ECON 303 : MIDTERM 2 Solution, Winter 2018

Instructions: The test is closed book/notes. Calculators are allowed. There are 100 points.

Problem 1. (35 points) Consider the two-period consumption-savings model we developed in class. As in class, maintain the simplifying assumption that $A_0 = 0$. Assume that $y_1$and y_2 stand for real income in period 1 and period 2, respectively, and r for the real interest rate. Suppose that the consumer consumes $c_1$in period 1 and c_2 in period 2 and saves or borrows A_1 optimally in period 1. (There is no government here).

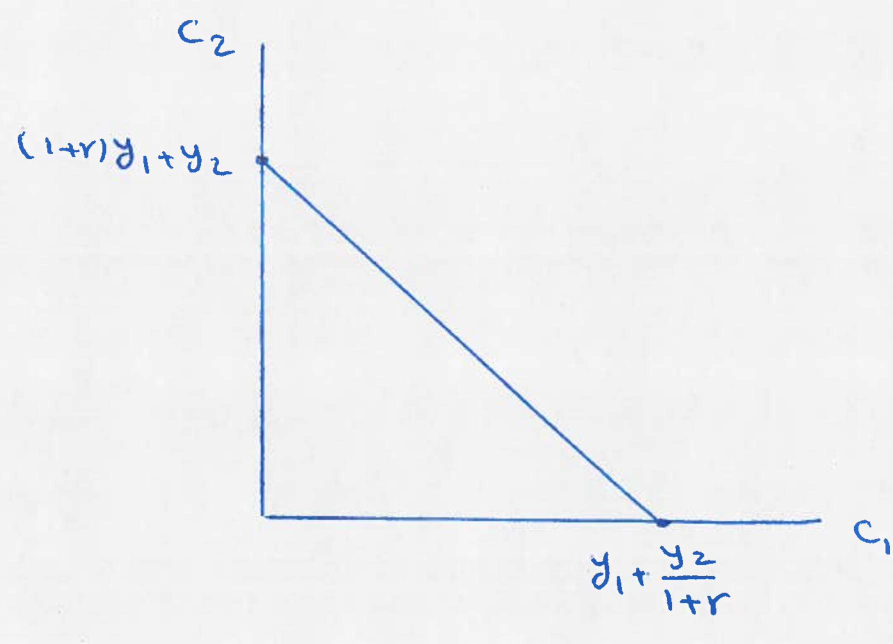
a. (7 points) Write down the period-1 and period-2 budget constraints and derive the Lifetime Budget Constraint (LBC). With c_2 on the vertical axis and c_1 on the horizontal axis, what is the slope of LBC? Why? Plot the LBC.

$$c_1 + A_1 = y_1$$

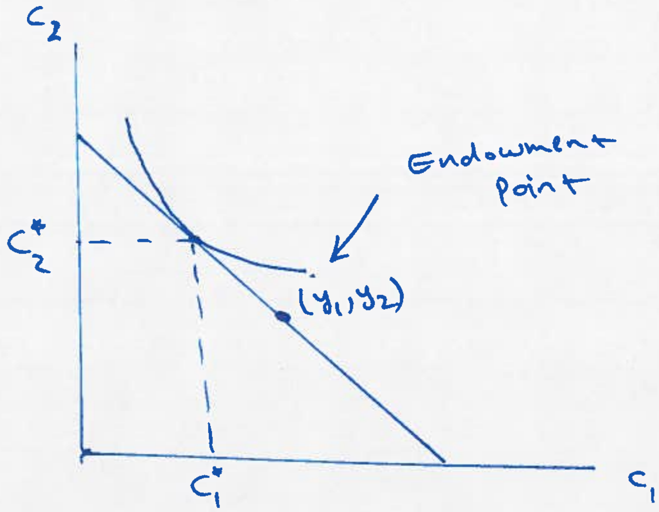
$$c_2 = y_2 + (1+r)A_1$$

Solve for A_1 $\Rightarrow$ $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$

The slope is $-(1+r)$, because it is simply $\frac{dc_2}{dc_1}$.



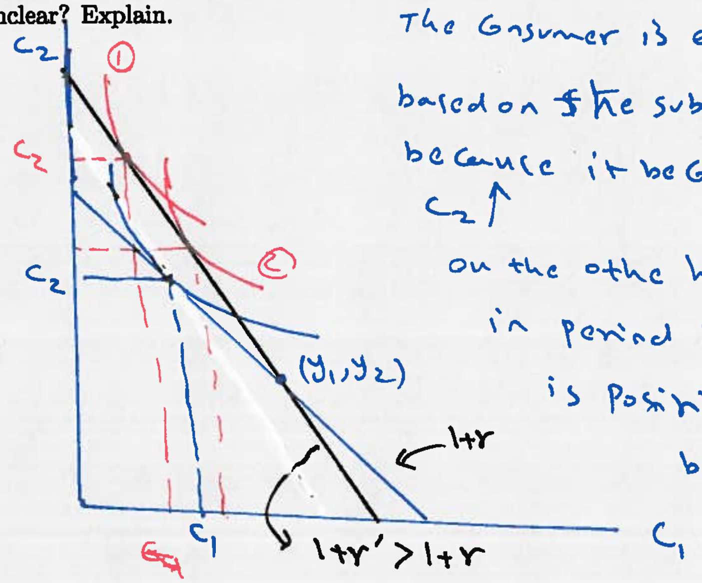
b. (5 points) Suppose that the household consumes **less than** its real income in period one. In a single carefully-labeled indifference-curve/budget constraint diagram show the optimal choice of c_1 and c_2 that is determined from the interaction of the budget constraint and the indifference map. **Highlight** the position of endowment point relative to the optimal choice. What is the price of c_1 in terms of c_2? Why?



The price of c_1 in terms of c_2 is $1+r$, because if the consumer gives up 1 unit of c_1 today he/she will receive $1+r$ units of consumption goods tomorrow.

c. (6 points) Continuing with what we assumed in part a, clearly describe/discuss, **in terms of the substitution effect, the income effect, and then the total effect**, the consequences of an increase in the real interest rate r on the optimal values of c_1 and c_2. Clearly indicate on a new diagram the positions of optimal c_1 and c_2 before and after the change in r where the position of the endowment point is also clear. (Remember that the position of optimal c_1 and c_2 before the change in r is exactly what you showed in part a). What can we say about the relationship between the real interest rate and private savings based on what we assumed in part a, and you derived here? Is it positive? Is it negative? Or unclear? Explain.

① & ② both c_2 are possible



The consumer is a saver, once $r \uparrow$ based on the substitution effect $c_1 \downarrow$ because it becomes more expensive. And $c_2 \uparrow$

on the other hand since interest income in period 2 $\uparrow$, the income effect is positive, therefore $c_1 \uparrow$ and $c_2 \uparrow$ because both are normal goods

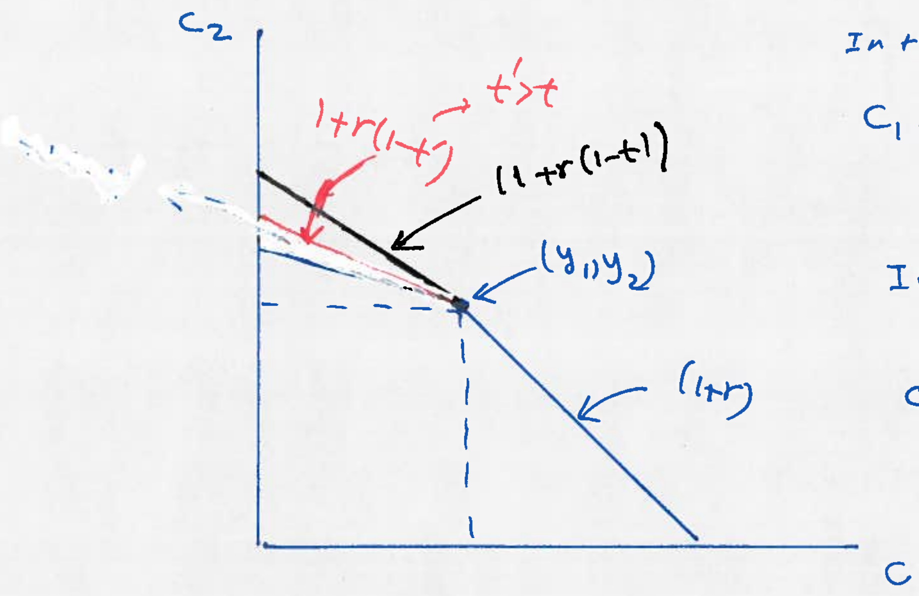
Therefore, on net, we cannot theoretically determine whether $c_1 \uparrow$ or $c_1 \downarrow$. But it is clear both effects imply $c_2 \uparrow$

Since it is not clear what happens for c_1 it is not clear what happens for Savings

d. Now suppose that the government introduces a tax on interest earnings. That is, **borrowers** face a real interest rate of r before and after the tax is introduced, but **lenders** receive an interest rate of $(1-t)r$ on their savings, where t is the tax rate. Therefore, we are looking at the effects of having t increase from zero to some values greater than zero, with r assumed to remain constant.

1. (7 points) Show the effects of the increase in the tax rate on a consumer's lifetime budget constraint. **(Graphically and mathematically)**.

The tax rate only affects lenders. As long as the consumer is a borrower, the slope of the LBC remains r, But in the case of lenders, the slope will be $1+(1-t)r$.



In the Case of borrower

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

In the Case of lender

$$c_1 + \frac{c_2}{1+r(1-t)} = y_1 + \frac{y_2}{1+r(1-t)}$$

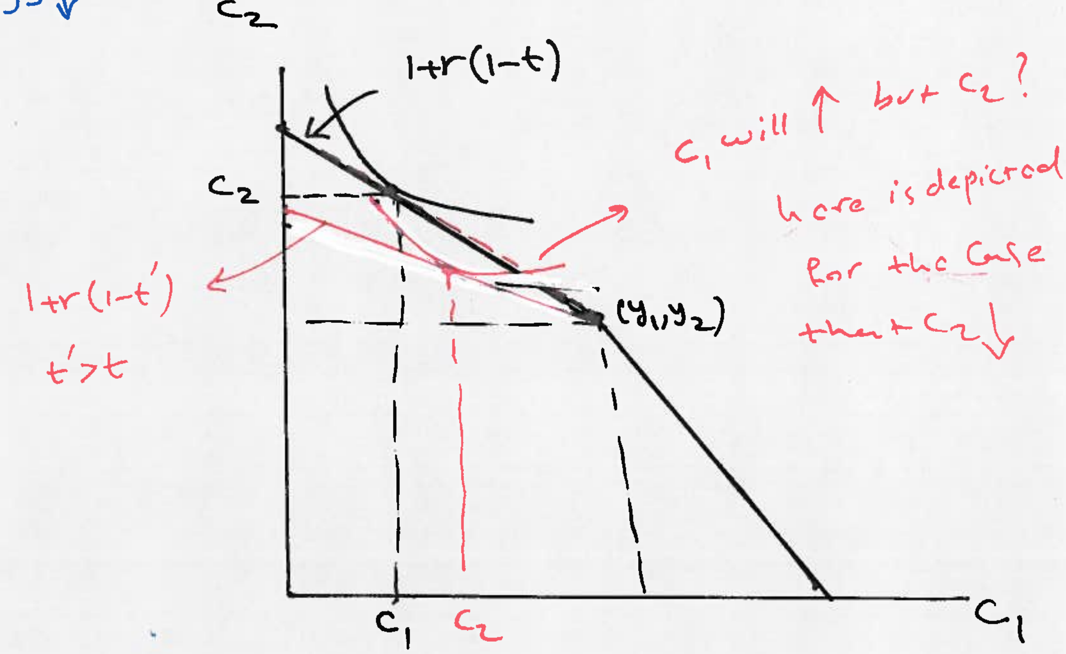
once $t \uparrow$ from t_1 to t_2

then $1+r(1-t_1) > 1+r(1-t_2)$

once $t \uparrow$, the part of LBC on the right hand side of (y_1, y_2) doesn't change, but the left hand side becomes flatter.

2. (10 points) How does the increase in t affect the optimal choices of c_1 and c_2 and saving for the consumer. Show how income and substitution effects matter for your answer, and show how it matters whether the consumer is initially a lender. **(Use graph to justify your answer)**.

If the Consumer is initially a borrower, then the change in t doesn't affect his/her behavior, But if the Consumer is initially a lender, then an increase in t lowers $1+r(1-t)$ and therefore, the BC becomes flatter. So c_1 becomes cheaper in terms of c_1 and therefore $c_1 \uparrow$ and $c_2 \downarrow$ (the SE). At the same time interest income received in period 2 $\uparrow$, therefore the IE implies that both c_1 and $c_2 \uparrow$. On net $c_1 \uparrow$ and c_2 not clear. since $c_1 \uparrow$ savings $\downarrow$.



Problem 2. (25 points) Suppose that in our standard one-period economy the representative household's preferences are described by the utility function $u(C, L) = C + \frac{1}{2}L^2 - \frac{1}{2}L$. Suppose that there is a representative firm that hires labor in a perfectly competitive labor market in order to produce output according $f(N) = N$. The (real) wage is w and is taken as given by the firm. We also assume that there is a government that must pay for all of its spending through labor income taxes. Therefore if G stands for government spending, then

$$G = t.w.N.$$

Assume that dividend payments are 0, $\pi = 0$. First, write down the budget constraint of the representative household and then, given the form of the utility function and the production function, solve the rep. household and the rep. firm problems. Then express total tax revenue as a function of labor tax rate $TR(t)$, and determine whether a Laffer curve arises or not. Explain your results. In the case of arising a Laffer curve, compute the optimal value of t.

The BC of the rep. household is $C = (1-t)w\cancel{N}$ or $C = (1-t)w(\overset{''}{h} - L)$

suppose $h=1$.

The household's problem is to maximize U given the BC $\Rightarrow$

$$MRS_{\ell,c} = (1-t)w \Rightarrow \frac{\frac{1}{2}2\ell - \frac{1}{2}}{1} = (1-t)w \Rightarrow \underline{\ell^* - \frac{1}{2} = (1-t)w}$$

The firm's problem is to maximize profits : $\pi = N - wN \Rightarrow$

$\underline{w = 1}$ $\quad G = twN$, therefore $TR(t) = twN$.

So
$$\left.\begin{array}{l} \ell^* - \frac{1}{2} = (1-t)w \\ w = 1 \\ C = (1-t)wN \\ N = 1 - L \end{array}\right\} \Rightarrow N = t - \frac{1}{2}$$

since N is a function of $t \rightarrow$ there will be a Laffer curve

$$TR(t) = w \cdot N \cdot t = 1 \cdot (t - \tfrac{1}{2}) \cdot t = t^2 - \tfrac{1}{2}t$$

$$\frac{\partial TR}{\partial t} = 0 \Rightarrow 2t - \tfrac{1}{2} = 0 \Rightarrow \underline{t^* = \tfrac{1}{2}}$$

Problem 3. (20 points) Consider the two-period consumption-savings model, augmented with a government sector. Each consumer has preferences described by the utility function

$$u(c_1, c_2) = c_1^{\frac{1}{2}} c_2^{\frac{1}{2}}$$

Suppose that both household and the government start with zero initial assets (i.e., $A_0 = 0$ and $B_0 = 0$), and that the real interest rate is always 0 percent. Assume that government purchases in the first period are 8 and in the second period are 16. In the first period the government imposes lump-sum taxes t_1 and in the second period it levies lump-sum taxes in the amount of 5. Finally, the real income of the household in periods 1 and 2 are, $y_1 = 21$ and $y_2 = 15$, respectively.

a. (10 points) Compute the optimal levels of private and national savings in period one.

$MRS_{c_1,c_2} = 1+r \Rightarrow \dfrac{\frac{1}{2} c_1^{-1/2} c_2^{1/2}}{\frac{1}{2} c_1^{1/2} c_2^{-1/2}} = 1 \Rightarrow \boxed{c_1 = c_2}$

$c_1 + c_2 = y_1 - T_1 + y_2 - T_2$, we don't have t_1, but from the gov. LBC we know $g_1 + g_2 = T_1 + T_2 \Rightarrow T_1 = 24 - 5 = 19$

So $2c_1 = 21 - 19 + 15 - 5 \Rightarrow c_1 = 6 \Rightarrow \boxed{c_2 = 6}$

$s_1^p = y_1 - T_1 - c_1 = 21 - 19 - 6 = -4$

$s_1^n = y_1 - g_1 - c_1 = 21 - 8 - 6 = 7$

b. (5 points) Now suppose that a credit constraint on the household is in place, with lenders stipulating that household cannot be in debt at the end of period one. Will this credit constraint affect consumers' optimal decisions? Explain why or why not. Is this credit constraint welfare-enhancing, welfare-diminishing, or welfare-neutral? Why?

since the rep. household wants to be optimally a borrower in period 1 ($s_1^p = -4$), the credit constraint affects its behavior because it cannot borrow. And do to this it can consume only 2 units in period 1, therefore the credit constraint is welfare-diminishing.

c. (5 points) Now with the credit constraint described above in place, consider a tax cut of 5 units in the first period, with no change in government purchases. Does the Ricardian Equivalence hold due to this tax cut, or it fails? What happens for the private savings? Provide economic intuition for the result you obtain.

The Consumer wanted optimally to be borrower, but due to the credit Constraint it Couldn't. So once there is a tax cut it simply will Consume the tax cut until the point of optimal consumption ($c_1^* = 6$).

therefore, due to the 5 units tax cut c_1 will increase by 4 units and this means national savings will change → r will change →

The Ricardian Equivalence doesn't hold.

Problem 4. (20 points) Suppose that in our two-period standard model the government does not have access to lump-sum taxes, only proportional consumption taxes in both periods. Denote by τ_1 the tax rate on period-1 consumption and by τ_2 the tax rate on period-2 consumption. Assume also that g_1 and g_2 stand for government spending in period 1 and period 2, respectively. suppose also that r is the real interest rate and $B_0 = 0$.

a. (3 points) Write down the government's period-1 and period-2 budget constraints.

$$g_1 + B_1 = \tau_1 c_1$$

$$g_2 = B_1(1+r) + \tau_2 c_2$$

b. (2 points) Construct the government's lifetime budget constraint (LBC), showing important steps.

Solve for B_1, plug in second-period BC

$$g_1 + \frac{g_2}{1+r} = \tau_1 c_1 + \frac{\tau_2 c_2}{1+r}$$

c. (3 points) Assuming that the consumer consumes c_1 in period 1 and c_2 in period 2 and earns y_1 in period 1 and y_2 in period 2, write down the consumer's period-1 and period-2 budget constraints.

$$(1+\tau_1) c_1 + A_1 = y_1$$

$$(1+\tau_2) c_2 = A_1(1+r) + y_1$$

d. (5 points) Construct the consumer's lifetime budget constraint (LBC), showing important steps. What is the slope of LBC of the consumer? Why?

$A_1 = y_1 - (1+\tau_1) c_1$ Plug in period 2 BC

$(1+\tau_2) c_2 = (y_1 - (1+\tau_1) c_1)(1+r) + y_1$

$\Rightarrow (1+\tau_1) c_1 + \frac{(1+\tau_2) c_2}{1+r} = y_1 + \frac{y_2}{1+r}$

$\frac{dc_2}{dc_1} = - \frac{(1+r)(1+\tau_1)}{(1+\tau_2)}$ → the slope

e. (7 points) Assuming that there are no credit constraints, suppose the government keeps its sequence of G_1 and G_2 unchanged, but decides to cut the tax rate **in period 1 on period-1 consumption** – that is, it lowers the tax rate τ_1. Is it possible for this economy to exhibit Ricardian Equivalence? If so, carefully show how/why and provide brief economic interpretation. If not, precisely explain why not.

If $\tau_1 \downarrow$, τ_2 must $\uparrow$ in order to have the LBC of the gov. be satisfied. therefore

Since the slope of the LBC of the household is $\frac{(1+r)(1+\tau_1)}{(1+\tau_2)}$ once $\tau_1 \downarrow$ and $\tau_2 \uparrow$

the slope $\downarrow$ and therefore c_1 will change and S_1^n will change → r changes the Ricardian Equivalence doesn't hold.