ANSWERS TO SAMPLE TUTORIAL QUESTIONS

1. For the past 20 years, the Medical College Admissions Tests (MCAT) has been given to students who have aspired for an M.D. Upon review of the statistics of this exam; you have noticed that the individual scores are normally distributed with a mean of 7.87, and a standard deviation of 1.83.

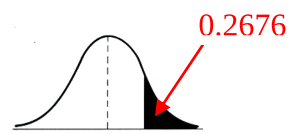
   What is the probability of scoring
   a) Over 9?
   b) Less than 11?
   c) Between 5.5 and 8.5?

   <u>Solution:</u>

$$z = \frac{x - \mu}{\sigma}$$

a) P (x > 9) → z = (9 – 7.87) / 1.83 → z = 0.62

   P (z > 0.62) = 1 - 0.7324 = 0.2676 or 26.76%



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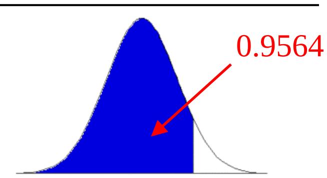
b) P (x < 11) → z = (11 – 7.87) / 1.83 → z = 1.71

   P (z < 1.71) → 0.9564 or 95.64%



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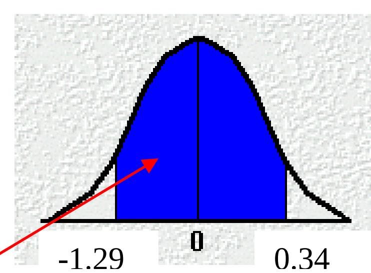
c) P (5.5 < x < 8.5)

   i. z = (5.5 – 7.87) / 1.83 → z = - 1.29
   P (z >-1.29) = 0.0985

   ii. z = (8.5 – 7.87) / 1.83 → z = 0.34
   P (z < 0.34) = 0.6331



   P (-1.29 < z < 0.34) = 0.6331 – 0.0985 = 0.5346 or 53.46%

2. Mike claims that Canadian students do better on the MCAT exam than others who write it. From a sample of 20 "Canucks", you find a mean of 8.12 with a standard deviation of 1.09. Is there any truth to his claim? Test this at the 95% level and please find the p-value if the population mean is 7.87.

Solution:

What kind of hypothesis testing is this? → t-distribution where σ is unknown.

$\bar{x} = 8.12$ $\quad n = 20$ $\quad s = 1.09$ $\quad \mu = 7.87$ $\quad$ Confidence Level = 0.95

$\alpha = 0.05$

Step 1: Set up the hypothesis

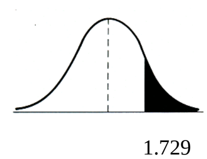
Ho: $\mu \leq 7.87$
Ha: $\mu > 7.87$

Step 2: Determining the Significance Level

Confidence Level = 0.95 → $\alpha = 0.05$

Step 3: Critical Region

Since n = 20, n < 30 therefore will we need to use the t-distribution table.
Since this is a one sided test, then α does not need to be divided into two.

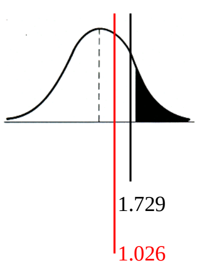


$t_{(n-1,\ \alpha)} = t_{(19,\ 0.05)} = 1.729$ ← Critical Value (positive because Ha is pointing to the right)

Step 4: Test Statistic

$t^* = \dfrac{\bar{x} - \mu}{s / \sqrt{n}} \longrightarrow t^* = \dfrac{8.12 - 7.87}{1.09 / \sqrt{20}} \longrightarrow t^* = 1.026$

Step 5: Conclusion



Since t* does not fall in the critical region, therefore we do not have enough evidence to reject Ho. Therefore, we can not support the claim that Canadians do better on the MCAT than other students who write it.

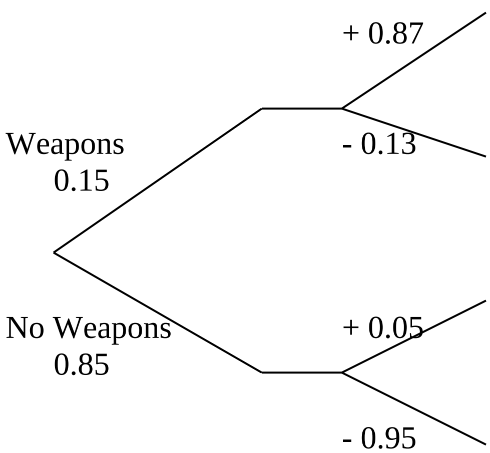
3. As captain of the star ship “Entrepreneur”, a Romulan inventor who claims to have constructed a superior weapons-detection device has approached you. The old detector, which is made by Acme Inc., can successfully detect a concealed weapon 87% of the time. But, for every 20 successful detections, 1 has been false…and an innocent citizen was unnecessarily vaporized. The new Romulan device can detect 93% of the time, with 1 in 18 being booboos. Knowing that 15% of citizens carry their weapons on them, please determine which device is better.

<u>Solution:</u>

What kind of problem is this? → Bayes’ Theorem

We are trying to determine whether or not the old or the new machine can correctly identify those who are carrying **weapons given that they are positive**.

<u>OLD DETECTOR</u>



Weapons and Positive = (0.15)(0.87) = 0.1305

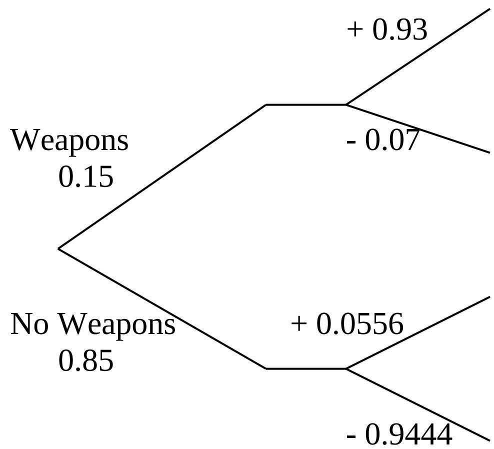
Weapons and Negative = (0.15)(0.13) = 0.0195

No Weapons and Positive = (0.85)(0.05) = 0.0425

No Weapons and Negative = (0.85)(0.95) = 0.8075

$$P\text{ (weapons | positive)} = \frac{0.1305}{0.1305 + 0.0425} = 0.7543 \text{ or } 75.43\%$$

NEW DETECTOR



Weapons and Positive = (0.15)(0.93) = 0.1395

Weapons and Negative = (0.15)(0.07) = 0.0105

No Weapons and Positive = (0.85)(0.0556) = 0.04726

No Weapons and Negative = (0.85)(0.9444) = 0.80274

$$P\ (\text{weapons} \mid \text{positive}) = \frac{0.1395}{0.1395 + 0.04726} = 0.7469 \text{ or } 74.69\%$$

Conclusion:

It would be better to stick with the old detector.

4. The more Sinisa studies, the better he will do on the final stats exam. His friends tell him otherwise, so he decided to do a little research of his own.

Sample	**Score**	**StDev**	**n**
Study	71.87	9.7	18
No Study	70.23	13.3	21

Put his worries to rest if the confidence level is set at 95%!

<u>Solution: Part I</u>

What kind of hypothesis testing is this? → T-Stat's Revenge because both populations are less than 30

<u>Step 1: Set up the hypothesis to determine whether or not this is case 1 or case 2</u>

$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1 \text{ (or } \sigma_1^2 = \sigma_2^2\text{)}$ Case 1

$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1 \text{ (or } \sigma_1^2 \neq \sigma_2^2\text{)}$ Case 2

Where σ_1 = No Study and σ_2 = Study

<u>Step 2: Determine the Critical Value</u>

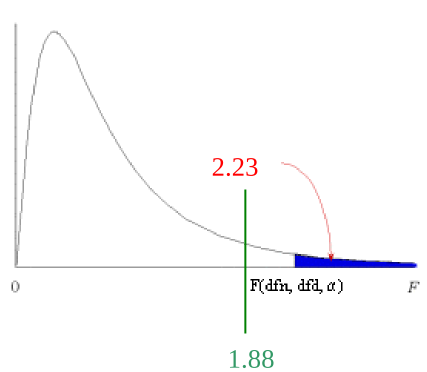
$n_1 = 21$ → $df_1 = 21-1 = 20$; $n_2 = 18$ → $df_2 = 18-1 = 17$; Confidence = 0.95 → $\alpha = 0.05$

$F(df_1, df_2, \alpha) = F(20, 17, 0.05) = 2.23$

<u>Step 3: Determining F*</u>

$F^* = \frac{s_1^2}{s_2^2}$ ⟶ $\frac{13.3^2}{9.7^2}$ ⟶ <u>1.88</u>

<u>Step 4: Conclusion</u>



Since F* falls in the Ho region then we can reject Ha and accept Ho. This is a case 1 where the variances are equal.

Solution: Part II

Step 1: Set up the hypothesis

Ho: $\mu_1 - \mu_2 \leq 0$ Where μ_1 = Study
Ha: $\mu_1 - \mu_2 > 0$ Where μ_2 = No Study

Step 2: Determining the Significance Level

Confidence Level = 0.95 → α = 0.05

Step 3: Critical Region

Since both sample sizes are n < 30 then we will use the t-distribution table, and since this has been proven to be a Case I scenario (see part 1), then the degrees of freedom used will be ($n_1 + n_2 - 2$).

df = $n_1 + n_2 - 2$ = 21 + 18 – 2 = 37

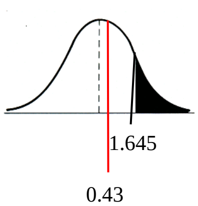
$t_{(37, 0.05)} = +1.645$

Step 4: Calculate t*

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}} \rightarrow \sqrt{\frac{(21-1)(13.3)^2 + (18-1)(9.7)^2}{21+18-2}} \rightarrow \underline{11.78}$$

$$t^* = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \rightarrow \quad t^* = \frac{(71.87 - 70.23) - 0}{11.78\sqrt{1/21 + 1/18}} \rightarrow t^* = 0.433$$

Step 5: Draw conclusions



Since t* does not fall in the critical region, we do not have enough evidence to reject Ho, therefore, studying will not increase your grade (please keep in mind that this is fictional data!)

5. Starting salaries for recent Concordia University graduates have always been a positive recruiting issue for the educational institution. Over the past 10 years, students have been able to find profitable jobs in a variety of fields. In fact, the values form a normal distribution (surprise!) with mean of $52,000, and a standard deviation of $6,500. After completing your degree, what are the chances that you will find a job that pays you:

a) Under $60,000?
b) More than $41,500?
c) Between $38,500 and $62,000?

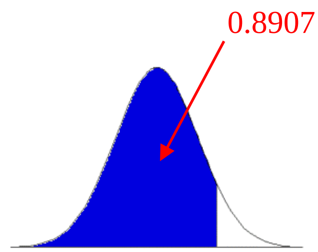
<u>Solution:</u>

$$z = \frac{x - \mu}{\sigma}$$

μ = $52,000
σ = $6,500

a) P (x < $60,000) → z = (60,000 – 52,000) / 6,500 = 1.231

P (x < $60,000) = P (z < 1.231) = 0.8907 or 89.07%



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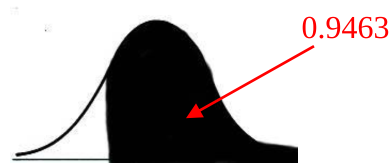
b) P (x > $41,500) → z = (41,500 – 52,000) / 6,500 = -1.61

P (x > $41,500) = P (z > -1.61) = 1 – 0.0537 = 0.9463



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c) P ($38,500 < x < $62,000)

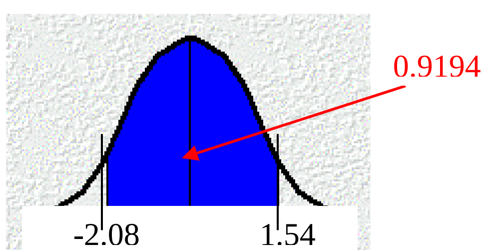
i. P (x < $38,500) → z = (38,500 – 52,000) / 6,500 = -2.08

P (x < $38,500) = P (z < -2.08) = 0.0188

ii. P (x > $62,000) → z = (62,000 – 52,000) / 6,500 = 1.54

P (x > $62,000) = P (z > 1.54) = 0.9382

iii. P ($38,500 < x < $62,000) = P (-2.08 < z < 1.54) = 0.9382 – 0.0188 = 0.9194



6. How much do you travel to get to school? This was one of the questions asked in a university-wide survey issued by the Dean of Students last month. With the bulk of the students coming from the island, the South Shore, and Laval, the original estimate had been 15 km. But upon analysis of the early results, the university seemed to be getting a different picture. Is there enough evidence to reject their estimate with 99% confidence? Please include the **p-value** in this complete hypothesis test, and don't forget your **conclusion**.

Sample Data for Distance Traveled by Students to get to School (in km.)

18	22	13	28	17	20	15	32	24	8

<u>Solution: Part I</u>

n = 10 Average = 197/10 = 19.7

x	**x^2**	**x - avg**	**$(x\text{-avg})^2$**	
18	324	-1.70	2.89	
22	484	2.30	5.29	
13	169	-6.70	44.89	
28	784	8.30	68.89	
17	289	-2.70	7.29	
20	400	0.30	0.09	
15	225	-4.70	22.09	
32	1024	12.30	151.29	
24	576	4.30	18.49	
8	64	-11.70	136.89	
197	4339	177.30	458.1	**Total**

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} \longrightarrow \frac{458.1}{10-1} \longrightarrow \underline{\underline{50.9}}$$

OR

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} \longrightarrow \frac{4339 - ((197)^2/10)}{10\text{-}1} \longrightarrow \underline{\underline{50.9}}$$

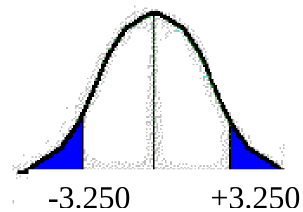
Therefore s = √50.9 = 7.134

<u>Solution: Part II</u>

What kind of problem is this? → Hypothesis Testing

<u>Step 1: Set up the Hypothesis</u>

Ho: $\mu = 15$ km
Ha: $\mu \neq 15$ km



<u>Step 2: Determine the critical region</u>

$\alpha = 0.01$

Since this is a two-sided test and n < 30, we will need to use the T-distribution Table

$t_{(9,0.005)} = \pm 3.250$

<u>Step 3: Determine t*</u>

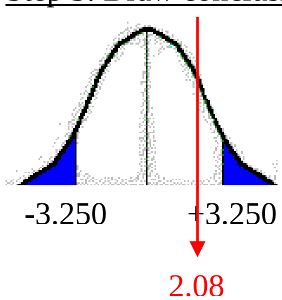
$$t^* = \frac{\overline{X} - \mu}{s / \sqrt{n}} \rightarrow t^* = \frac{19.7 - 15}{7.134/\sqrt{10}} \rightarrow t^* = 4.7/2.26 = 2.08$$

<u>Step 4: Determine the P-value</u>

P (t > 2.08) = 0.0188

Therefore the p-value, since it is a two sided test, is 2 x 0.0188 = 0.0376

<u>Step 5: Draw conclusions</u>



Since t* does not fall in the critical region, and since our p-value is greater than α (0.01), we do not have enough evidence to reject their estimate that students travel 15 km. to get to school.

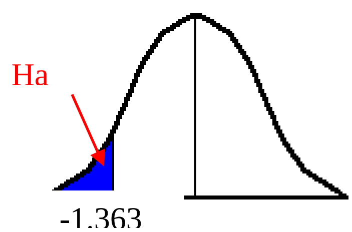
7. After using “Kickbutt” for one week, smokers in a controlled experiment seemed to have cut down on the number of cigarettes they were smoking. Using the data from the first week’s results, please determine if the drug had **indeed helped them to cut down on their tobacco intake** at the 90% confidence level.

Number of Cigarettes Smoked

Before	19	22	32	17	37	20	23	24	28	21	15	18
After	15	7	33	10	28	12	23	17	19	24	11	16

Solution:

What kind of hypothesis testing is this? → Dependent



Ho: Ud ≥ 0
Ha: Ud < 0

If d = after - before

$\alpha = 0.10$
n = 12 therefore we need to use the t-distribution table

$t_{(n-1,\, \alpha)} = t_{(11,\, 0.1)} = -1.363$ ← Critical Value (negative because Ha is pointing to the left)

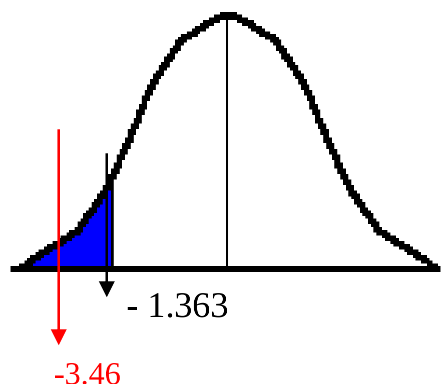
Before	**After**	**d**	**d^2**
19	15	-4	16
22	7	-15	225
32	33	1	1
17	10	-7	49
37	28	-9	81
20	12	-8	64
23	23	0	0
24	17	-7	49
28	19	-9	81
21	24	3	9
15	11	-4	16
18	16	-2	4
276	**215**	**-61**	**595**

$$\bar{d} = \frac{\sum d}{n} \longrightarrow \frac{(-4-15+1-7-9-8+0-7-9+3-4-2)}{12} \longrightarrow \underline{\underline{-5.08}}$$

$$s_d = \sqrt{\frac{\sum d^2 - \left[\frac{\left(\sum d\right)^2}{n}\right]}{n-1}} \longrightarrow \sqrt{\frac{595 - [(-61)^2 / 12]}{(12-1)}} \longrightarrow \underline{\underline{5.09}}$$

$$t^* = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \longrightarrow t^* = \frac{-5.08- 0}{5.09 \; / \sqrt{12}} \longrightarrow t^* = \underline{\underline{-3.46}}$$

P value = P (t < -3.46) = 0.003
α = 0.1



Conclusion:
t* falls in the Ha critical region and the P value is smaller than alpha, therefore we reject Ho and accept Ha.

There is enough evidence to determine that the drug has indeed helped them to cut down on their tobacco intake

8. Attitudes toward mathematics were measured for two different groups. The attitude scores ranged from 0 to 80 with higher scores indicating a more positive attitude. One group consisted of elementary education majors and the group consisted of majors from several other areas. The results are as follows:

Group (major)	Sample Size	Mean	St Dev
Elementary	25	42.7	15.5
Non Elementary	45	49.3	17.0

Is there enough evidence to suggest that their **attitudes are different** at the 99% level?

Solution:

What kind of problem is this? → T-Stat's Revenge because at least one population is less than 30

Step 1: Set up the hypothesis to determine whether or not this is case 1 or case 2

$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1 \text{ (or } \sigma_1^2 = \sigma_2^2\text{)}$ Case 1

$H_a: \frac{\sigma_1^2}{\sigma_2^2} \neq 1 \text{ (or } \sigma_1^2 \neq \sigma_2^2\text{)}$ Case 2

Where σ_1 = Non Elementary and σ_2 = Elementary

Step 2: Determine the Critical Value

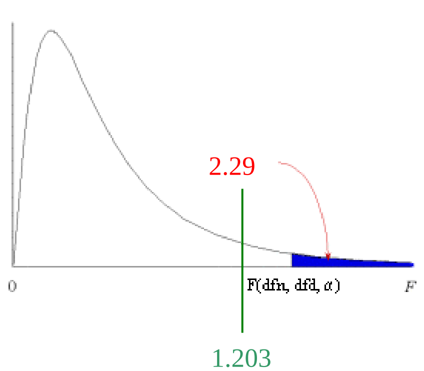
$n_1 = 45$ → $df_1 = 45-1 = 44$; $n_2 = 25$ → $df_2 = 25-1 = 24$; Confidence = 0.99 → $\alpha = 0.01$

$F(df_1, df_2, \alpha) = F(44, 24, 0.10) = 2.29$

Step 3: Determining F*

$$F^* = \frac{s_1^2}{s_2^2} \longrightarrow \frac{17.0^2}{15.5^2} \longrightarrow \underline{\underline{1.203}}$$

Step 4: Conclusion



Since F* falls in the Ho region then we can reject Ha and accept Ho. This is a case 1 where the variances are equal.

Solution: Part II

Step 1: Set up the hypothesis

Ho: $\mu_1 - \mu_2 = 0$ Where μ_1 = No Elementary
Ha: $\mu_1 - \mu_2 \neq 0$ Where μ_2 = Elementary

Step 2: Determining the Significance Level

Confidence Level = 0.99 → $\alpha = 0.01$

Step 3: Critical Region

Since this is a two sided test, then $\alpha = 0.01$ → $\alpha/2 = 0.005$

Since at least one sample size is n < 30 then we will use the t-distribution table

$df = n_1 + n_2 - 2 = 45 + 25 - 2 = 68$

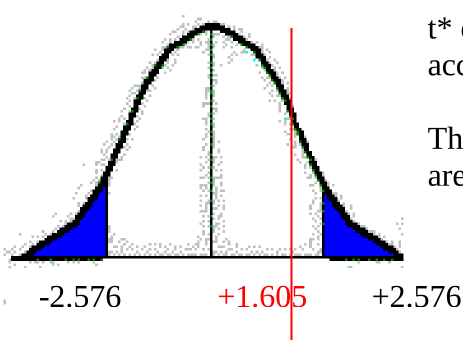
$t_{(68, 0.005)} = +/- 2.576$

Step 4: Calculate t*

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}} \rightarrow \sqrt{\frac{(45-1)(17.0)^2 + (25-1)(15.5)^2}{45+25-2}} \rightarrow \underline{\underline{16.486}}$$

$$t^* = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \rightarrow \quad t^* = \frac{(49.3-42.7) - 0}{16.486 \sqrt{1/45 + 1/25}} \rightarrow t^* = \underline{\underline{1.6049}}$$

Step 5: Draw conclusions



t* does not fall in the critical region, therefore, we can accept Ho.

There is not enough evidence to suggest that their attitudes are different