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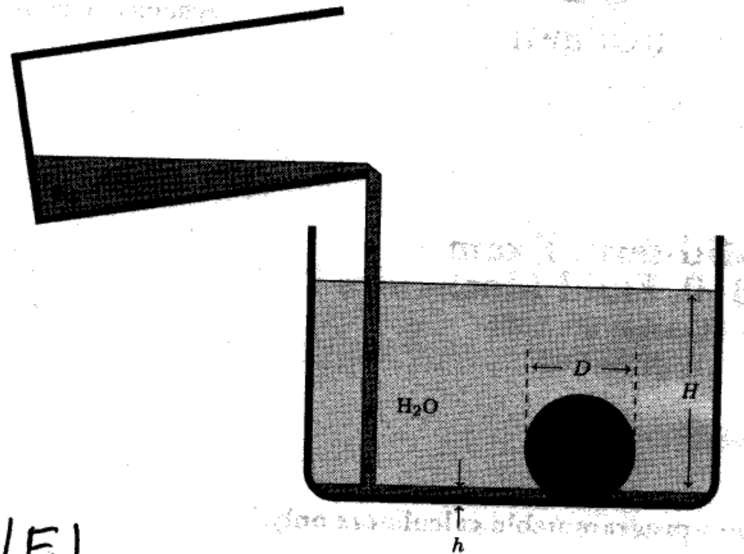
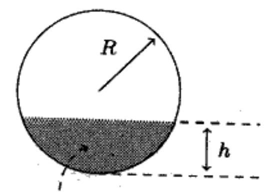
1) A solid sphere sits at the bottom of a beaker that is full with water to a depth of H. Mercury, $\rho_{Hg} = 13\,534\,\mathrm{kg/m^3}$, is slowly added to the beaker. When the depth of mercury, h, reaches $\frac{3}{4}D$, the sphere lifts from the bottom. What is the density of the sphere?



Volume of part of a sphere:



$$V = \frac{\pi h^2}{3}(3R - h)$$

① $|F_g| = |F_b|$

② $\rho_s \cancel{g} V_s = \rho_{H_2O} \cancel{g} V_{H_2O} + \rho_{Hg} \cancel{g} V_{Hg}$

① $\rho_s\left(\frac{4}{3}\pi R^3\right) = \rho_{H_2O}\left[\frac{4}{3}\pi R^3 - \frac{\pi h^2}{3}(3R - h)\right] + \rho_{Hg}\left[\frac{\pi h^2}{3}(3R - h)\right]$

$\rho_s\left(\frac{4}{3}\cancel{\pi}\cancel{R^3}\right) = \rho_{H_2O}\left[\frac{4}{3}\cancel{\pi}\cancel{R^3} - \frac{\cancel{\pi}\left(\frac{3}{2}\cancel{R}\right)^2}{3}\left(3\cancel{R} - \frac{3}{2}\cancel{R}\right)\right] + \rho_{Hg}\left[\frac{\cancel{\pi}\left(\frac{3}{2}\cancel{R}\right)^2}{3}\left(3\cancel{R} - \frac{3}{2}\cancel{R}\right)\right]$

$\frac{4}{3}\rho_s = \rho_{H_2O}\left[\frac{4}{3} - \frac{3}{4}\left(3 - \frac{3}{2}\right)\right] + \rho_{Hg}\left[\frac{3}{4}\left(3 - \frac{3}{2}\right)\right]$

$\frac{4}{3}\rho_s = \rho_{H_2O}\left[\frac{4}{3} - \frac{3}{4}\left(\frac{3}{2}\right)\right] + \rho_{Hg}\left[\frac{3}{4}\,\frac{3}{2}\right]$

$\frac{4}{3}\rho_s = (\rho_{Hg} - \rho_{H_2O})\frac{9}{8} + \frac{4}{3}\rho_{H_2O}$

① $\boxed{\rho_s = (\rho_{Hg} - \rho_{H_2O})\frac{27}{32} + \rho_{H_2O}}$ or $= (13534\,\mathrm{kg/m^3} - 1000\,\mathrm{kg/m^3})\frac{27}{32} + 1000\,\mathrm{kg/m^3}$

$\boxed{= 11576\,\mathrm{kg/m^3}}$

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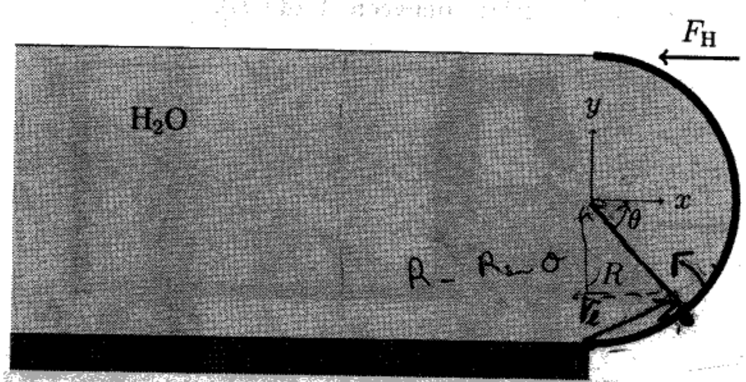
2) A gate is formed from a half cylinder. It has a width, W; a height, $2R$; and is hinged at the bottom. What horizontal force, F_{H} is needed to hold the gate in equilibrium?



$x = R\cos\theta$
$y = -R\sin\theta$
$z = z$ ①

$d\vec{F}_p = -p\hat{n}\,dS$ ① $p = \rho g(R-y) = \rho g(R + R\sin\theta) = \rho g R(1+\sin\theta)$

$\vec{r} = R\cos\theta\,\hat{\imath} - R\sin\theta\,\hat{\jmath} + z\hat{k}$

② $\hat{n}\,dS = \left(\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial z}\right) d\theta\,dz = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -R\sin\theta & -R\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} d\theta\,dz = (-R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath})\,d\theta\,dz$

① $d\vec{F}_p = -\rho g R(1+\sin\theta)(-R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath})\,d\theta\,dz$

① "lever arm": $\vec{r}_l = R\cos\theta\,\hat{\imath} + R(1-\sin\theta)\hat{\jmath} + z\hat{k}$

$d\vec{M} = \vec{r}_l \times d\vec{F}_p$ ① $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ R\cos\theta & R(1-\sin\theta) & z \\ -\cos\theta & \sin\theta & 0 \end{vmatrix} (-\rho g R^2(1+\sin\theta)\,d\theta\,dz) =$

$= \left[-z\sin\theta\,\hat{\imath} - z\cos\theta\,\hat{\jmath} + \left(\cancel{R\cos\theta\sin\theta} + R\cos\theta - \cancel{R\cos\theta\sin\theta}\right)\hat{k}\right](-\rho g R^2(1+\sin\theta)\,d\theta\,dz)$

only look at moment around "z"

$M_z = \int_0^W \int_{-\pi/2}^{\pi/2} -\rho g R^3(1+\sin\theta)\cos\theta\,d\theta\,dz$ ① $= -\rho g R^3 W \int_{-\pi/2}^{\pi/2} (1+\sin\theta)\cos\theta\,d\theta$

$= -\rho g R^3 W \left[\sin\theta + \frac{\sin^2\theta}{2}\right]_{-\pi/2}^{\pi/2} = -2\rho g R^3 W$ ① ① $F = \frac{M_z}{2R} = \rho g R^2 W$

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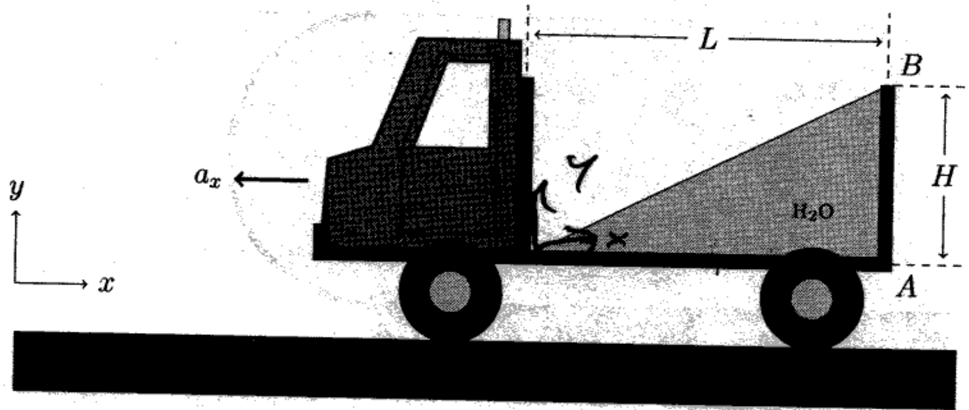
3) A truck is half filled with water and undergoes constant acceleration. What is the maximum acceleration, a_x, that the truck can have without spilling any water? At this acelleration, what is the net pressure force on the back wall of the truck (*i.e.* the surface between A and B).



Width = W

$$-\nabla p + \rho \vec{g} = \rho \vec{a}$$

a_x is a negative number

① x: $-\frac{\partial p}{\partial x} = \rho a_x \qquad p = -\rho a_x x + C_1(y)$

y: $-\frac{\partial p}{\partial y} - \rho g = 0 \qquad p = -\rho g y + C_2(x)$

$$① \; p = -\rho[a_x x + g y] + C_3$$

on surface $p = 0,\ x = x_s,\ y = y_s$

$$0 = -\rho[a_x x_s + g y_s] + C_3$$

@ $y = 0 \quad x = 0 \quad p = 0 \quad C_3 = 0$ ①

$$0 = \cancel{\rho}[a_x x_s + g y_s] \qquad y_s = -\frac{a_x x_s}{g} \; ①$$

slope must $= H/L$ $\therefore$ $\frac{H}{L} = -\frac{a_x}{g}$ $\quad \boxed{a_x = \frac{-Hg}{L}}$

②

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$F_p = -p\hat{n}\,ds$

$p = -\rho[a_x x + g y]$

$a_x = -\frac{Hg}{L}$

$\hat{n} = -\hat{\imath}$

$x = L$

$ds = W\,dy$

$p\big|_{x=L} = -\rho[-Hg + gy]$ (1)

$$F_p = \int_0^H \rho[Hg - gy]\,W\,dy \quad (1)$$

$$= \rho W g \int_0^H [H - y]\,dy$$

$$= \rho W g \left[Hy - \frac{y^2}{2}\right]_0^H$$

$$= \rho W g \left[H^2 - \frac{H^2}{2}\right]$$

$$\boxed{F_p = \frac{\rho W g H^2}{2}\hat{\imath}} \quad (2)$$