1. (4 points) An 8-member research group is to be formed from a group of 12 mathematicians, 8 physicists, 9 chemists and 11 biologists. How many ways are there to form a committee that

(You do not need to evaluate fully. Express your answers in terms of combinations and/or permutations whenever appropriate. Use the adequate notation.)

(a) contains exactly 4 mathematicians?

$\binom{12}{4}\binom{28}{4}$ ✓

(b) contains at most 2 biologists?

0 biologists: $\binom{29}{8}\binom{11}{0}$

1 biologist: $\binom{29}{7}\binom{11}{1}$

2 biologists: $\binom{29}{6}\binom{11}{2}$

$\binom{29}{8} + \binom{29}{7}\binom{11}{1} + \binom{29}{6}\binom{11}{2}$ ✓

(c) contains exactly 3 chemists but no mathematicians?

$\binom{9}{3}\binom{19}{5}$ ✓

(d) contains 2 physicists and 4 mathematicians where one of these four mathematicians takes up the position of research chair and another mathematician among these four takes up the position of assistant to research chair?

$\binom{8}{2}\binom{10}{2}\binom{20}{2}\cdot 12\cdot 11$

↑ phyisists ↑ remaining math ↑ other 2 ↑ chair ↑ assistant

✓

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2. (4 points) Use the Binomial Theorem

(a) to expand $(x+y)^7$.

$(x+y)^7$

$= \binom{7}{0}x^7 + \binom{7}{1}x^6y + \binom{7}{2}x^5y^2 + \binom{7}{3}x^4y^3 + \binom{7}{4}x^3y^4 + \binom{7}{5}x^2y^5 + \binom{7}{6}xy^6 + \binom{7}{7}y^7$

$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^6 + 7xy^6 + y^7$

(b) to expand $(x+2y)^6$.

$= \binom{6}{0}x^6 + \binom{6}{1}x^5(2y) + \binom{6}{2}x^4(2y)^2 + \binom{6}{3}x^3(2y)^3 + \binom{6}{4}x^2(2y)^4 + \binom{6}{5}x(2y)^5 + \binom{6}{6}(2y)^6$

$= x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6$

(c) to find the coefficient of $x^{30}y^{20}$ in $(x+y)^{50}$.

$\binom{50}{20}x^{30}y^{20}$

$\rightarrow 4.713 \times 10^{13}$

coefficient of $x^{30}y^{20}$ in $(x+y)^{50}$ is $\binom{50}{20}$.

(d) to find the coefficient of $x^{20}y^{40}$ in $(x-3y)^{60}$.

$\binom{60}{40}x^{20}(-3y)^{40}$

$= \binom{60}{40}(-3)^{40}x^{20}y^{40}$

$= \binom{60}{40}(3)^{40}x^{20}y^{40}$

$\therefore$ The coefficient is $\binom{60}{40}(3)^{40}$

$= \binom{60}{40}3^{40}$

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Assignment 4

3) For $n \geq 4$ define $P(n)$ to be the proposition $n^2 \leq 2^n$.

BI: Consider $n=4$.
Since $4^2 = 16 = 2^4$, $4^2 \leq 2^4$ is true. So, $P(4)$ is true.

IS: For an arbitrary integer $k \geq 4$ assume that $P(k)$ is true.
Then $k^2 \leq 2^k$.
We want to show that $\underbrace{(k+1)^2 \leq 2^{k+1}}_{P(k+1)}$.

$(k+1)^2 = k^2 + 2k + 1$
Since $2k+1 \leq k^2$ for $k \geq 4$, we have

$\underline{\underline{(k+1)^2}} \leq k^2 + k^2 = 2k^2 \underset{(IS)}{\leq} 2 \cdot 2^k = \underline{\underline{2^{k+1}}}$

We showed that $P(k) \rightarrow P(k+1)$ for $k \geq 4$.

BI and IS together imply that $P(n)$ is true for all integers $n \geq 4$.

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