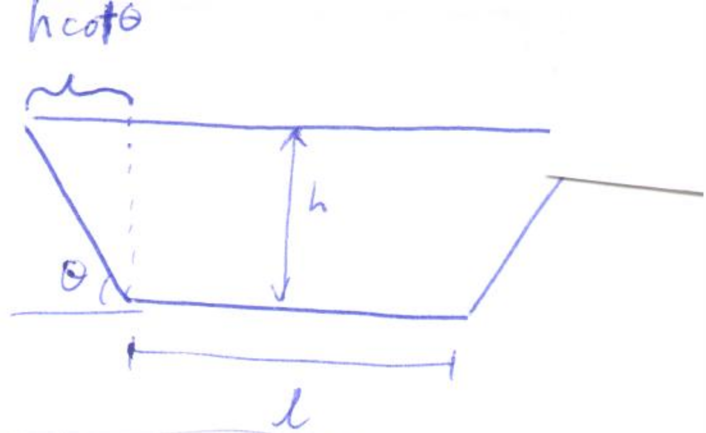
1) Weight:

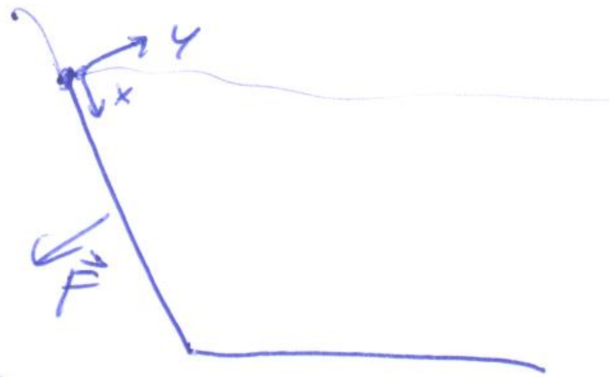


$$\vec{F}_g = -\rho g V \hat{k} = -\rho g [l + h\cot\theta] h w \hat{k}$$

Pressure force:

bottom $\vec{F}_g = -pA\hat{k} = \boxed{-\rho g h l w \hat{k}}$

one side:



$$\vec{F}_p = \iint_S -p\hat{n}\, dS \qquad \hat{n} = -\hat{j}$$

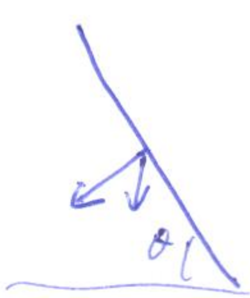
$$p = \rho g h$$

$$h = x \sin\theta$$

$$\vec{F}_p = \int_0^w \int_0^{h/\sin\theta} \rho g x \sin\theta \, \hat{j} \, dx\, dz$$

$$= w \rho g \sin\theta \frac{x^2}{2}\Big]_0^{h/\sin\theta} = \frac{1}{2} w \rho g \frac{h^2}{\sin\theta}$$

$\Rightarrow$ global coordinate system



vertical component $= -\frac{1}{2} w \rho g \frac{h^2}{\sin\theta} \cos\theta \hat{k} = -\frac{1}{2} w \rho g h^2 \cot\theta \hat{k}$

total force = bottom + 2×sides

$= -\rho g h w [l + h\cot\theta]\hat{k}$

2) $\frac{dp}{dz} = \rho g$ $\qquad \rho = \frac{p}{RT}$ $\qquad g = -9.81\ m/s^2$

2 choices : $T = \text{constant}$

$T = T_0 + Az$

$A = \frac{dT}{dz}$, slope

$T = \text{const}$

$$\frac{dp}{dz} = \frac{p}{RT} g \Rightarrow \int_{p_1}^{p_2} \frac{dp}{p} = \int_{z_1}^{z_2} \frac{g}{RT} dz$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{g}{RT}(z_2 - z_1) \Rightarrow \boxed{\frac{p_2}{p_1} = e^{\frac{g(z_2-z_1)}{RT}}}$$

$T = T_0 + Az$

$$\frac{dp}{dz} = \frac{p}{R(T_0 + Az)} g \Rightarrow \int_{p_1}^{p_2} \frac{dp}{p} = \int_{z_1}^{z_2} \frac{g}{R(T_0+Az)} dz$$

$$\boxed{\ln\left(\frac{p_2}{p_1}\right) = \frac{g}{RA} \ln\left(\frac{T_0 + Az_2}{T_0 + Az_1}\right)}$$

$$\frac{p_2}{p_1} = \left(\frac{T_0 + Az_2}{T_0 + Az_1}\right)^{g/RA}$$

layer 1: $T = T_0 + Az$

$P_0 = 101325\ Pa \quad A = -6.5\times10^{-3}\ K/m \quad T_0 = 288K$

$$P = P_1 \left(\frac{288K - 6.5\times10^{-3}\ K/m\ z}{288K}\right)^{\frac{9.81\ m/s^2}{(287\ J/kgK)(-6.5\times10^{-3}\ K/m)}}$$

$P_1 = 101325\ Pa \rightarrow P_2\big|_{11\ km} = \boxed{22594\ Pa}$

layer 2: $T = -56.5\ °C = 216.5\ K$

$$P = P_2\, e^{\left[\frac{9.81\ m/s^2\ (z - 11000\ m)}{287\ J/kgK\ (216.5K)}\right]}$$

$$\boxed{P_3\big|_{20km} = 5456\ Pa}$$

layer 3: $T(z) = 216.5K + 1\times10^{-3}\ K/m\ (z - 20000\ m)$

$$P = P_3 \left(\frac{216.5K + 1\times10^{-3}\ K/m\ (z - 20000\ m)}{216.5\ K}\right)^{\frac{-9.81\ m/s^2}{(287\ J/kgK)(1\times10^{-3}\ K/m)}}$$

$$P_4 = 5456\ Pa \left(\frac{228.5K}{216.5K}\right)^{\frac{-9.81\ m/s^2}{(287\ J/kgK)(1\times10^{-3}\ K/m)}} = \boxed{850\ Pa}$$

layer 4: $T(z) = 228.6\,K + 2.8 \times 10^{-3}\,K/m\,(z - 32000\,m)$

$$P = P_4 \left(\frac{228.6\,K + 2.8 \times 10^{-3}\,K/m\,(z - 32000\,m)}{228.6} \right)^{\frac{-9.81\,m/s^2}{287\,J/kgK\,(2.8 \times 10^{-3}\,K/m)}}$$

$$P_5 = 450\,Pa \left(\frac{270.6}{228.6} \right)^{\frac{-9.81\,m/s^2}{(287\,J/kgK)(2.8 \times 10^{-3}\,K/m)}}$$

$$\boxed{P_5 = 108.5\,Pa}$$

layer 5: $T = -2.5°C = 270.6\,K$

$$P = P_5\, e^{\left[\frac{-9.81\,m/s^2\,(z - 47000\,m)}{287\,J/kgK\,(270.6\,K)}\right]}$$

$$P_6 = 108.5\, e^{\left(\frac{-9.81\,m/s^2\,(4000\,m)}{287\,J/kgK\,(270.6\,K)}\right)}$$

$$\boxed{P_6 = 65.5\,Pa}$$

layer 6: $T = 270.6\,K - 2.8\times10^{-3}\,K/m\,(z - 51000\,m)$

$$P = P_6 \left(\frac{270.6\,K - 2.8\times10^{-3}\,K/m\,(z-51000\,m)}{270.6\,K}\right)^{\frac{-9.81\,m/s^2}{(287\,J/kg\,K)(-2.8\times10^{-3}\,K/m)}}$$

$$P_7 = 65.5\,Pa\left(\frac{214.6\,K}{270.6\,K}\right)^{\frac{-9.81\,m/s^2}{(287\,J/kg\,K)(-2.8\times10^{-3}\,K/m)}}$$

$$\boxed{P_7 = 3.9\,Pa}$$

layer 7: $T = 214.6\,K - 2\times10^{-3}\,K/m\,(z - 71000\,m)$

$$P = P_7 \left(\frac{214.6\,K - 2\times10^{-3}\,K/m\,(z-71000\,m)}{214.6\,K}\right)^{\frac{-9.81\,m/s^2}{(287\,J/kg\,K)(-2\times10^{-3}\,K/m)}}$$

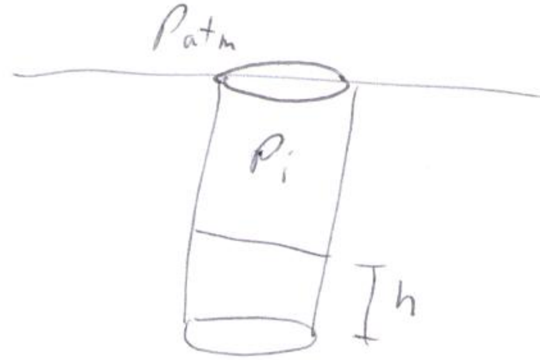
$$P_8 = 3.9\,Pa\left(\frac{186.6\,K}{214.6\,K}\right)^{\frac{-9.81\,m/s^2}{287\,J/kg\,K\,(-2\times10^{-3}\,K/m)}}$$

$$\boxed{P_8 = 0.4\,Pa}$$

Rounding errors can pile up in this calculation. Your answer may differ a bit.

You can find ρ from the ideal gas law

3) a) $P_i = P_{atm} + \rho_{H_2O}\, g (L-h)$



Boyle's law: $PV = \text{const}$

$$P_{atm} A L = P_i A (L-h)$$

$$P_i = \frac{P_{atm} \cancel{A} L}{\cancel{A}(L-h)}$$

$$P_{atm} + \rho_{H_2O}\, g (L-h) = \frac{P_{atm} L}{L-h} \Rightarrow \boxed{h = 0.082\,m}$$

b) $\vec{F} = P_i A \hat{k} - P_{atm} A \hat{k}$

$$P_i = P_{atm} + \rho g (L-h) = 110\,334\,Pa$$

$$\vec{F} = (110\,334\,Pa - 101\,325\,Pa)(0.1\,m^2) = \boxed{901\,N}$$

4) $d\vec{M} = d\vec{M}_1 + d\vec{M}_2$

$= \vec{r}_l \times (d\vec{F}_1 + d\vec{F}_2)$

$d\vec{F}_1 = -p_1 \hat{n}_1 dS \quad dF_2 = -p_2 \hat{n}_2 dS$

$\hat{n}_1 = \hat{j} \quad \hat{n}_2 = -\hat{j}$

$p_1 = \rho g(8m - \frac{4}{5}x) \quad p_2 = \rho g(5m - \frac{4}{5}x) \quad r_l = x\hat{i} + z\hat{k}$

$d\vec{M} = (x\hat{i} + z\hat{k}) \times [-\rho g(8m - \frac{4}{5}x)\hat{j} + \rho g(5m - \frac{4}{5}x)\hat{j}]\,dx\,dz$

$= (x\hat{i} + z\hat{k}) \times [(-3m)\rho g \hat{j}]\,dx\,dz$

$= (-3m)\rho g \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 0 & z \\ 0 & 1 & 0 \end{vmatrix} dx\,dz = (-3m\,\rho g)\begin{bmatrix} -z\hat{i} \\ 0\hat{j} \\ x\hat{k} \end{bmatrix} dx\,dz$

need M_z

$M_z = \int_0^{2m}\int_0^{5m} (-3m)\rho g\, x\, dx\, dz = -(3m)(2m)(1000\frac{kg}{m^3})(9.81 m/s)(\frac{25m^2}{2})$

$= -735\,750$ Nm

$\vec{F} = \left(\frac{735\,750\text{ Nm}}{5m}\right)\hat{j} = \boxed{147150 N \hat{j}}$

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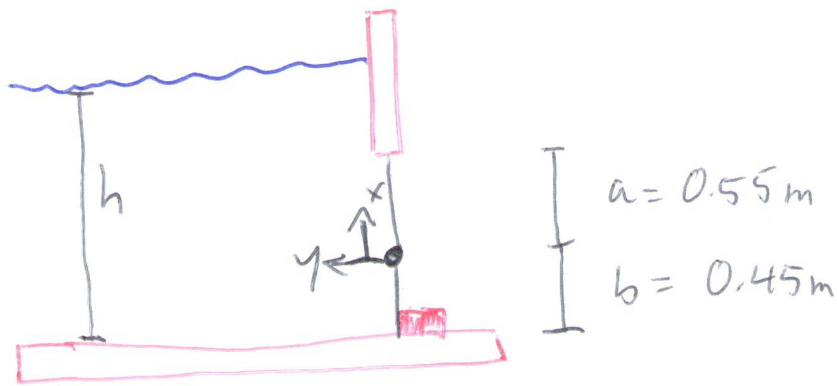

$r_l = x\hat{i} + z\hat{k}$

$p = [h - b - x]\rho g$

$\hat{n} = \hat{j}$

$dS = dx\,dz$

$-0.45m < x < 0.55m$

$0 < z < 2m$

$d\vec{F}_p = -p\hat{n}\,dS$

$d\vec{M} = \vec{r}_l \times d\vec{F}_p$

$$d\vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & 0 & z \\ 0 & 1 & 0 \end{vmatrix} (h-b-x)\rho g\,dx\,dz$$

$$= [-z\hat{i} + x\hat{k}](h-b-x)\rho g\,dx\,dz$$

only interested in z component

$$dM_z = (h-b-x)x\rho g\,dx\,dz$$

$$M_z = \int_0^{2m}\int_{-0.45m}^{0.55m} (h-b-x)x\rho g\,dx\,dz$$

$$= (2m)(\rho g)\int_{-0.45m}^{0.55m} (h-b)x - x^2\,dx\,dz$$

$$M_z = (2m)(\rho g)\left[\frac{h-b}{2}x^2 - \frac{x^3}{3}\right]_{-0.45m}^{0.55m}$$

need to find "h" so that $M_z = 0$

$\therefore$

$$\left[\frac{h-b}{2}x^2 - \frac{x^3}{3}\right]_{-0.45m}^{0.55m} = 0$$

$$0 = \frac{h-(0.45m)}{2}\left(0.3025m^2 - 0.2025m^2\right) - \frac{1}{3}\left(0.166375m^3 + 0.091125m^3\right)$$

$$0 = \frac{h-0.45m}{2}(0.1m^2) - \frac{1}{3}(0.2575m^3)$$

$$\boxed{h = 2.17m}$$