CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE

Department of Mechanical, Industrial and Aerospace Engineering

MIDTERM

ENGR 311/AA SUMMER 2018

Total Marks = 30

Instructor: Dr. Nabil Esmail

Date: 24 May 2018 Time: 75 minutes

NAME: ______________________________
(Please Print) SURNAME FIRST NAME

STUDENT ID: ______________ SECTION: AA

SIGNATURE: ______________________________

INSTRUCTOR: ______________________________

Name and student I/D must be written in INK.

All work and solution steps must be illustrated in order to gain full marks assigned to the question.

INDIVIDUAL WORK - Closed Book Test

Material allowed: Approved calculator only

Answer the questions in the space provided.

Return the paper at the end of the scheduled time.

Q1 (10)	**Q2 (12)**	**Q3 (8)**	**Total (30)**

Question No. 1. (10 Marks) Using the Laplace Transformation method, solve the integral equation to find $f(t)$:

$$f(t) = 2t - 4\int_0^t \cos\tau \,.\, f(t-\tau)d\tau$$

$$g(\tau) \equiv \cos\tau \qquad h(t-\tau) = f(t-\tau) \quad V(s) = \mathcal{L}\left\{\int_0^t \cos\tau \,.\, f(t-\tau)d\tau\right\}$$

$$G(s) = \frac{s}{s^2+1} \qquad H(s) = F(s)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{2t\} - 4\mathcal{L}\left\{\int_0^t \cos\tau \,.\, f(t-\tau)d\tau\right\}$$

$$F(s) = \frac{2}{s^2} - 4\left[\frac{s}{s^2+1}\right].F(s)$$

$$F\left[1 + \frac{4s}{s^2+1}\right] = F\left[\frac{s^2+4s+1}{s^2+1}\right] = \frac{2}{s^2}$$

$$F = \frac{2(s^2+1)}{s^2(s^2+4s+1)} = \frac{2}{s^2+4s+1} + \frac{2}{s^2(s^2+4s+1)}$$

$$PF \qquad \frac{2}{s^2(s^2+4s+1)} \equiv \frac{As+B}{s^2} + \frac{(Cs+D)}{(s^2+4s+1)}$$

$$2 \equiv As^3 + 4As^2 + As + Bs^2 + 4Bs + B + Cs^3 + Ds^2$$

$$A + C = 0 \quad 4A + B + D = 0 \quad A + 4B = 0 \quad B = 2$$

$$A = -8 \quad B = 2 \quad C = 8 \quad D = 30$$

$$PF \qquad \frac{2}{s^2(s^2+4s+1)} \equiv -\frac{-8s+2}{s^2} + \frac{8s+30}{(s^2+4s+1)}$$

$$F = \frac{2}{s^2+4s+1} - \frac{-8s+2}{s^2} + \frac{8s+30}{(s^2+4s+1)} = \frac{8}{s} - \frac{2}{s^2} + \frac{8s+32}{(s^2+4s+1)}$$

$$s^2 + 4s + 1 = (s+2)^2 - 3$$

$$F = \frac{8}{s} - \frac{2}{s^2} + \frac{8s+16+16}{(s+2)^2-3} = \frac{8}{s} - \frac{2}{s^2} + \frac{8(s+2)}{(s+2)^2-3} + \frac{16}{(s+2)^2-3}$$

$$f(t) = 8 - 2t + 8e^{-2t}\cosh\sqrt{3}t + \frac{16}{\sqrt{3}}e^{-2t}\sinh\sqrt{3}t$$

For the last two terms we used the Table Transforms 20 and 21.

Question No. 2. (12 Marks) Using the Laplace Transformation method, solve the differential equation:

$$\boldsymbol{y'' + 6y' + 10y = f(t), \quad y = y(t), \quad y(0) = 0, \quad y'(0) = 0}$$

$$\boldsymbol{f(t) = \begin{cases} 0 & for \quad 0 \le t < 5 \\ 1 & for \quad 5 \le t \le 7 \\ 0 & for \quad t > 7 \end{cases}}$$

$$(s^2Y - sy_0 - y_0') + (6sY - 6y_0) + 10Y = U(t-5) - U(t-7)$$

$$s^2Y + 6sY + 10Y = \mathcal{L}\{U(t-5)\} - \mathcal{L}\{U(t-7)\}$$

$$s^2Y + 6sY + 10Y = \frac{e^{-5s}}{s} - \frac{e^{-7s}}{s}$$

$$Y = \frac{e^{-5s}}{s(s^2+6s+10)} - \frac{e^{-7s}}{s(s^2+6s+10)}$$

$$PF \qquad \frac{1}{s(s^2+6s+10)} = \frac{A}{s} + \frac{Bs+C}{(s^2+6s+10)}$$

$$1 \equiv As^2 + 6As + 10A + Bs^2 + Cs$$

$$A + B = 0 \qquad 6A + C = 0 \qquad 10A = 1$$

$$A = \frac{1}{10} \qquad B = -\frac{1}{10} \qquad C = -\frac{3}{5}$$

$$\frac{1}{s(s^2+6s+10)} = \frac{1}{10s} - \frac{s+6}{10(s^2+6s+10)}$$

$$= \frac{1}{10s} - \frac{s+3+3}{10[(s+3)^2+1]}$$

$$Y = \left[\frac{1}{10s} - \frac{s+3+3}{10[(s+3)^2+1]}\right](e^{-5s} - e^{-7s})$$

$$Y = \left[\frac{1}{10s} - \frac{(s+3)}{10[(s+3)^2+1]} - \frac{3}{10[(s+3)^2+1]}\right](e^{-5s} - e^{-7s})$$

$$y(t) = \frac{1}{10}[U(t-5) - U(t-7)]$$

$$-\frac{1}{10}\left[e^{-3(t-5)}\cos(t-5)U(t-5) - e^{-3(t-7)}\cos(t-7)U(t-7)\right]$$

$$-\frac{3}{10}\left[e^{-3(t-5)}\sin(t-5)U(t-5) - e^{-3(t-7)}\sin(t-7)U(t-7)\right]$$

Question No. 3. (8 Marks) Using the Laplace Transformation method, solve the differential equation:

$$y'' + y = 3\cos t + \delta(t-\pi) \qquad y = y(t),\quad y(0) = 1 \quad y'(0) = 0$$

Take Laplace of the equation

$$s^2Y - sy(0) - y'(0) + Y = \mathcal{L}\{3\cos t + \delta(t-\pi)\}$$

Rearrange

$$Y = \frac{s}{(s^2+1)} + \frac{3s}{(s^2+1)^2} + \frac{e^{-\pi s}}{(s^2+1)}$$

$$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3s}{(s^2+1)^2}\right\} + \mathcal{L}^{-1}\left\{e^{-\pi s}\frac{1}{(s^2+1)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos t$$

$$\mathcal{L}^{-1}\{\, e^{-as}F(s)\} = U(t-a)f(t-a)$$

$$If\ a = \pi \quad F(s) = \frac{1}{(s^2+1)} \qquad \mathcal{L}^{-1}\left\{e^{-\pi s}\frac{1}{(s^2+1)}\right\} = \sin(t-\pi)U(t-\pi)$$

$$\mathcal{L}^{-1}\left\{\frac{3s}{(s^2+1)^2}\right\}$$

$$f(t) = 3t\sin t \;\; Table\ \#22$$

$$t\sin kt = \mathcal{L}^{-1}\left\{\frac{2ks}{\left(s^2+k^2\right)^2}\right\}$$

$$y(t) = \cos t + \frac{3}{2}t\sin t + \sin(t-\pi)U(t-\pi)$$