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University of Ottawa
Department of
mechanical engineering

Final Exam
MCG 3340, Fluid Mechanics I

December 11th, 2016
7:00–10:00 p.m.

Closed-book, non-programmable calculators only.

Name: Solutions

Student number: ______________________

Question	**Possible**	**Result**
1	10	
2	10	
3	10	
4	10	
5	10	
Total	**50**	

Name:

1) The drag force, F_D, that acts on a ship is a function of the forward speed, V, the fluid density, ρ, the fluid viscosity, μ, gravitational acceleration, g, the ship's length, L, as well as its width, W. A new ship being developed has a length of 20 m, a width of 5 m, and is expected to travel in fresh water at 20 m/s.

a) Find suitable non-dimensional groups for the problem.

b) The expected drag on a proposed ship design is to be studied using a scale model in a tank full of fresh water. It is hoped that a model of length 0.5 m can be used. Is this possible?

c) If the experiment is possible, what is the expected ratio between the measured drag on the model and the drag on the real boat? If the experiment is not possible, what could be changed to make such a study feasible?

a) $\Pi_1 = Re = \frac{\rho V L}{\mu}$ (1) $\quad \Pi_2 = Fr = \frac{V}{\sqrt{gL}}$ (1) $\quad \Pi_3 = \frac{W}{L}$ (1)

$\Pi_4 = \frac{F_D}{\rho V^2 L^2}$ (1)

b) $\left(\frac{W}{L}\right)_p = \left(\frac{W}{L}\right)_m \qquad W_m = \left(\frac{5m}{20m}\right)(0.5m) = 0.125m$

$Re_m = Re_p \qquad \frac{\cancel{\rho}(20m/s)(20m)}{\cancel{\mu}} = \frac{\rho V_m(0.5m)}{\mu} \Rightarrow V_m = 800 m/s$

(3)

$Fr_m = Fr_p \qquad \frac{V_m}{\sqrt{g(0.5m)}} = \frac{20m/s}{\sqrt{g(20m)}} \qquad V_m = 3.16 m/s$

can't satisfy both! impossible

c) One would need to use a fluid with a different $\frac{\mu}{\rho} = \nu$.

(3)

Name:

2) For laminar viscous flow in a circular conduit, the velocity, $v_x(r)$, has a parabolic profile.

a) Find an expression for the velocity profile, v_x, as a function of volumetric flow rate, Q, pipe diameter, D, and distance from the pipe centre, r.

b) A small Pitot tube is used to measure the flow rate of a laminar, incompressible, viscous flow in a circular pipe. The Pitot tube is very small and is placed exactly in the centre of the pipe. Find an expression for the volumetric flow rate, Q, as a function of the difference between the static and stagnation pressures measured by the Pitot tube, Δp, the density of the fluid, ρ, and the diameter of the pipe, D.

a) $Q = \iint_S V_x \, dS$ $\qquad V_x = Ar^2 + B$ $\quad$ 2

$$= \int_0^{2\pi}\int_0^R (Ar^2 + B)\, r\, dr\, d\theta = 2\pi \int_0^R [Ar^3 + Br]\, dr$$

$$= 2\pi \left[\frac{A}{4} r^4 + \frac{B}{2} r^2\right]_0^R = 2\pi \left(\frac{AR^4}{4} + \frac{BR^2}{2}\right) \quad 2$$

$@ r = R \quad V_x = 0$ (no slip) $\therefore 0 = AR^2 + B \quad B = -AR^2$

$$Q = 2\pi A\left(\frac{R^4}{4} - \frac{R^4}{2}\right) = -\cancel{2}\pi A \frac{R^4}{\cancel{4}\,2} \therefore A = -\frac{2Q}{\pi R^4}$$

$$\boxed{V_x = \frac{2Q}{\pi R^2}\left[1 - \frac{r^2}{R^2}\right]} \quad 2$$

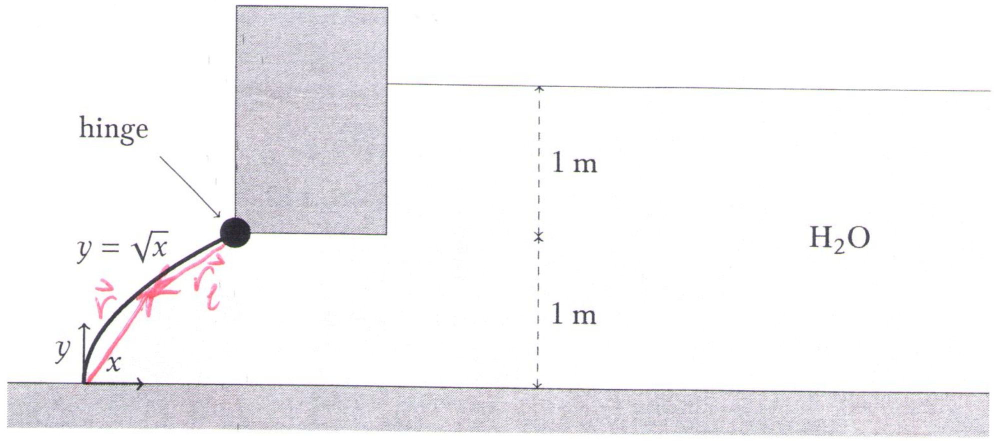
$$\boxed{\text{or } V_x = \frac{8Q}{\pi D^2}\left[1 - \frac{4r^2}{D^2}\right]}$$

b) $@ r = 0 \qquad V_x = \dfrac{2Q}{\pi R^2}$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_0}{\rho} \Rightarrow \Delta p = \frac{\rho V_1^2}{2} = \frac{\rho}{2}\left(\frac{2Q}{\pi R^2}\right)^2$$

$$\boxed{Q = \frac{\pi D^2}{4}\sqrt{\frac{\Delta p}{2\rho}}} \quad 4$$

3) The gate shown below has a width of 2 m. It's shape is described by the expression $y = \sqrt{x}$. What moment does the pressure force generate around the hinge?



$\vec r = u^2\hat\imath + u\hat\jmath + v\hat k$

$0 \le v \le 2\,m$ (2)

$0 \le u \le 1\,m$

$\frac{\partial \vec r}{\partial u} = 2u\hat\imath + \hat\jmath$

$\frac{\partial r}{\partial v} = \hat k$

$\hat n\,dS = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 2u & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\hat\imath - 2u\hat\jmath)\,du\,dv$ (2) points in correct direction

$p = \rho g(2-y)$
$= \rho g(2-u)$ (2)

$d\vec F_p = -p\hat n\,dS = -\rho g(2-u)\,(\hat\imath - 2u\hat\jmath)\,du\,dv$

$d\vec M = \vec r_\ell \times d\vec F_p$ $\qquad \vec r_\ell = (-\hat\imath - \hat\jmath) + \vec r = (u^2-1)\hat\imath + (u-1)\hat\jmath + v\hat k$

$d\vec M = \begin{vmatrix} \hat\imath & \hat\jmath & \hat k \\ 1 & -2u & 0 \\ u^2-1 & u-1 & v \end{vmatrix} [-\rho g(2-u)]\,du\,dv = [-\rho g(2-u)] \begin{bmatrix} -2uv\hat\imath \\ -v\hat\jmath \\ (u-1) + 2u(u^2-1)\hat k \end{bmatrix} du\,dv$ (2)

oops

only want z component.

$M_z = \int_0^2\int_0^1 -\rho g(2-u)\,[u-1+2u(u^2-1)]\,du\,dv = \int_0^2\int_0^1 -\rho g(2-u)(u-1+2u^3-2u)\,du\,dv$

$= -2\rho g\int_0^1 [-2u^4+4u^3+u^2-u-2]\,du = -2\rho g\left[-\frac{2}{5}u^5 + u^4 + \frac{u^3}{3} - \frac{u^2}{2} - 2u\right]_0^1$

should have -ve

$\boxed{M_z = \frac{47}{15}\rho g}$ (2)

$\boxed{M_z = -30\,738\,Nm}$

~~check soln~~

Name:

4) Two tanks are connected by a piping system made of commercial steel. The system is composed of pipes with diameters of 0.1 m and 0.05 m. The viscosity of the water is $\mu = 1.2 \times 10^{-3}$ Pa s. When the gate valve is opened completely, what is the expected volumetric flow rate and in which direction will the flow be?

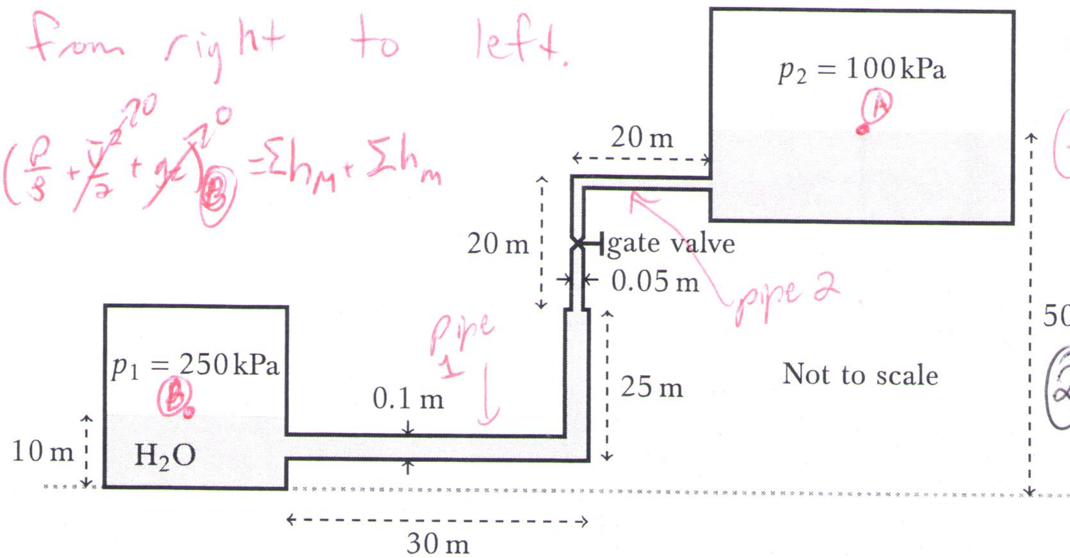
flows from right to left.

$\varepsilon = 0.046\,mm$

$\left(\frac{P}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_A - \left(\frac{P}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_B = \sum h_M + \sum h_m$

(2)

$\left(\frac{\varepsilon}{D}\right)_1 = \frac{0.046\,mm}{100\,mm}$



$\left(\frac{\varepsilon}{D}\right)_1 = 4.6\times10^{-4}$

(2)

$\left(\frac{\varepsilon}{D}\right)_2 = 9.2\times10^{-4}$

$\uparrow z = 0$

Major losses: - 55m of 10cm pipe - 40m of 5cm pipe

Minor losses 1 90° elbow in 10cm pipe 1 gate valve in 5cm pipe

1 entry in 10cm pipe 1 90° elbow in 5cm pipe

1 exit loss in 5cm pipe

1 expansion (.05m to 0.1m)

$$\left(\frac{P}{\rho} + gz\right)_A - \left(\frac{P}{\rho}\right)_B = f_1\left(\frac{55m}{0.1m}\right)\left(\frac{\bar{V}_1^2}{2}\right) + f_1(30)\left(\frac{\bar{V}_1^2}{2}\right) + 0.5\left(\frac{\bar{V}_1^2}{2}\right) + f_2\left(\frac{40m}{.05m}\right)\left(\frac{\bar{V}_2^2}{2}\right)$$

(2)

$V_2 = 4V_1$

$$+ f_2(8)\left(\frac{\bar{V}_2^2}{2}\right) + f_2(30)\left(\frac{\bar{V}_2^2}{2}\right) + 1\left(\frac{\bar{V}_2^2}{2}\right) + 0.55\left(\frac{\bar{V}_2^2}{2}\right)$$

$$= \left\{ f_1\left[(550) + 30\right] + 0.5 + f_2(16)\left(\frac{40}{.05} + 8 + 30\right) + 16 + 8.8 \right\}\frac{\bar{V}_1^2}{2}$$

13408

$$= \left(f_1(580) + f_2(13408) + 25.3\right)\frac{\bar{V}_1^2}{2}$$

$$\bar{V}_1 = \sqrt{\frac{2\left(\frac{P_A - P_B}{\rho} + g z_A\right)}{580 f_1 + 13408 f_2 + 25.3}}$$

$Re_2 = 2 Re_1$

2

$$= \sqrt{\frac{2\left[(100000 - 250000)/1000 + 9.81(\cancel{50m}\ 40m)\right]}{580 f_1 + 13408 f_2 + 25.3}}$$

$$= \sqrt{\frac{\cancel{681\,m^2/s^2}\ 484.8\,m^2/s^2}{580 f_1 + 13408 f_2 + 25.3}}$$

guess $Re_1 = 100\,000$ $Re_2 = 200\,000$

$f_1 = 0.0205$ $f_2 = 0.021$

$\bar{V}_1 = \cancel{1.45\,m/s}\ 1.23\,m/s \Rightarrow Re_1 = 102\,500$

$\cancel{Re_1 = 121\,667}$ $\cancel{Re_2 = 243\,333}$ Good guess

$\cancel{f_1 = 0.02}$ $\cancel{f_2 = 0.0205}$

$\cancel{\bar{V}_1 = 1.48\,m/s}$

close enough

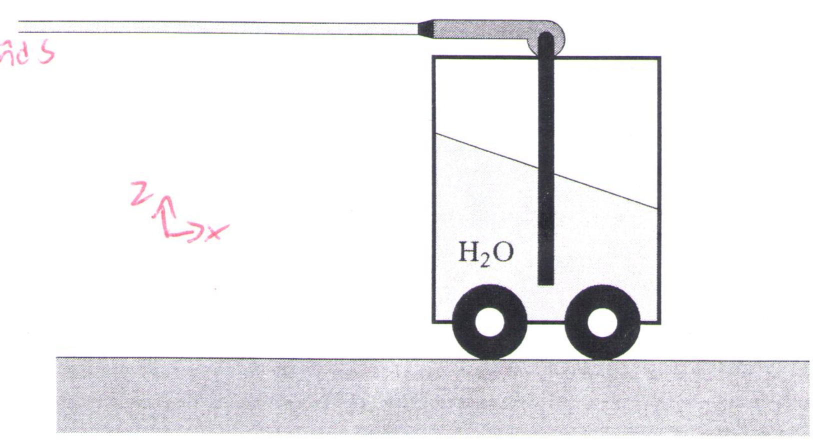
2

$$Q = V_1 S_1 = 1.23\,m/s\ \pi \frac{(0.1m)^2}{4} \quad \boxed{0.0097\,m^3/s} = 9.6\,L/s$$

$\cancel{0.012\,m^3/s}$

Name:

5) A cart is partially filled with water. A jet of water is sprayed horizontally from the cart through the use of a pump. The jet has a speed of 20 m/s and a diameter of 5 cm. The water level within the cart changes slowly enough that the liquid in the cart can be considered a static fluid. All forms of friction may be neglected. At the instant when the cart has a mass of 200 kg, what is the slope of the water surface within the cart?



$$\cancel{\sum \vec{F}}^{\,0} - \iiint_V \rho \vec{a}\, dV = \cancel{\frac{d}{dt}\iiint_V \rho \vec{v}}^{\,0} + \oiint_S \rho \vec{v}\, \vec{v}\cdot\hat{n}\, dS$$

x:

$$-Ma_x = -\rho V_x^2 S$$

$$\boxed{a_x = \frac{\rho V_x^2 S}{M}} = 3.93\ m/s^2$$

(3)

$$\vec{\nabla} p = \rho(\vec{g} - \vec{a})$$

$$x: \frac{\partial p}{\partial x} = -\rho a_x = -\rho\left(\frac{\rho V_x^2 S}{M}\right) \Rightarrow p = -\frac{\rho^2 V_x^2 S}{M} x + f_1(z)$$

$$z: \frac{\partial p}{\partial z} = -\rho g \Rightarrow p = -\rho g z + f_2(x)$$

$$p = -\frac{\rho^2 V_x^2 S}{M} - \rho g z + C$$

(3)

@ surface $P_{rel} = 0$, $z = z_s$

$$0 = -\frac{\rho^2 V_x^2 S x}{M} - \rho g z_s + C$$

(1)

$$\boxed{slope = -0.4}$$

$$z_s = \frac{\frac{\rho^2 V_x^2 S}{M} x - C}{(-\rho g)}$$

(3)

$$z_s = \boxed{-\frac{\rho V_x^2 S}{Mg}}\, x + \frac{C}{\rho g}$$