Université d'Ottawa
Département de
génie mécanique

uOttawa

University of Ottawa
Department of
mechanical engineering

Mid-term Exam
MCG 3340, Fluid Mechanics I

November 18th, 2016
11:30–12:50

Closed-book, non-programmable calculators only.

Name: Solutions

Student number:

Question	Possible	Result
1	10	
2	10	
3	10	
Total	**30**	

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\iiint\limits_V \rho\,\mathrm{d}V + \oiint\limits_S \rho\vec{v}\cdot\hat{n}\,\mathrm{d}S$$

$$\vec{\nabla}p = \rho(\vec{g}-\vec{a})$$

$$\sum\vec{F} - \iiint\limits_V \rho\vec{a}\,\mathrm{d}V = \frac{\mathrm{d}}{\mathrm{d}t}\iiint\limits_V \rho\vec{v}\,\mathrm{d}V + \oiint\limits_S \rho\vec{v}\,\vec{v}\cdot\hat{n}\,\mathrm{d}S$$

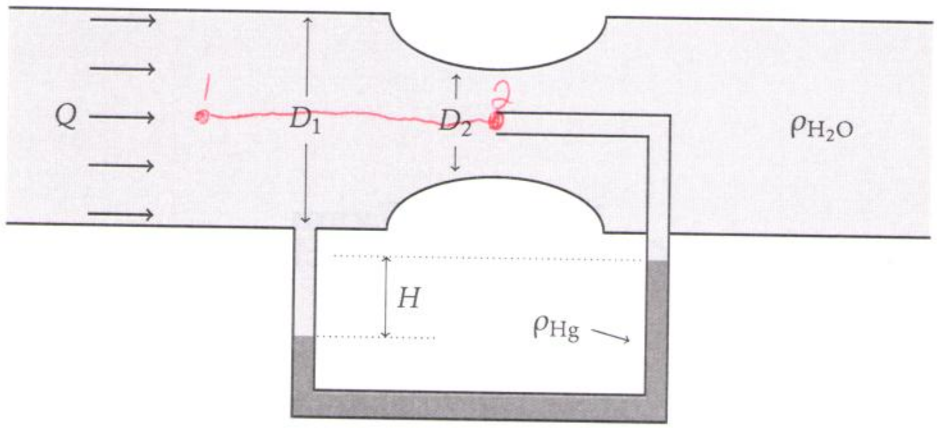
$$\mathrm{d}\vec{F} = -p\hat{n}\,\mathrm{d}A$$

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{const}$$

$$\mathrm{d}\vec{M} = \vec{r}_\ell \times \mathrm{d}\vec{F}$$

$$\frac{k}{k-1}\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{const}$$

1) An enterprising engineer has invented a new "super" flow meter for water flow in circular pipes that combines a Pitot tube and a Venturi flow meter. In this new design, how is the difference in height of mercury in the manometer, H, related to the volumetric flow rate, Q, the diameter of the pipe, D_1, and the diameter of the throat, D_2?



$$P_1 + \frac{\rho V_1^2}{2} + \cancel{\rho g z_1} = P_2 + \cancel{\frac{\rho V_2^2}{2}} + \cancel{\rho g z_2}$$

$$P_1 - P_2 = \frac{\rho V_1^2}{2} \qquad V_1 = \frac{Q}{\pi D_1^2/4} = \frac{4Q}{\pi D_1^2}$$

$$= \frac{\rho\left(4Q/\pi D_1^2\right)^2}{2} = \frac{16\rho Q^2}{2\pi^2 D_1^2}$$

$$\Delta p = gH(\rho_{Hg} - \rho_{H_2O}) = \frac{16\rho Q^2}{2\pi^2 D_1^2}$$

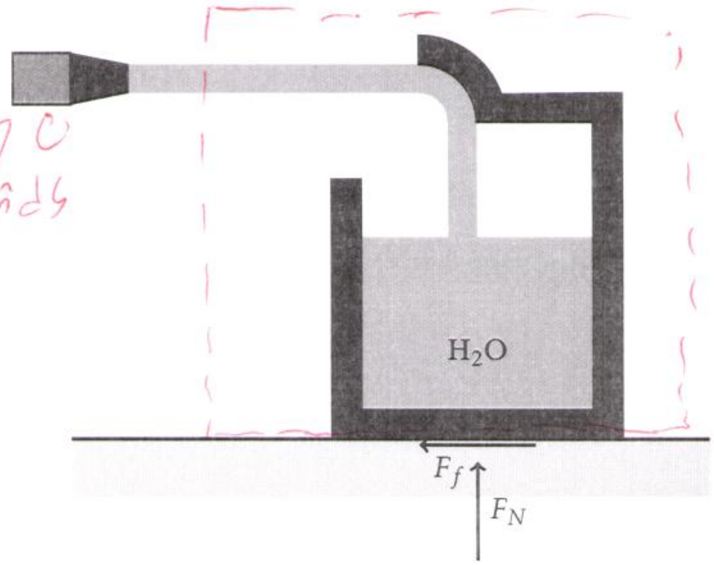
$$\boxed{H = \frac{8\rho Q^2}{\pi^2 D_1^2 g(\rho_{Hg} - \rho_{H_2O})}}$$

Name:

2) A jet of water with a diameter of 5 cm and velocity of 15 m/s strikes an object and is redirected into an attached container, as illustrated.

   a) When the container and contained water have a mass of 100 kg, what must be the normal force, F_N exerted by the ground?

   b) What is the minimum coefficient of static friction, μ_s, between the container and the ground such that the container does not move?



y dir:

a) $\sum F_y = \frac{d}{dt}\iiint_V \cancel{\rho v_y} \, dV + \cancel{\oint\!\!\oint_S \rho v_y \vec{v}\cdot\hat{n}\, dS}$ = 0

$\sum F_y = 0$

$F_N = -F_g = \boxed{981\,N}$

x-dir

b) $\sum F_x = \frac{d}{dt}\iiint_V \rho v_x \, dV + \oint\!\!\oint_S \rho v_x \vec{v}\cdot\hat{n}\, dS$

$F_f = \rho V_x(-V_x) S$

$S = \frac{\pi D^2}{4} = 0.00196\,m^2$

$-100\,kg\,(9.81\,m/s^2)\,\mu_s = -1000\,kg/m^3\,(15\,m/s)^2\,(0.00196\,m^2)$

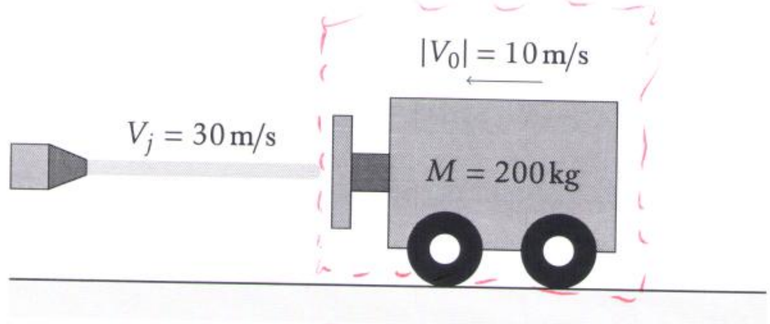
$\boxed{\mu_s = 0.45}$

3) Un bloc de masse, $M = 200\,\mathrm{kg}$, se déplace avec une vitesse initiale de $10\,\mathrm{m/s}$ vers un tuyaux d'incendie. Lorsque le bloc est à $10\,\mathrm{m}$ de l'extrémité du tuyaux, un jet d'eau avec une vitesse, $V_j = 30\,\mathrm{m/s}$, et une section, $S = 0{,}005\,\mathrm{m}^2$, est dirigé ver le bloc. Le bloc frappera-t-il le tuyaux? Sinon, quelle est la distance la plus proche qu'il atteindra?

Négliger le frottement dans les roues.

hint:

$$\frac{x-A}{x+1} = \frac{x+1-(A+1)}{x+1} = 1 - \frac{A+1}{x+1}$$



$$\cancel{\sum F} - \iiint_V \rho \vec{a}\, dV = \cancel{\frac{d}{dt}\iiint_V \rho \vec{v}\, dV} + \iint_S \rho \vec{v}\, \vec{v}\cdot\hat{n}\, dS$$

x-dir:

$$-M a_x = \rho (V_j - u)[-(V_j-u)] S$$

$$M \frac{du}{dt} = \rho (V_j - u)^2 S$$

$$\int_{u_0}^{u} \frac{du}{(V_j - u^2)} = \int_0^t \frac{\rho S}{M}\, dt$$

$$\left[\frac{1}{V_j - u}\right]_{u_0}^{u} = \frac{\rho S}{M} t \Rightarrow \frac{1}{V_j-u} - \frac{1}{V_j - u_0} = \frac{\rho S}{M} t$$

$$\frac{1}{V_j - u} = \frac{1}{V_j - u_0} + \frac{gS}{M} t = \frac{1 + \frac{gS}{M}[V_j - u_0] t}{V_j - u_0}$$

$$V_j - u = \frac{V_j - u_0}{1 + \frac{gS}{M}[V_j - u_0] t}$$

define

$$Z = \frac{gS}{M}(V_j - u_0)$$

$$u = V_j - \frac{(V_j - u_0)}{1 + Zt} = \frac{\cancel{V_j} + V_j Z t - \cancel{V_j} + u_0}{1 + Zt}$$

$$u = \frac{u_0 + V_j Z t}{1 + Zt} = \frac{u_0 - V_j + V_j + V_j Z t}{1 + Zt}$$

$$u = \frac{dx}{dt} = \frac{u_0 - V_j}{1 + Zt} + V_j$$

$$\int_{x_0}^{x} dx = \int_0^t \left[\frac{u_0 - V_j}{1 + Zt} + V_j\right] dt$$

$$x - x_0 = (u_0 - V_j) Z \ln(1 + Zt) + V_j t$$

$$x = V_j t + (u_0 - V_j) z \ln(1 + z t) + x_0$$

$$z = \frac{(1000\,Kg/m^3)(0.005\,m^2)}{200\,kg}\left[30\,m/s - (-10\,m/s)\right] = 1\,s^{-1}$$

$$u = \frac{u_0 + V_j z t}{1 + z t}$$ $\therefore$ speed is zero when $u_0 + V_j z t = 0$

$$t = \frac{-u_0}{V_j z} = \frac{10\,m/s}{30\,m/s\,(1\,s^{-1})} = \frac{1}{3}\,s$$

If we assume the jet hits the block when it is 10 m away, $x_0 = 10\,m$

If we assume the jet is started when the block is 10 m away, it will take $\frac{\Delta x}{\Delta v} = \frac{10\,m}{40\,m/s} = \frac{1}{4}\,s$ to hit the block. In this case $x_0 = 9.75\,m$

we accept both answers.

$$x = (30\,m/s)\left(\tfrac{1}{3}\,s\right) + (-10\,m/s - 30\,m/s)(1\,s^{-1}) \ln\left[1 + (1\,s^{-1})\left(\tfrac{1}{3}\,s\right)\right] + 10\,m$$

$$\boxed{x = 8.5\,m}$$

does not hit.