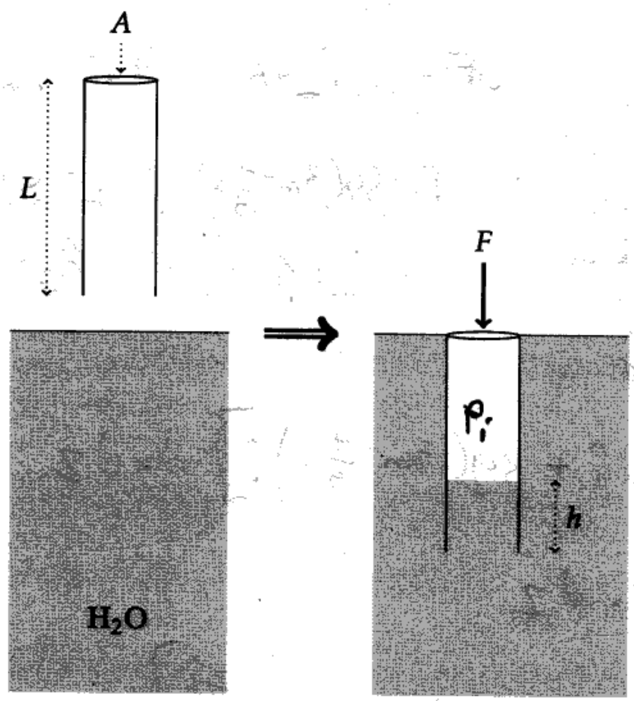
Name:

1) A thin-walled cylinder has a cross-sectional area, $A = 0.1\,\mathrm{m}^2$ and a length, $L = 1\,\mathrm{m}$. It is closed on the top and open on the bottom. The cylinder is submerged in water such that its top is level with the water's surface.

a) To what height, h, will the water enter the cylinder.

b) What is the magnitude of the downward force, F, needed to hold the cylinder in place.

Neglect the weight of the cylinder.



$$P_i = P_{atm} + \rho_{H_2O}\, g L - \rho_{H_2O}\, g h$$

$$= P_{atm} + \rho_{H_2O}\, g (L - h)$$

Boyle's law $PV = \text{const}$

$$P_i A (L-h) = P_{atm} A L$$

$$P_i = P_{atm} \frac{L}{L-h}$$

$$P_{atm} + \rho g (L-h) = P_{atm} \frac{L}{L-h}$$

$$\rho g (L-h)^2 + P_{atm}(L-h) - P_{atm} L = 0$$

$$L - h = \frac{-P_{atm} \pm \sqrt{P_{atm}^2 + 4\rho g P_{atm} L}}{2\rho g}$$

$$h = L + \frac{P_{atm}}{2\rho g} \pm \frac{\sqrt{P_{atm}^2 + 4\rho g P_{atm} L}}{2\rho g}$$

$h = 12.25\,\mathrm{m}$ or $\boxed{0.082\,\mathrm{m}}$

↑ spurious root

b) $F = P_i A - P_{atm} A$

$P_i = P_{atm} + \rho g (L-h)$

$= 101325\,\mathrm{Pa} + (1000\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(1 - .082)\,\mathrm{m}$

$= 110334\,\mathrm{Pa}$

$F = 110334\,\mathrm{Pa}\,(0.1\,\mathrm{m^2}) - 101325\,\mathrm{Pa}\,(0.1\,\mathrm{m^2})$

$\boxed{= 901\,\mathrm{N}}$

or

$F = F_b = \rho_{H_2O}\, g V_d$

$= (1000\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(0.1\,\mathrm{m^2})(1\,\mathrm{m} - 0.082\,\mathrm{m})$

$\boxed{= 901\,\mathrm{N}}$

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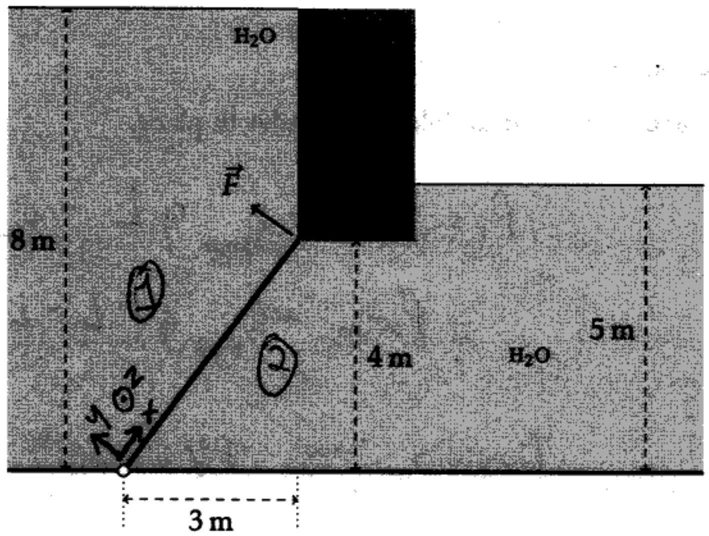
2) What is the smallest magnitude force, $\vec{F}$, needed to open the gate? The gate has a width of 2 m.



$d\vec{M} = d\vec{M}_1 + d\vec{M}_2$

$= \vec{r}_\ell \times (d\vec{F}_1 + d\vec{F}_2)$

$dF_1 = -p_1 \hat{n}_1 dA \qquad dF_2 = -p_2 \hat{n}_2 dA$

$p_1 = \rho g(8m - \frac{4}{5}x) \qquad p_2 = \rho g(5m - \frac{4}{5}x)$

$\hat{n}_1 = \hat{j} \qquad \hat{n}_2 = -\hat{j}$

$r_\ell = x\hat{i} + z\hat{k}$

$d\vec{M} = (x\hat{i} + z\hat{k}) \times \left[-\rho g(8m - \frac{4}{5}x)\hat{j} + \rho g(5m - \frac{4}{5}x)\hat{j}\right] dx\,dz$

$= (x\hat{i} + z\hat{k}) \times \left[(-3m)\rho g\,\hat{j}\right] dx\,dz$

$= -(3m)\rho g \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 0 & z \\ 0 & 1 & 0 \end{vmatrix} dx\,dz = -(3m)\rho g \begin{bmatrix} -z\hat{i} \\ +0\hat{j} \\ +x\hat{k} \end{bmatrix} dx\,dz$

$M_z = \int_0^{2m}\int_0^{5m} -(3m)\rho g\, x\, dx\,dz = -(3m)(2m)(1000\,kg/m^3)(9.81\,m/s^2)\left[\frac{x^2}{2}\right]_0^{5m}$

$= -(3m)(2m)(1000\,kg/m^3)(9.81\,m/s^2)\left(\frac{25m^2}{2}\right) = -735\,750\,Nm$

$\vec{F} = \frac{735\,750\,Nm}{5m}\hat{j} = \boxed{147\,150\,N\hat{j}}$

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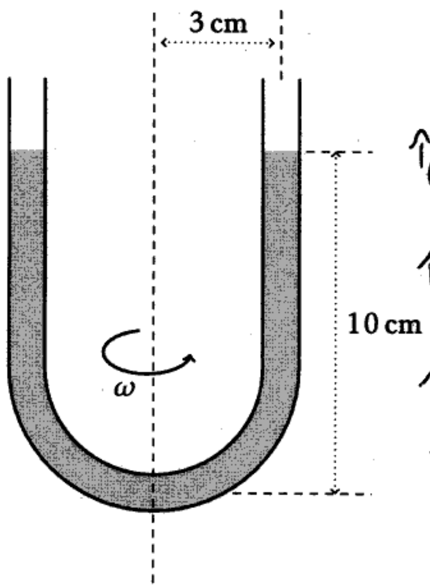
3) If the pressure of a liquid falls below its vapour pressure, it will spontaneously vaporize. When this happens for mechanical reasons, this is known as *cavitation.*

Consider the tube below that is partially filled with water, open on both ends, and rotates around its central axis. Determine the angular velocity, ω, that will cause cavitation in the water. Assume the vapour pressure of water is 1.77 kPa.

hint: $\vec{\nabla} p = \frac{\partial p}{\partial r}\hat{i}_r + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{i}_\theta + \frac{\partial p}{\partial z}\hat{i}_z$



$$\vec{\nabla} p = \rho \vec{g} - \rho \vec{a}$$

$$\hat{i}_\theta: \frac{1}{r}\frac{\partial p}{\partial \theta} = 0 \Rightarrow p \neq f(\theta)$$

$$\hat{i}_z: \frac{\partial p}{\partial z} = -\rho g \Rightarrow p = -\rho g z + f_2(r)$$

$$\hat{i}_r: \frac{\partial p}{\partial r} = \rho r \omega^2 \Rightarrow p = \frac{\rho r^2 \omega^2}{2} + f_3(z)$$

$$p = \frac{\rho r^2 \omega^2}{2} - \rho g z + C$$

@ $r=0$ & $z=0$ $p = p_v = 1770$ Pa $\Rightarrow C = 1770$ Pa

@ $z = 0.1$m & $r = 0.03$m $p = p_{atm}$

$$p_{atm} = \frac{\rho r^2 \omega^2}{2} - \rho g z + p_v$$

$$\omega^* = \sqrt{\frac{2}{\rho r^2}\left[p_{atm} + \rho g z - p_v\right]}$$

$$\omega = \sqrt{\frac{2}{(1000 kg/m^3)(0.03m)^2}\left[101325 Pa + (1000 kg/m^3)(9.81 m/s^2)(0.1m) - 1770 Pa\right]}$$

$$\boxed{\omega = 473 \frac{rad}{s} = 4514 rpm}$$