

Final Exam
MCG 3340, Fluid Mechanics I
December 17th, 2015
2:00–5:00 p.m.

Closed-book, non-programmable calculators only.

Name: _____

Student number: _____

Question	Possible	Result
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

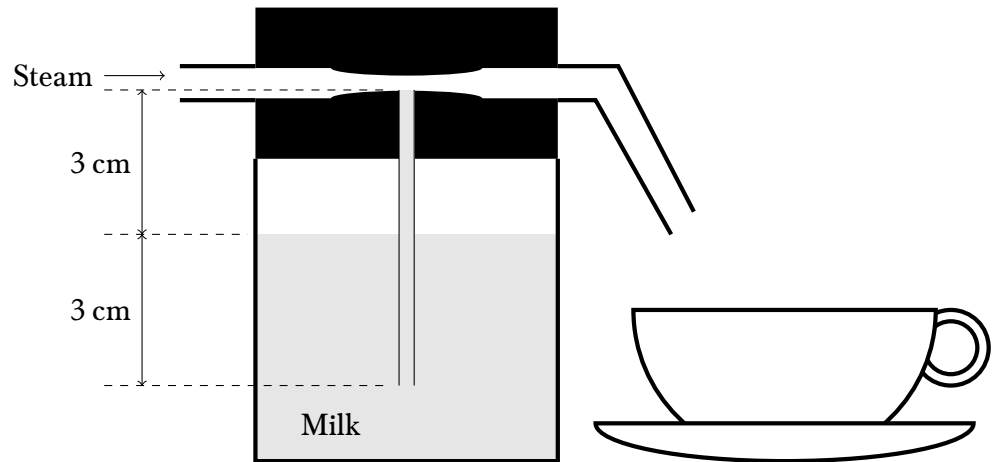
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- 1) A scale-model test of a proposed new aircraft design is to be conducted in a wind tunnel. The lift force, F_L , on the real aircraft is expected to be a function of the aircraft length, L , flow speed, V , fluid density, ρ , fluid viscosity, μ , and speed of sound, c . The proposed aircraft has a length of 30 m and is expected to cruise at a speed of 900 km/h at an altitude, h , of 11 km. Two wind-tunnel facilities are available for testing, both can accommodate a scale model with a maximum length of 2 m. One uses air at standard conditions and the other uses high-pressure, cryogenic nitrogen.
- a) Find suitable non-dimensional groups for the problem.
 - b) Is it possible to use either wind tunnel? Why?
 - c) If it were possible to use both tunnels, what is the ratio of the expected lift on the real aircraft to that measured in the test? Compute the ratio for each tunnel.

	Air, $h = 11$ km	Air, wind tunnel	Nitrogen, wind tunnel
Temperature, (K)	216	288	180
Density, (kg/m^3)	0.365	1.225	7.482
Speed of sound, (m/s)	295	340	274
Viscosity, (Pa s)	1.4×10^{-5}	1.8×10^{-5}	1.2×10^{-5}

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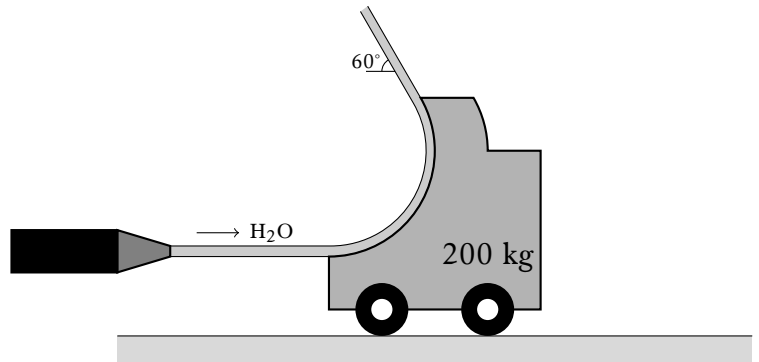
- 2) An automatic cappuccino machine uses a Venturi to draw milk, $\rho_{\text{milk}} = 1035 \text{ kg/m}^3$, vertically into a stream of hot steam through a straw with a diameter of 0.5 cm and a length of 6 cm. The viscosity of the milk is $\mu_{\text{milk}} = 3 \times 10^{-3} \text{ Pa s}$. The steam has a density, $\rho_{\text{steam}} = 1.7 \text{ kg/m}^3$ and it enters the Venturi with a relative pressure of 200 Pa and velocity of 3 m/s. Assume the loss coefficient, K , at both the straw entrance and exit is 0.5. Neglect viscosity in the steam flow. What must be the ratio between the maximum and minimum diameters in the Venturi to achieve a milk flow of 12 ml/s?



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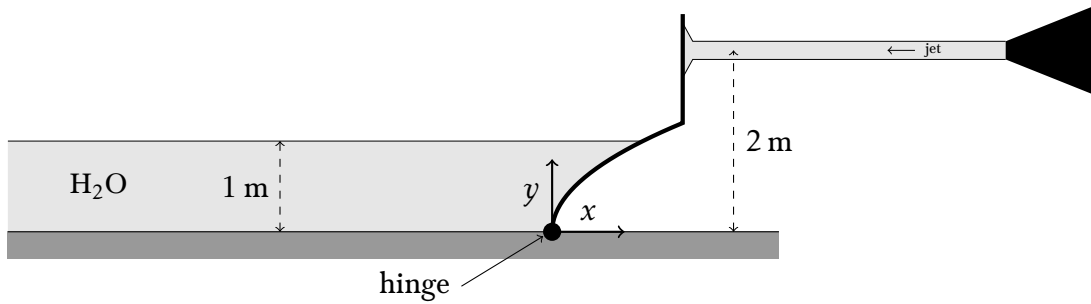
- 3) A jet of water with speed, $V_j = 20\text{ m/s}$ hits a 200 kg cart and is turned. The jet maintains a cross-sectional area, $S_j = 0.02\text{ m}^2$. At time, $t = 0\text{ s}$, the cart is at rest at a position, $x = 0\text{ m}$. Find an expression for the speed of the cart for $t > 0\text{ s}$.

You can neglect friction in the wheels, variation in the momentum of the water that is in contact with the cart, and gravitational effects.



Name: _____

- 4) The gate shown below has a width of 1.5 m. Its shape is described by the expression, $y = \sqrt{x}$. It is held in place by a jet of water that strikes it 2 m above the hinge. The diameter of the jet is 5 cm. What must the speed of the jet be to maintain equilibrium?

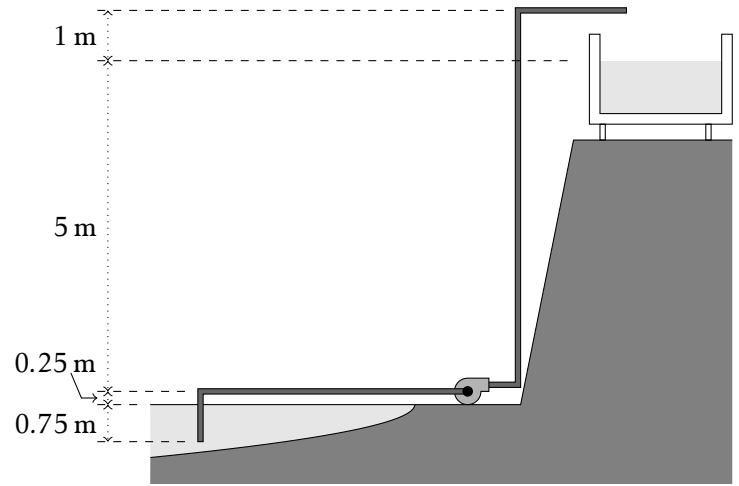


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5) A piping system is used to deliver water from a lake to a storage tank, as shown. The piping system is made of commercial steel and is 20 m long in total. The pipes have an inner diameter of 4 cm. The system has an electric pump and three 90° elbows. The pressure difference provided by the pump is 66 kPa. The viscosity of the water is, $\mu_{\text{H}_2\text{O}} = 1.3 \times 10^{-3} \text{ Pa s}$. Treat the pipe entrance as “reentrant” (see page 11).

a) What is the expected volumetric flow rate?

b) If the pump has an efficiency of 75%, how much power does it consume?



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$$\vec{\nabla} p = \rho \vec{g} - \rho \vec{a}$$

$$d\vec{F} = -p \hat{n} dS$$

$$d\vec{M} = \vec{r}_\ell \times d\vec{F}$$

$$0 = \frac{\partial}{\partial t} \iiint_{\text{C.V.}} \rho dV + \oint_S \rho \vec{V} \cdot \hat{n} dS$$

$$\sum \vec{F} - \iiint_{\text{C.V.}} \rho \vec{a} dV = \frac{\partial}{\partial t} \iiint_{\text{C.V.}} \rho \vec{V} dV + \oint_S \rho \vec{V} \vec{V} \cdot \hat{n} dS$$

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t} \iiint_{\text{C.V.}} \rho e dV + \iint_S \rho \left(e + \frac{p}{\rho} \right) \vec{V} \cdot \hat{n} dS$$

$$e = u + \frac{1}{2} V^2 + gz$$

$$\frac{p}{\rho} + gz + \frac{V^2}{2} = \text{constant}$$

$$\frac{k}{k-1} \frac{p}{\rho} + gz + \frac{V^2}{2} = \text{constant}$$

$$p = \rho RT \quad \text{and} \quad \frac{p}{\rho^k} = \text{constant}$$

$$\left(\frac{p}{\rho} + \frac{1}{2} \alpha \bar{V}^2 + gz \right)_1 - \left(\frac{p}{\rho} + \frac{1}{2} \alpha \bar{V}^2 + gz \right)_2 = h_{l_T} - h_{\text{pompe}}$$

$$h_{\text{pompe}} = \frac{\dot{W}_{\text{pompe}}}{\dot{m}}$$

$$h_{l_M} = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$h_{l_m} = K \frac{\bar{V}^2}{2} \quad \text{or} \quad h_{l_m} = f \frac{L_e}{D} \frac{\bar{V}^2}{2}$$

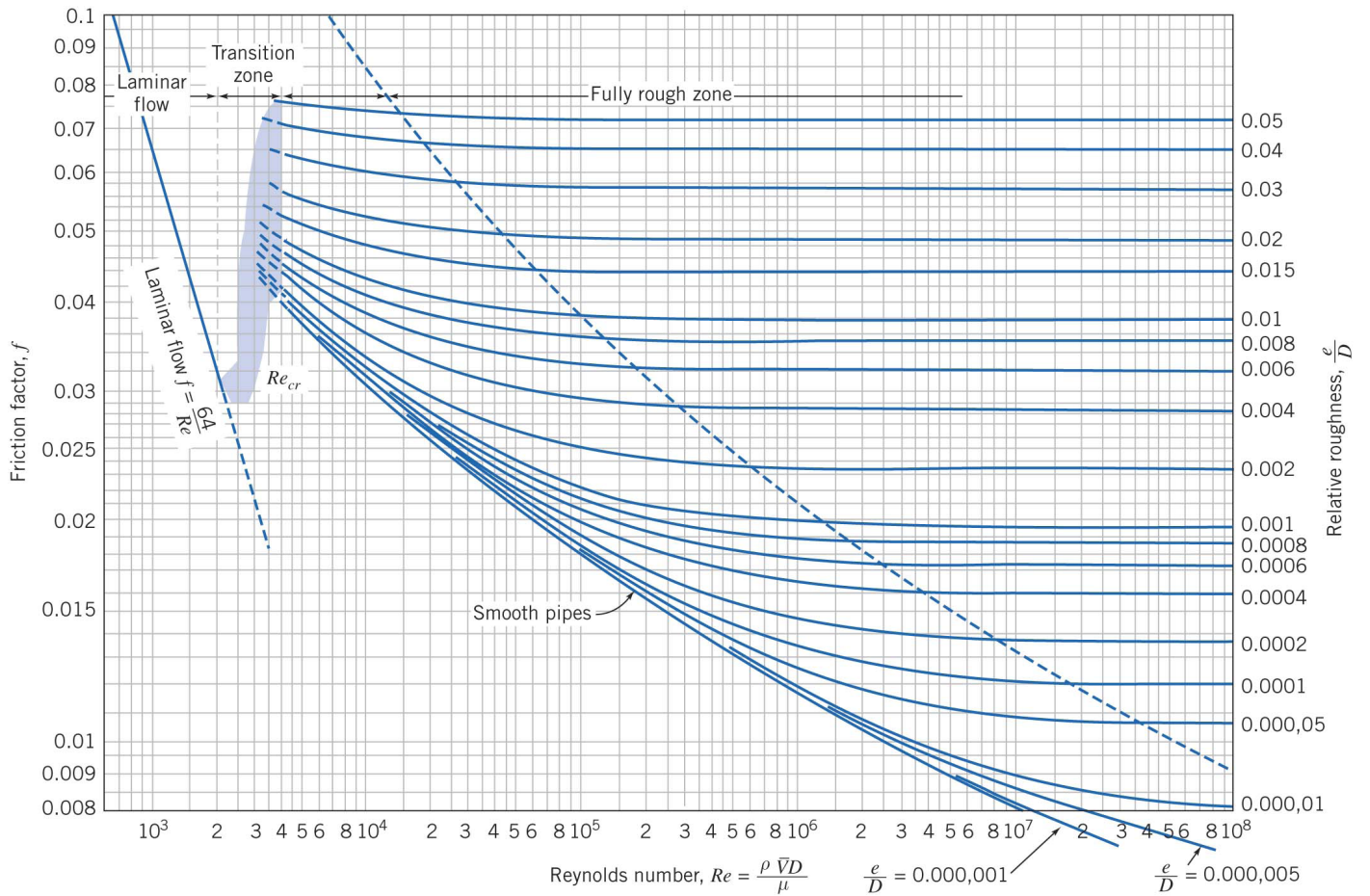


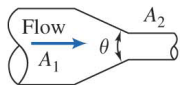
Figure 1: Moody diagram; Pritchard (2011).

Roughness for Pipes of Common Engineering Materials

Pipe	Roughness, e	
	Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9
Concrete	0.001–0.01	0.3–3
Wood stave	0.0006–0.003	0.2–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Asphalted cast iron	0.0004	0.12
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015



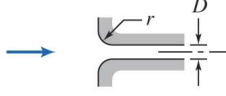
Figure 2: Roughness of common materials; Pritchard (2011).

Loss Coefficients (K) for Gradual Contractions: Round and Rectangular Ducts

	A_2/A_1	Included Angle, θ , Degrees						
		10	15–40	50–60	90	120	150	180
	0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
	0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
	0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{lm} = K(\bar{V}_2^2/2)$.

Minor Loss Coefficients for Pipe Entrances

Entrance Type	Minor Loss Coefficient, K^a								
Reentrant 	0.78								
Square-edged 	0.5								
Rounded 	<table border="1"> <tr> <td>r/D</td> <td>0.02</td> <td>0.06</td> <td>≥ 0.15</td> </tr> <tr> <td>K</td> <td>0.28</td> <td>0.15</td> <td>0.04</td> </tr> </table>	r/D	0.02	0.06	≥ 0.15	K	0.28	0.15	0.04
r/D	0.02	0.06	≥ 0.15						
K	0.28	0.15	0.04						

^aBased on $h_{lm} = K(\bar{V}^2/2)$, where \bar{V} is the mean velocity in the pipe.

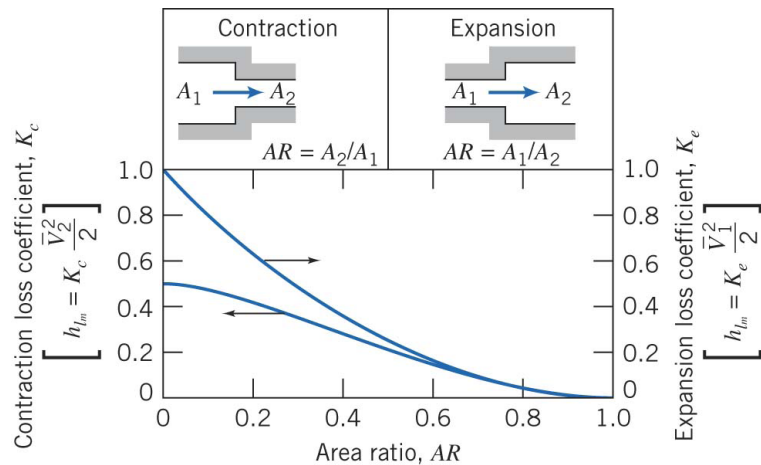


Figure 3: Loss coefficients for certain geometries; Pritchard (2011).

