1)

$$\frac{k}{k-1}\frac{p_0}{\rho_0} = \frac{k}{k-1}\frac{p}{\rho} + \frac{V^2}{2}$$

$$\frac{p_0}{\rho_0} = \frac{p}{\rho} + \frac{k-1}{k}\frac{V^2}{2}$$

$$\frac{p_0}{\rho_0}\frac{\rho}{p} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

isentropic : $\frac{p}{\rho^k} = \frac{p_0}{\rho^k}$

$$\rho_0 = \left(\frac{p_0}{p}\right)^{1/k}\rho$$

$$\frac{p_0}{\cancel{\rho}}\left(\frac{p}{p_0}\right)^{1/k}\frac{\cancel{\rho}}{p} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

$$\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

$$\boxed{p_0 = p\left[1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}\right]^{k/k-1}}$$

Assy # ~~5~~ #2

$$\vec{V}_1 = \frac{Q}{A_1} = \frac{0.1\,m^3/s}{\frac{\pi}{4}(0.15m)^2}\,\hat{\imath} = 5.66\,m/s\,\hat{\imath}$$

$$\vec{V}_2 = \frac{Q}{A_2}(-\hat{\jmath}) = \frac{0.1\,m^3/s}{\pi/4(0.1m)^2}(\hat{\jmath}) = -12.73\,m/s\,\hat{\jmath}$$

$(P_2 = P_a)$

Bernoulli: $\frac{P_1}{\rho} + gz_1 + \frac{|V_1|^2}{2} = \frac{P_a}{\rho} + gz_2 + \frac{|V_2|^2}{2}$ $\quad z_1 \approx z_2$

$$\frac{P_1 - P_a}{\rho} = \frac{1}{2}\left(|V_1|^2 - |V_2|^2\right)$$

$$P_1 - P_a = \underbrace{(500\,kg/m^3)}_{\rho/2}\left[(12.73m/s)^2 - (5.66m/s)^2\right]$$

$$P_1 - P_a = 65\,kPa$$

$$\sum F = \cancel{\frac{\partial}{\partial t}\iiint_V \rho\vec{v}\,dV}^{\,0} + \iint_S \rho\vec{v}\,\vec{v}\cdot\hat{n}\,ds$$

$$F_R + F_P = \iint_S \rho\vec{v}\,\vec{v}\cdot\hat{n}\,ds$$

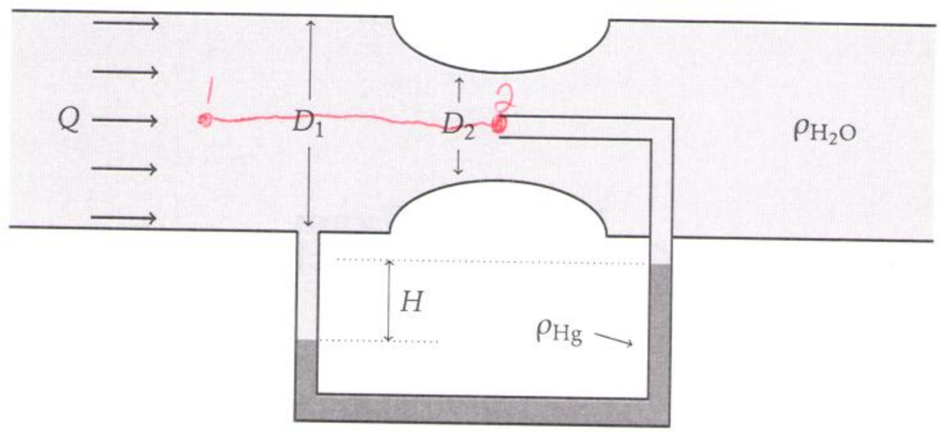
$$F_R = -\rho\vec{V}_1 Q + \rho\vec{V}_2 Q - (P_1 - P_a)S_1\,\hat{\imath}$$

$$= (1000\,kg/m^3)(0.1m^3/s)\left[-(5.66m/s\,\hat{\imath}) + (-12.73m/s)\hat{\jmath}\right]$$

$$- 65000\,Pa\,\frac{\pi}{4}(0.15m)^2\,\hat{\imath}$$

$$= \left[-1715\,\hat{\imath} - 1273\,\hat{\jmath}\right]N$$

1) An enterprising engineer has invented a new "super" flow meter for water flow in circular pipes that combines a Pitot tube and a Venturi flow meter. In this new design, how is the difference in height of mercury in the manometer, H, related to the volumetric flow rate, Q, the diameter of the pipe, D_1, and the diameter of the throat, D_2?



$$P_1 + \frac{\rho V_1^2}{2} + \cancel{\rho g z_1} = P_2 + \cancel{\frac{\rho V_2^2}{2}} + \cancel{\rho g z_2}$$

$$P_1 - P_2 = \frac{\rho V_1^2}{2} \qquad V_1 = \frac{Q}{\pi D_1^2/4} = \frac{4Q}{\pi D_1^2}$$

$$= \frac{\rho \left(4Q/\pi D_1^2\right)^2}{2} = \frac{16\rho Q^2}{2\pi^2 D_1^2}$$

$$\Delta p = gH(\rho_{Hg} - \rho_{H_2O}) = \frac{16\rho Q^2}{2\pi^2 D_1^2}$$

$$\boxed{H = \frac{8\rho Q^2}{\pi^2 D_1^2 g(\rho_{Hg} - \rho_{H_2O})}}$$