Université d'Ottawa
Département de
génie mécanique

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University of Ottawa
Department of
mechanical engineering

Final Exam
MCG 3340, Fluid Mechanics I

December 17th, 2015
2:00–5:00 p.m.

Closed-book, non-programmable calculators only.

Name: Solutions

Student number: ____________________

Question	Possible	Result
1	10	
2	10	
3	10	
4	10	
5	10	
Total	**50**	

1) A scale-model test of a proposed new aircraft design is to be conducted in a wind tunnel. The lift force, F_L, on the real aircraft is expected to be a function of the aircraft length, L, flow speed, V, fluid density, ρ, fluid viscosity, μ, and speed of sound, c. The proposed aircraft has a length of 30 m and is expected to cruise at a speed of 900 km/h at an altitude, h, of 11 km. Two wind-tunnel facilities are available for testing, both can accommodate a scale model with a maximum length of 2 m. One uses air at standard conditions and the other uses high-pressure, cryogenic nitrogen.

a) Find suitable non-dimensional groups for the problem.

b) Is it possible to use either wind tunnel? Why?

c) If it were possible to use both tunnels, what is the ratio of the expected lift on the real aircraft to that measured in the test? Compute the ratio for each tunnel.

	Air, $h = 11$ km	Air, wind tunnel	Nitrogen, wind tunnel
Temperature, (K)	216	288	180
Density, (kg/m^3)	0.365	1.225	7.482
Speed of sound, (m/s)	295	340	274
Viscosity, (Pa s)	1.4×10^{-5}	1.8×10^{-5}	1.2×10^{-5}

$F_L = f(L, V, \rho, \mu, c)$ use: $\rho V L$ as repeated variables

a) $\Pi_1 = \dfrac{F_L}{\rho V^2 L^2}$ (1) $\Pi_2 = \dfrac{\rho V L}{\mu}$ (1) $= Re$ $\Pi_3 = \dfrac{V}{c} = Ma$ (1)

b) Real: $Re = \dfrac{\rho V L}{\mu} = \dfrac{(0.365\ kg/m^3)(250\ m/s)(30\ m)}{1.4\times10^{-5}\ Pa\,s} = 1.96\times10^8$

$Ma = \dfrac{V}{c} = \dfrac{250\ m/s}{295\ m/s} = 0.847$

Air: $Ma = 0.847 = \dfrac{V}{c} \Rightarrow V = 0.847(340\ m/s) = 288\ m/s$

$Re = 1.96\times10^8 \Rightarrow L = \dfrac{1.96\times10^8(1.8\times10^{-5}\ Pa\,s)}{(1.225\ kg/m^3)(288\ m/s)} = \boxed{10\ m}$

(2)

Cannot use

10m > 2m

N_2: $Ma = 0.847 = \frac{V}{c} \Rightarrow V = 0.847(274\,m/s)$

$= 232\,m/s$

$Re = 1.96\times10^8 \Rightarrow L = \frac{1.96\times10^8\,(1.2\times10^{-5}\,Pa\,s)}{(7.482\,kg/m^3)(232\,m/s)}$

$\boxed{= 1.35\,m}$ (2)

$\boxed{\text{Can use } 1.35\,m < 2\,m}$

c) $\left(\frac{F_L}{\rho U^2 L^2}\right)_{Real} = \left(\frac{F_L}{\rho U^2 L^2}\right)_{Model}$

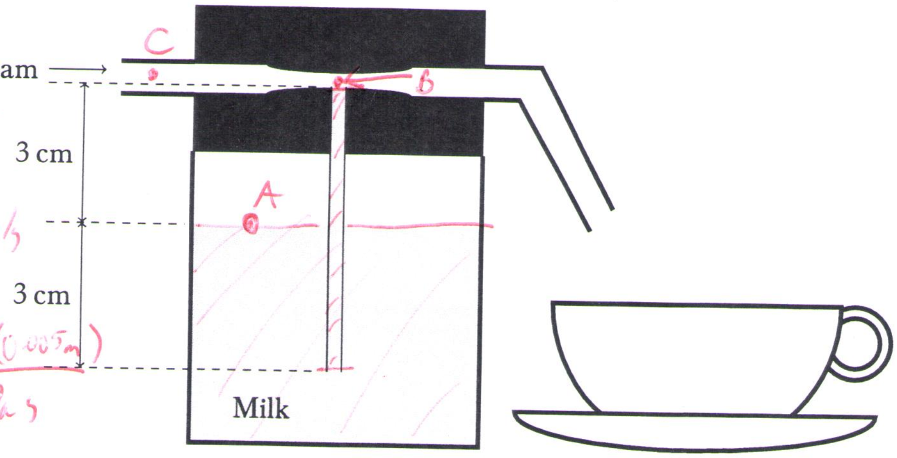
$$\frac{F_{L\,R}}{F_{L\,M}} = \frac{(\rho U^2 L^2)_R}{(\rho U^2 L^2)_M}$$

Air: $\frac{(0.365\,kg/m^3)(250\,m/s)^2(30\,m)^2}{(1.225\,kg/m^3)(288\,m/s)^2(10\,m)^2} = \boxed{2.02}$ (3)

N_2: $\frac{(0.365\,kg/m^3)(250\,m/s)^2(30\,m)^2}{(7.482\,kg/m^3)(232\,m/s)^2(1.35\,m)^2} = \boxed{27.97}$

Name:

2) An automatic cappuccino machine uses a Venturi to draw milk, $\rho_{\text{milk}} = 1035\,\text{kg/m}^3$, vertically into a stream of hot steam through a straw with a diameter of 0.5 cm and a length of 6 cm. The viscosity of the milk is $\mu_{\text{milk}} = 3 \times 10^{-3}$ Pa s. The steam has a density, $\rho_{\text{steam}} = 1.7\,\text{kg/m}^3$ and it enters the Venturi with a relative pressure of 200 Pa and velocity of 3 m/s. Assume the loss coefficient, K, at both the straw entrance and exit is 0.5. Neglect viscosity in the steam flow. What must be the ratio between the maximum and minimum diameters in the Venturi to achieve a milk flow of 12 ml/s?



$$Q = 12\,\text{ml/s} = 1.2\times10^{-5}\,\text{m}^3/\text{s}$$

$$A = \frac{\pi}{4}(0.005\,\text{m})^2 = 1.96\times10^{-5}\,\text{m}^2$$

$$V = \frac{Q}{A} = \frac{1.2\times10^{-5}\,\text{m}^3/\text{s}}{1.96\times10^{-5}\,\text{m}^2} = 0.61\,\text{m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(1035\,\text{kg/m}^3)(0.61\,\text{m/s})(0.005\,\text{m})}{3\times10^{-3}\,\text{Pa s}}$$

$$= 1052 \rightarrow \text{laminar}$$

$$f = \frac{64}{Re} = \frac{64}{1052} = 0.06 \quad (2)$$

$$\left(\frac{\cancel{P_A}^{0}}{\rho_m} + \alpha\frac{\cancel{V_A^2}^{0}}{2} + \cancel{g z_A}^{0}\right) - \left(\frac{P_B}{\rho_B} + \alpha\frac{V_B^2}{2} + g z_B\right) = \frac{64}{Re}\frac{L}{D}\frac{V_B^2}{2} + 2K\frac{V_B^2}{2}$$

$$P_B = -\rho\left\{\left[\frac{64}{Re}\frac{L}{D} + 2K + \alpha\right]\frac{V_B^2}{2} + g z_B\right\}$$

$$= -(1035\,\text{kg/m}^3)\left\{\left[\frac{64}{1052}\frac{(0.06\,\text{m})}{(0.005\,\text{m})} + 2(0.5) + \overset{\alpha=2}{2}\right]\frac{(0.61\,\text{m/s})^2}{2} + 9.81\,\text{m/s}^2 \times 0.03\,\text{m}\right\}$$

$$P_B = -1023\,\text{Pa} \quad (3)$$

$$\frac{P_C}{\rho_{\text{steam}}} + \frac{V_C^2}{2} = \frac{P_B}{\rho_{\text{steam}}} + \frac{V_B^2}{2}$$

$$V_B = \sqrt{2\left(\frac{P_C - P_B}{\rho_{\text{steam}}} + \frac{V_C^2}{2}\right)}$$

$$V_B = \sqrt{2\left(\frac{200\,\text{Pa} - (-1023\,\text{Pa})}{1.7\,\text{kg/m}^3}\right) + \frac{(3\,\text{m/s})^2}{2}}$$

$$V_B = 38\,\text{m/s} \quad (3)$$

$$\frac{A_C}{A_B} = \frac{V_B}{V_C} = \frac{38\,\text{m/s}}{3\,\text{m/s}} = 12.7$$

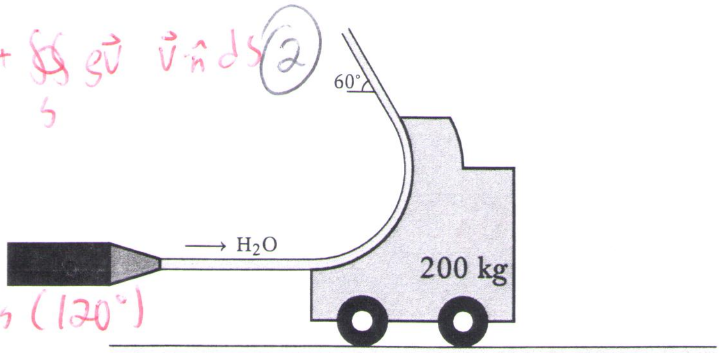
$$\left(\frac{D_C}{D_B}\right) = \sqrt{\frac{A_C}{A_B}} = \sqrt{12.7} = \boxed{3.6} \quad (2)$$

Name:

3) A jet of water with speed, $V_j = 20\,\mathrm{m/s}$ hits a $200\,\mathrm{kg}$ cart and is turned. The jet maintains a cross-sectional area, $S_j = 0.02\,\mathrm{m}^2$. At time, $t = 0\,\mathrm{s}$, the cart is at rest at a position, $x = 0\,\mathrm{m}$. Find an expression for the speed of the cart for $t > 0$ s.

You can neglect friction in the wheels, variation in the momentum of the water that is in contact with the cart, and gravitational effects.

$$\cancelto{0}{\sum \vec F} = \iiint \rho \vec a \, dV = \cancelto{0}{\frac{\partial}{\partial t}\iiint \rho \vec v \, dV} + \iint_S \rho \vec v \, \vec v \cdot \hat n \, dS \quad (2)$$



$$-\left(\frac{dU}{dt}\right) M = -V_{rel}^2 \rho S + V_{rel}^2 \rho S \cos(120°)$$

$$= -\frac{3}{2} V_{rel}^2 \rho S \quad (2)$$

$$V_{rel} = V_j - U \quad (1)$$

(2)

$$\frac{dU}{dt} = \frac{3}{2}\frac{\rho S}{M}(V_j - U)^2$$

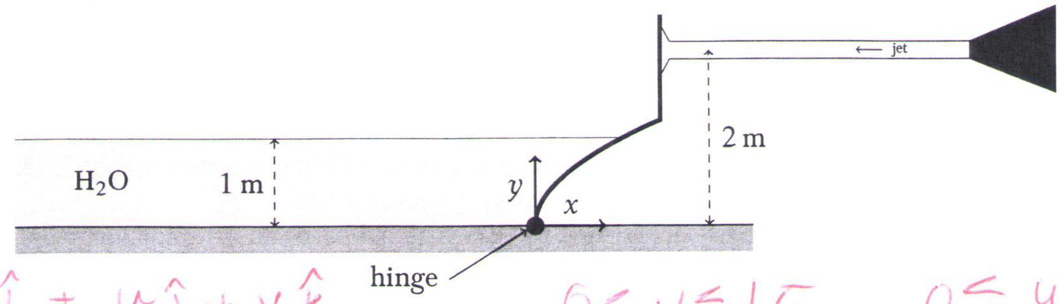
$$\int_0^u \frac{dU}{(V_j - U)^2} = \int_0^t \frac{3}{2}\frac{\rho S}{M} dt$$

$$\frac{1}{V_j - U} - \frac{1}{V_j} = \frac{3}{2}\frac{\rho S t}{M} \quad (3)$$

$$U = V_j \left[\frac{\frac{3}{2}\frac{V_j \rho S}{m} t}{1 + \frac{3}{2}\frac{V_j \rho S t}{m}}\right] = 20\,\mathrm{m/s}\left[\frac{(3\,\mathrm{s}^{-1})t}{1 + (3\,\mathrm{s}^{-1})t}\right]$$

Name:

4) The gate shown below has a width of 1.5 m. It's shape is described by the expression, $y = \sqrt{x}$. It is held in place by a jet of water that strikes it 2 m above the hinge. The diameter of the jet is 5 cm. What must the speed of the jet be to maintain equilibrium?



$r = u^2\hat{i} + u\hat{j} + v\hat{k}$ $\quad 0 \le v \le 1.5 \quad 0 \le u \le 1$

$\frac{\partial r}{\partial u} = 2u\hat{i} + \hat{j} \quad \frac{\partial r}{\partial v} = \hat{k}$

$\left|\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right| = \begin{vmatrix} i & j & k \\ 2u & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{i} - 2u\hat{j}$ points right @ $u = 0$, switch sign

$\therefore \hat{n}\, dA = (-\hat{i} + 2u\hat{j})\, du\, dv$ (2)

$p = \rho g(1-u)$ (2)

$dM = \vec{r} \times (-p\hat{n}\, dA) = -\rho g(1-u) \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u^2 & u & v \\ -1 & 2u & 0 \end{vmatrix} du\, dv$

$= -\rho g(1-u) \begin{bmatrix} -2uv\,\hat{i} \\ -v\,\hat{j} \\ (2u^3+u)\hat{k} \end{bmatrix} du\, dv$ (2) $\quad M_z = \int_0^w \int_0^1 -\rho g(1-u)(2u^3+u)\, du\, dv$

$M_z = -W\rho g \int_0^1 \left[2u^3 + u - 2u^4 - u^2\right] du = -W\rho g\left[\frac{2u^4}{4} + \frac{u^2}{2} - \frac{2u^5}{5} - \frac{u^3}{3}\right]_0^1$

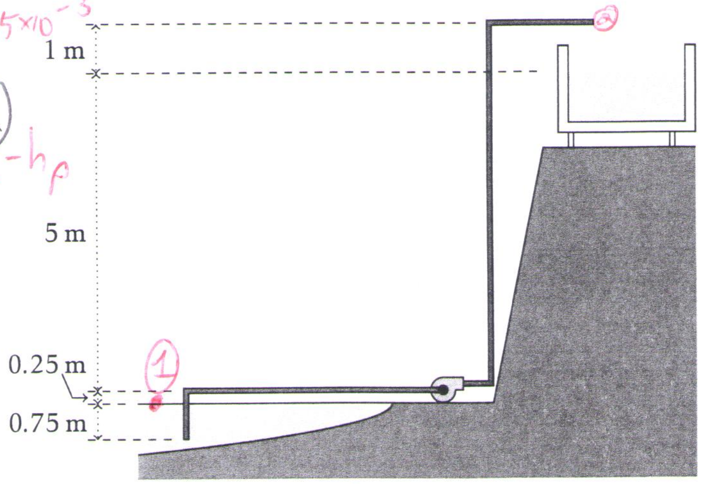
$M_z = -\frac{4}{15} W\rho g = -3924\ \text{Nm}$ (2) $\quad F_j = \frac{3924\ \text{Nm}}{2\ \text{m}} = \rho V_j^2 A$ (2)

$V_j = \sqrt{\frac{3924\ \text{Nm}}{(2\text{m})(1000\ \text{kg/m}^3)\left(\pi \frac{0.05^2}{4}\right)}} = \boxed{31.61\ \text{m/s}}$

5) A piping system is used to deliver water from a lake to a storage tank, as shown. The piping system is made of commercial steel and is 20 m long in total. The pipes have an inner diameter of 4 cm. The system has an electric pump and three 90° elbows. The pressure difference provided by the pump is 66 kPa. The viscosity of the water is, $\mu_{H_2O} = 1.3 \times 10^{-3}$ Pa s. Treat the pipe entrace as "reentrant" (see page 11).

a) What is the expected volumetric flow rate?

b) If the pump has an efficiency of 75%, how much power does it consume?



$\varepsilon = 0.046\,mm \qquad \frac{\varepsilon}{D} = \frac{0.046\,mm}{40\,mm} = 1.15\times10^{-3}$

$$\left(\frac{P}{\rho} + \alpha\frac{\bar{V}^2}{2} + gz\right)_1 - \left(\frac{P}{\rho} + \alpha\frac{\bar{V}^2}{2} + gz\right)_2 = h_{\ell t} - h_p \quad (2)$$

$$h_p = \frac{\Delta p}{\rho} = \frac{66000\,Pa}{1000\,kg/m^3} = 66\,m^2/s^2 \quad (2)$$

$$h_{\ell t} = f\frac{L}{D}\frac{\bar{V}^2}{2} + 3f\frac{L_e}{D}\frac{\bar{V}^2}{2} + K\frac{\bar{V}^2}{2} \quad (2)$$

$$-\left(\frac{\alpha\bar{V}^2}{2} + gz_2\right) = \frac{\bar{V}^2}{2}\left[f\left(\frac{L}{D} + 3\frac{L_e}{D}\right) + K\right] - 66\,m^2/s^2$$

$$-gz_2 + 66\,m^2/s^2 = \frac{\bar{V}^2}{2}\left[f\left(\frac{L}{D} + 3\frac{L_e}{D}\right) + K + \alpha\right]$$

30 0.78

$\frac{20\,m}{0.04\,m} = 500$

$(9.81\,m/s^2)(6.25\,m)$

$$4.6875\,m^2/s^2 = \frac{\bar{V}^2}{2}\left[f(590) + \alpha + 0.78\right]$$

$$\bar{V} = \sqrt{\frac{2(4.6875\,m^2/s^2)}{f(590) + \alpha + 0.78}}$$

assume $Re = 100\,000$

$f = 0.022$

$\alpha = 1$

$$\bar{V} = \sqrt{\frac{2(4.6875\,m^2/s^2)}{0.022(590) + 1.78}} = 0.8\,m/s \rightarrow Re = 24600$$

$f = 0.027$

$\alpha = 1$

close enough

$$\bar{V} = \sqrt{\frac{2(4.6875\,m^2/s^2)}{0.027(590) + 1.78}} = 0.73\,m/s \rightarrow Re = 22500$$

$\bar{V} = 0.73\,m/s$

$$Q = \bar{V}A = \bar{V}\frac{\pi D^2}{4} = (0.73\,m/s)\pi\frac{(0.04\,m)^2}{4}$$

$$\boxed{= 9.2\times10^{-4}\,m^3/s = 0.92\,L/s} \quad (2)$$

$$h_p = \frac{\dot{W}}{Q\rho}$$

$$\dot{W} = h_p Q\rho = 66\,m^2/s^2(9.2\times10^{-4})(1000) = 60.72\,W \quad (2)$$

$$\dot{W} = \frac{60.72}{.75} \boxed{= 81\,W}$$