Name:

1) a) List four assumptions necessary to apply Bernoulli's equation for either compressible or incompressible flow.

(1) - steady state

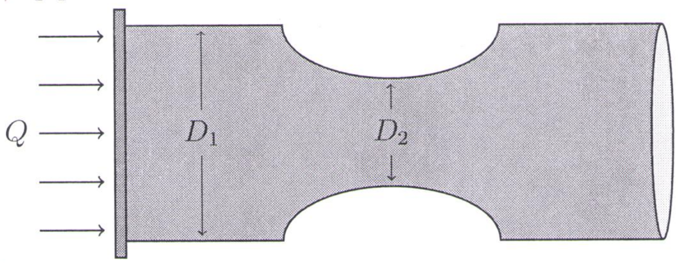
(1) - inviscid ($\mu = 0$)

(1) - along stream line

(1) - no other sources/sinks of energy

b) Water enters a pipe of diameter, D_1 at a relative pressure P_1. The pipe contains a contraction to a diameter, D_2. What is the maximum flow rate, Q, possible such that the water will not cavitate in the throat of the contraction. The water will cavitate if its pressure drops below its saturation pressure at a given temperature, P_{sat}.
Neglect friction losses in the pipe.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2}$$



(1) $V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2}$ $\quad V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2}$ $\quad P_{1abs} = P_1 + P_{atm}$, $P_{2abs} = P_{sat}$ (1)

$$\frac{P_1 + P_{atm}}{\rho} + \frac{1}{2}\left(\frac{4Q}{\pi D_1^2}\right)^2 = \frac{P_{sat}}{\rho} + \frac{1}{2}\left(\frac{4Q}{\pi D_2^2}\right)^2$$

(1) $$\frac{P_1 + P_{atm} - P_{sat}}{\rho} = \frac{1}{2}\left(\frac{16Q^2}{\pi^2}\right)\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right) = \frac{1}{2}\left(\frac{16Q^2}{\pi^2}\right)\left(\frac{D_1^4 - D_2^4}{D_2^4 D_1^4}\right)$$

$$Q^2 = \frac{(P_1 + P_{atm} - P_{sat})\, D_1^4 D_2^4 \pi^2}{8\rho\,(D_1^4 - D_2^4)}$$

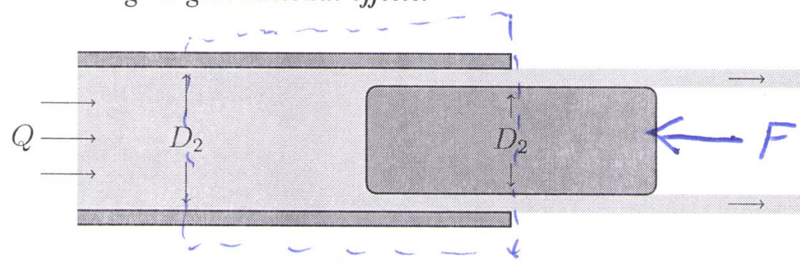
(1) $$Q = \frac{D_1^2 D_2^2 \pi}{2}\sqrt{\frac{P_1 + P_{atm} - P_{sat}}{2\rho(D_1^4 - D_2^4)}}$$

2) Find the force required to hold a cylindrical plug of diameter, D_2, in place at the end of a circular water pipe of diameter, D_1, as a function of flow rate, Q.

Assume frictionless flow and neglect gravitational effects.

$A_1 = \frac{\pi D_1^2}{4}$

$A_2 = \frac{\pi(D_1^2 - D_2^2)}{4}$



$A_1 - A_2 = \frac{\pi D_2^2}{4}$ $\quad P_2 = P_{atm} \approx 0$ $\quad V_1 = \frac{Q}{A_1}$ $\quad V_2 = \frac{Q}{A_2}$ (1)

$P_1 + \frac{\rho V_1^2}{2} = \cancel{P_2}^{0} + \frac{\rho V_2^2}{2} \Rightarrow P_1 = \frac{\rho}{2}(V_2^2 - V_1^2)$ (1)

(1) $\sum F_x = \sum v_x \rho \vec{V}\cdot\hat{n} A$

(1) $-F + F_p = -V_1^2 \rho A_1 + V_2^2 \rho A_2$

$F_p = \frac{\rho}{2}(V_2^2 - V_1^2) A_1$

(1) $F = F_p + V_1^2 \rho A_1 - V_2^2 \rho A_2$

$= \rho Q^2 \left\{ \left[\frac{1}{2A_2^2} - \frac{1}{2A_1^2}\right] A_1 + \frac{1}{A_1} - \frac{1}{A_2} \right\}$

$= \rho Q^2 \left\{ \frac{A_1}{2A_2^2} - \frac{1}{2A_1} + \frac{1}{A_1} - \frac{1}{A_2} \right\}$

$= \rho Q^2 \left\{ \frac{A_1^2 - 2A_1A_2 + A_2^2}{2A_2^2 A_1} \right\}$

$= \rho Q^2 \left\{ \frac{(A_1 - A_2)^2}{2A_1 A_2^2} \right\}$

$F = \frac{\rho Q^2}{2} \left\{ \frac{\left(\frac{\pi}{4} D_2^2\right)^2}{\frac{\pi}{4} D_1^2 \left(\frac{\pi}{4}[D_1^2 - D_2^2]\right)^2} \right\}$

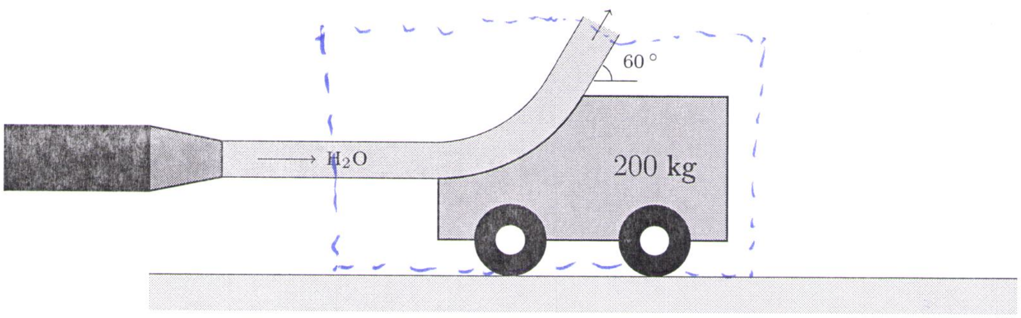
(1) $F = \frac{2\rho Q^2}{\pi} \left(\frac{D_2^4}{D_1^2 (D_1^2 - D_2^2)^2} \right)$

in the negative x direction.

Name:

3) A jet of water with speed, $V_j = 20$ m/s hits a 200 kg cart and is turned by 60°. The jet maintains a cross-sectional area, $S_j = 0,02$ m^2. At time, $t = 0$ s, the cart is at rest at a position, $x = 0$ m. Find an expression for the speed of the cart for $t > 0$ s.

You can neglect friction in the wheels, variation in the momentum of the water that is in contact with the cart, and gravitational effects.



$$\cancel{\sum F_x}^{\,0} - \int_{C.V.} a_x \rho\, dV = \cancel{\frac{\partial}{\partial t}\int_{C.V.} V_x \rho\, dV}^{\,0} + \int_{C.S.} V_x \rho \vec{V}\cdot\hat{n}\, dS$$

(1) $$-\frac{du}{dt} M = \int_{C.S.} V_x \rho \vec{V}\cdot\hat{n}\, dS$$

$$\boxed{\begin{aligned}\vec{V}_1 &= (V_j - u)\hat{\imath} \quad (1)\\ \vec{V}_2 &= (V_j - u)[\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}]\end{aligned}}$$

(1) $$\begin{aligned}-\frac{du}{dt} M &= -V_{1x}\rho|\vec{V}_1| S + V_{2x}\rho|\vec{V}_2| S\\ &= -V_1^2 \rho S + V_2^2 \rho S \cos\theta\\ &= -\frac{1}{2} V_1^2 \rho S\end{aligned}$$

$$\frac{du}{dt} = \frac{\rho S}{2M}(V_j - u)^2$$

(1) $$\int_0^u \frac{d\hat{u}}{(V_j - \hat{u})^2} = \int_0^t \frac{\rho S}{2M}\, d\hat{t}$$

$$\int_0^u \frac{-d(V_j - \hat{u})}{(V_j - \hat{u})^2} = \frac{1}{V_j - \hat{u}}\Big]_0^u = \frac{1}{V_j - u} - \frac{1}{V_j} = \frac{\rho S}{2M} t \quad (1)$$

$$\Rightarrow u = V_j\left[\frac{\frac{V_j \rho S}{2M} t}{1 + \frac{V_j \rho S}{2M} t}\right]$$

$$\frac{V_j \rho S}{2M} = \frac{(20\,m/s)(1000\,kg/m^3)(0.02\,m^2)}{2\,(200\,kg)} = 1\,s^{-1}$$

(1) $$\boxed{u = 20\,m/s\left[\frac{t}{1s + t}\right]}$$