Introduction to Quadratic Surfaces - Mathispower4u

Linear Function :

$$z = ax + by + c$$

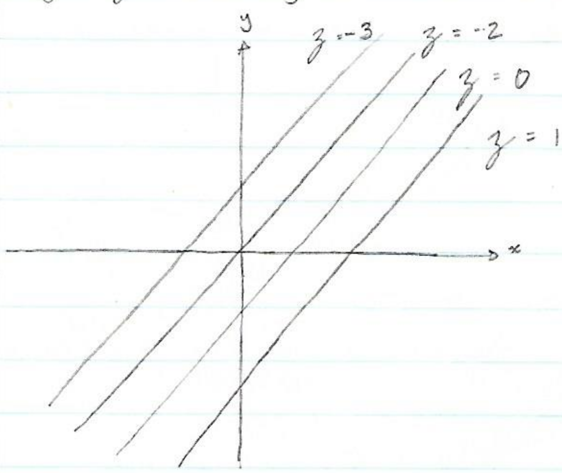
If $z = 0$, $ax + by + c = 0$

If $z = 1$, $ax + by + c = 1$

If $z = p$, $ax + by + c = p$

} All straight lines

Eg $z = 2x - 3y + 1$

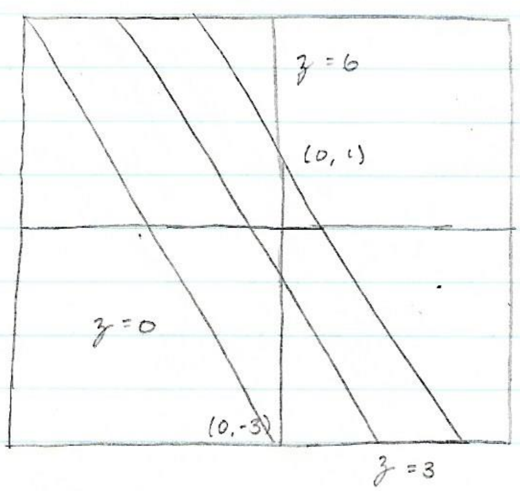


$z = 0$, $2x - 3y = -1$

$z = 1$, $2x - 3y = 0$

$z = 5$ $2x - 3y = 4$

Eg.



$y = ax + by + c$

$a = \frac{3}{2}$ $b = \frac{3}{4}$

$z = \frac{3}{2}x + \frac{3}{4}y + c$

$3 = \frac{3}{4}(1) + c$

$c = \frac{9}{4}$

$z = \frac{3}{2}x + \frac{3}{4}y + \frac{9}{4}$

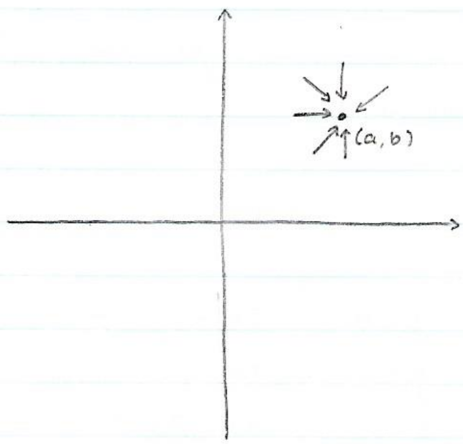
$4z = 6x + 3y + 9$

$6x + 3y - 4z = 9$

Limits of Two Variable Functions

$z = f(x, y)$; Let (a, b) be a point in the domain of $f(x, y)$

$\lim_{(x,y)\to(a,b)} f(x, y) = L$; L is a finite number. When (x, y) and (a, b) are close enough, then $|f(x,y) - L|$ can be arbitrarily small



; Infinitely many ways to approach (a, b), so the limit must be consistent in all directions in order for the limit to exist

Suppose we have a two variable function ; $z = f(x, y)$.
$z = f(x, y)$ is continuous at (a, b) if $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$ only if

i) $\lim_{(x,y)\to(a,b)} f(x, y)$ exists

ii) (a, b) exists in the domain of $f(x, y)$

iii) $f(a, b)$ exists

Then, if $f(x, y)$ is continuous everywhere, $f(x, y)$ is continuous

14.3 Partial Derivatives

Instantaneous Rate of Change at x

$$\lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h} = f_x(x, y)$$; Partial derivative of x

Notation :

$$f_x(x, y) = \frac{\partial z}{\partial x}(x, y) = D_x(f)$$

Partial Derivative of y

$$\lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h}$$; Treat x as a constant

Eg. $z = e^{x/y}$

$z = e^{x/y}$; $u = \frac{x}{y}$ $u' = \frac{1}{y}$

$= e^u$; $z_u = e^u$

$z_x = \frac{1}{y} e^{x/y}$

$z = e^{x/y}$; $u = \frac{x}{y}$ $u'_y = -\frac{x}{y^2}$

$z_y = -\frac{x}{y^2} e^{x/y}$

Eg. $z = x^2y^2 + 3xy + x - y$

$z_x = 2xy^2 + 3y + 1$

$z_y = 2x^2y + 3x - 1$

Eg $z = x \ln(y^2 - x)$

$z_x = \ln(y^2 - x) + x \frac{1}{y^2 - x}(-1)$

$= \ln(y^2 - x) - \frac{x}{y^2 - x}$

$z_y = x \cdot \frac{1}{y^2 - x}(2y)$

$= \frac{2xy}{y^2 - x}$