x + 3x - 2x^2 + 4x - 6 + 5x^2$
   $= x^2 - 2x^2 + 5x^2 + 3x + 3x + 4x - 6$
   $= 4x^2 + 10x - 6$

7. Simplify the following expressions using the properties of exponents so that there are only positive exponents:

   (a) $\dfrac{m^{-3}\, n^{6}}{(m^{2}\, n)^{3}}$

   $$= \frac{m^{-3}\, n^{6}}{(m^{2})^{3}\, (n)^{3}}$$
   $$= \frac{m^{-3}\, n^{6}}{m^{2(3)}\, n^{3}}$$
   $$= \frac{m^{-3}}{m^{6}} \cdot \frac{n^{6}}{n^{3}}$$
   $$= m^{-3-6} \cdot n^{6-3}$$
   $$= m^{-9} \cdot n^{3}$$
   $$= \frac{1}{m^{9}} \cdot n^{3}$$
   $$= \frac{n^{3}}{m^{9}}$$

   (b) $\dfrac{(x^{-1}\, y^{-2})^{-3}}{x^{2}\, y^{4}}$

   $$= \frac{(x^{-1})^{-3}\, (y^{-2})^{-3}}{x^{2}\, y^{4}}$$
   $$= \frac{x^{-1(-3)}\, y^{-2(-3)}}{x^{2}\, y^{4}}$$
   $$= \frac{x^{3}}{x^{2}} \cdot \frac{y^{6}}{y^{4}}$$
   $$= x^{3-2} \cdot y^{6-4}$$
   $$= xy^{2}$$

   (c) $\dfrac{21\, a^{3}\, b^{7}}{-7\, a^{4}\, b}$

   $$= \frac{21}{-7} \cdot \frac{a^{3}}{a^{4}} \cdot \frac{b^{7}}{b}$$
   $$= -3 \cdot a^{3-4} \cdot b^{7-1}$$
   $$= -3 \cdot a^{-1} \cdot b^{6}$$
   $$= -3 \cdot \frac{1}{a} \cdot b^{6}$$
   $$= \frac{-3\, b^{6}}{a}$$

8. True or False:

   (a) $\dfrac{(a^5)^3}{(b^3)^{-5}} = (ab)^{15}$

   True, and we can prove it by simplifying the left hand side:

   $$\frac{(a^5)^3}{(b^3)^{-5}}$$
   $$= \frac{a^{5(3)}}{b^{3(-5)}}$$
   $$= \frac{a^{15}}{b^{-15}}$$
   $$= a^{15}b^{15}$$
   $$= (ab)^{15}$$

   (b) $(x+y)^2 = x^2 + y^2$

   False, and we can prove it by showing an example where it is not true. To do this, we let $x = 1$, and $y = 2$. Then by the order of operations we get:

   $(x+y)^2 = (1+2)^2 = (3)^2 = 9$

   But:

   $x^2 + y^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

   And since $9 \neq 5$, this statement is not always true, which makes it false.

   (c) $-2^2 = (-2)^2$

   False.

   The expression on the left says to take the negative of 2 squared, while the expression on the right says to square the number "negative 2".

   $-2^2 = -(2 \times 2) = -(4) = -4$

   But:

   $(-2)^2 = (-2)(-2) = 4$

9. Today I worked twice as many hours as yesterday. Tomorrow I will work three times as many hours as yesterday. Using a variable to represent the number of hours I worked yesterday, write an algebraic expression for the total number of hours I will have worked.

   Let's use h represent the numbers of hours I worked yesterday. Then the numbers of hours I worked today is: $2h$. And the numbers of hours I will work tomorrow is: $3h$. The total number of hours worked is: $h + 2h + 3h = 6h$

10. Calvin and Travis are having an argument over a post they saw on Facebook. The post asked for the value of the following mathematical expression:

$$4 - 4 \div 4 + 4 \times 4 + 4$$

Calvin believes the answer is 23, while Travis claims it is 20. Who is correct? Explain why.

Calvin is correct, the answer is 23. This is because Calvin followed the proper order of operations, while Travis did not. You must perform multiplication and division *before* addition and subtraction. You must also perform them as they appear from left to right:

$4 - 4 \div 4 + 4 \times 4 + 4$

$= 4 - 1 + 4 \times 4 + 4$

$= 4 - 1 + 16 + 4$

$= 3 + 16 + 4$

$= 19 + 4$

$= 23$

This is what Travis did:

$4 - 4 \div 4 + 4 \times 4 + 4$

$= 0 \div 4 + 4 \times 4 + 4$

$= 0 + 4 \times 4 + 4$

$= 4 \times 4 + 4$

$= 16 + 4$

$= 20$

Travis viewed the expression as a list of instructions to be performed one after another, but that is not the correct order of operations.

11. A "magician" on YouTube claims to be able to read your mind by making you follow these instructions. Play along:

- Choose the number of your birth month: January = 1, February = 2, etc.
- Multiply that number by 2.
- Now add 10 to the result.

- Divide this new number by 2.
- Subtract the original number you chose in the first step.
- Think very hard about this final number and I will attempt to read your mind.

  ... you are thinking of the number 5.

Using a variable, such as x or n, write an algebraic expression that represents the steps of the "magic trick". Simplify your expression to show that the result is always 5, no matter what number you start with.

Following the steps of the magic trick we can create an algebraic expression:

- Choose a number: n
- Multiply by 2: $2n$
- Now add 10: $2n + 10$
- Divide by 2: $(2n + 10) \div 2$
- Subtract the original number: $(2n + 10) \div 2 - n$

Simplifying this expression gives us the result:

$(2n + 10) \div 2 - n$

$= \dfrac{2n + 10}{2} - n$

$= \dfrac{2n}{2} + \dfrac{10}{2} - n$

$= n + 5 - n$

$= 5$

12. Recall the expression from question 10 :

$$4 - 4 \div 4 + 4 \times 4 + 4$$

The value of this expression is 23. Now consider the following expression:

$$(4 - 4) \div 4 + 4 \times (4 + 4)$$

Notice that there are now brackets placed around certain numbers. What is the value of this new expression?

(i) 23 (ii) 20 (iii) 32 (iv) 0

Brackets create groups that force certain operations to be performed first. We must now do the operations inside the brackets before we do anything else:

$(4-4) \div 4 + 4 \times (4+4)$

$= (0) \div 4 + 4 \times (8)$

Now we perform multiplication and division, as they appear from left to right:

$= 0 + 4 \times (8)$

$= 0 + 32$

$= 32$

The correct answer is option (iii).

13. Which of the following expressions represents the subtraction of $x+3$ from $2x+1$?

(i) $2x+1-x-3$

(ii) $(2x+1)-(x+3)$

(iii) $2x+1-x+3$

(iv) $(x+3)-(2x+1)$

To subtract $x+3$ **from** $2x+1$, we must write $2x+1$ first, and then write the minus sign, followed by $x+3$.

But, we must subtract **all** of $x+3$ from $2x+1$. To do this, we have to group $x+3$ into brackets, like so:

$2x+1-(x+3)$

This is what's shown in option (ii) so that is a correct answer.

Now we can simplify this expression by distributing the minus over the bracket containing $x+3$, giving us this:

$2x+1-x-3$

This is option (i), so that is also a correct answer.

Lastly, we can keep simplifying by combining the like terms:

$x-2$

This expression isn't one of the options, so let's simply look at the other two.

The expression in option (iii) subtracts x from $2x+1$, but then **adds** 3. We need the 3 to also be subtracted from $2x+1$, not added. The lack of brackets around all of $(x+3)$ makes this option incorrect.

Lastly, option (iv) shows $2x + 1$ being subtracted from $x + 3$. That's completely backwards from what we were asked to do. So option (iv) is incorrect.

14. Which of the following expressions represents the division of $12x^2 + 9x$ by $3x$?

    (i) $\dfrac{12x^2 + 9x}{3x}$ (ii) $4x + 3$ (iii) $12x^2 + 9x \div 3x$ (iv) $\dfrac{12x^2}{3x} + \dfrac{9x}{3x}$

    Without too much effort we can see that in option (i) the expression $12x^2 + 9x$ is being divided by $3x$. So that is a correct answer.

    At first glance it may seem like option (iii) is also correct, but if you remember the order of operations, in option (iii) only $9x$ is being divided by $3x$, and that is not what we want. We want both $12x^2$ and $9x$ to be divided by $3x$. So option (iii) is not correct.

    Option (iv) has two fractions in it, but they have the same denominator. This means that we can add them together by simply adding the numerators. Option (iv) can be rewritten as:

    $$\frac{12x^2}{3x} + \frac{9x}{3x} = \frac{12x^2 + 9x}{3x}$$

    And this is exactly the same as option (i), so it is also correct.

    The last one to check is option (ii), but now that we know that options (i) and (iv) are correct, we can show that option (ii) is as well, by simplifying option(iv):

    $$\frac{12x^2}{3x} + \frac{9x}{3x} = \frac{12}{3} \cdot \frac{x^2}{x} + \frac{9}{3} \cdot \frac{x}{x} = 4 \cdot x + 3 \cdot 1 = 4x + 3$$

    So option (ii) is also correct.

15. Which of the following expressions represents the product of $x + 7$ and $x - 3$?

    (i) $x^2 - 3x + 7x - 21$ (ii) $(x - 3)(x + 7)$

    (iii) $x(x - 3) + 7(x - 3)$ (iv) $(x + 7)(x - 3)$

    A product is the result of multiplication, so in this problem we want to multiply $x + 7$ and $x - 3$. Doing this will require putting brackets around the two expressions, as showing in option (iv), which is correct.

    Option (ii) is also correct, because it still shows $x + 7$ and $x - 3$ being multiplied, just in a different order. The result will still be the same though (isn't it true that $4 \cdot 5$ is equal to $5 \cdot 4$?)

Option (iii) is the result of taking the expression in option (iv) and distributing the multiplication. The x and the 7 in the first bracket both have to multiply $(x-3)$, and the expression shown in option (iii) is exactly this:

$$(\mathbf{x}+\mathbf{7})(x-3)$$

$$= \mathbf{x}(x-3)+\mathbf{7}(x-3)$$

Lastly, option (i) is the result of further distributing the expression from option (iii):

$$\mathbf{x}(x-3)+\mathbf{7}(x-3)$$

$$= \mathbf{x}(x)-\mathbf{x}(3)+\mathbf{7}(x)-\mathbf{7}(3)$$

$$= x^2-3x+7x-21$$

So option (i) is also correct. All four options in this case are correct answers.