$$z = 9 - x^2 + y^2$$

$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + 9 - r^2\,\hat{k}$$

$$\frac{\partial \vec{r}}{\partial \theta} = -r\sin\theta\,\hat{i} + r\cos\theta\,\hat{j} + 0\,\hat{k}$$

$$\frac{\partial \vec{r}}{\partial r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} + -2r\,\hat{k}$$

$$\left|\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial r}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -r\sin\theta & r\cos\theta & 0 \\ \cos\theta & \sin\theta & -2r \end{vmatrix} = \begin{matrix} -2r^2\cos\theta\,\hat{i} \\ -2r^2\sin\theta\,\hat{j} \\ -(r\sin^2\theta - r\cos^2\theta)\,\hat{k} \end{matrix}$$

points into water ✓

x & y are symmetric $\therefore F_x = F_y = 0$

$$F_z = \iint_S -p\hat{n}\,dS$$

$$p_{rel} = \rho g(9 - z)$$
$$= \rho g(9 - 9 - r^2)$$
$$= \rho g r^2$$

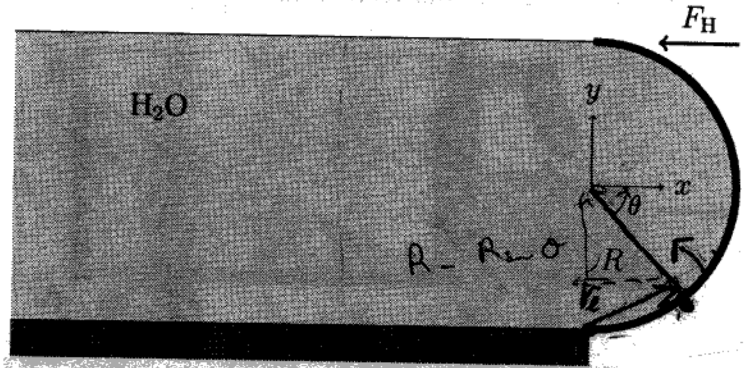
$$= \int_0^{2\pi}\int_0^{3m} -\rho g r^2 \left(-r\underbrace{(\sin^2\theta + \cos^2\theta)}_{1}\right) dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{3m} \rho g r^3\,dr\,d\theta = 2\pi\left[\frac{\rho g r^4}{4}\right]_0^{3m} = \frac{\pi}{2}\rho g r^4$$

$$= \frac{\pi}{2}(1000\,kg/m^3)(9.8\,m/s^2)(3m)$$
$$= 1248\,kN\ \hat{k}$$

Name:

2) A gate is formed from a half cylinder. It has a width, W; a height, $2R$; and is hinged at the bottom. What horizontal force, F_H is needed to hold the gate in equilibrium?



$$x = R\cos\theta \qquad y = -R\sin\theta \qquad z = z \qquad 1$$

$$d\vec{F}_p = -p\,\hat{n}\,dS \qquad 1 \quad p = \rho g(R-y) = \rho g(R + R\sin\theta) = \rho g R(1+\sin\theta)$$

$$\vec{r} = R\cos\theta\,\hat{\imath} - R\sin\theta\,\hat{\jmath} + z\,\hat{k}$$

$$2 \quad \hat{n}\,dS = \left(\frac{\partial r}{\partial \theta} \times \frac{\partial r}{\partial z}\right) d\theta\, dz = \begin{vmatrix} i & j & \hat{k} \\ -R\sin\theta & -R\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} d\theta\, dz = \left(-R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}\right) d\theta\, dz$$

$$1 \quad d\vec{F}_p = -\rho g R(1+\sin\theta)\left(-R\cos\theta\,\hat{\imath} + R\sin\theta\,\hat{\jmath}\right) d\theta\, dz$$

$$1 \quad \text{"lever arm"}: \vec{r}_\ell = R\cos\theta\,\hat{\imath} + R(1-\sin\theta)\,\hat{\jmath} + z\,\hat{k}$$

$$d\vec{M} = \vec{r}_\ell \times d\vec{F}_p \quad 1 \quad = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & k \\ R\cos\theta & R(1-\sin\theta) & z \\ -\cos\theta & \sin\theta & 0 \end{vmatrix} \left(-\rho g R^2(1+\sin\theta)\, d\theta\, dz\right) =$$

$$= \left[-z\sin\theta\,\hat{\imath} - z\cos\theta\,\hat{\jmath} + \left(\cancel{R\cos\theta\sin\theta} + R\cos\theta - \cancel{R\cos\theta\sin\theta}\right)\hat{k}\right]\left(-\rho g R^2(1+\sin\theta)\, d\theta\, dz\right)$$

only look at moment around "z"

$$M_z = \int_0^W \int_{-\pi/2}^{\pi/2} -\rho g R^3 (1+\sin\theta)\cos\theta\, d\theta\, dz \quad 1 \quad = -\rho g R^3 W \int_{-\pi/2}^{\pi/2} (1+\sin\theta)\cos\theta\, d\theta$$

$$= -\rho g R^3 W \left[\sin\theta + \frac{\sin^2\theta}{2}\right]_{-\pi/2}^{\pi/2} = -2\rho g R^3 W \quad 1 \qquad 1 \quad F = \frac{M_z}{2R} = \rho g R^2 W$$