Phys 259 Winter 2018 – Group Lecture Activity01 – Jan 19, 2018

Group #	Student	Last Name	First Name
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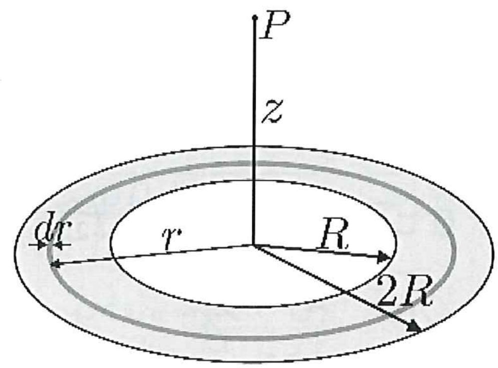
This activity will be graded out of 10 marks to provide feedback, but will not be counted in your grade.

- The purpose of this activity is for students to employ mathematical methods to determine the equation for the electric field on the axis of a uniformly charged annulus at point P.

- The situation is shown in the figure to the right. The annulus is positively charged with surface charge density σ (note that σ = annulus's total charge divided by its area). The point of interest, P, is a distance *z* from the center of the annulus along the axis of symmetry.

- We start with a known equation; the equation for the magnitude of the electric field on the axis of a thin ring of charge;



$$E_{ring} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+r^2)^{3/2}}$$

The parts below walk you through related questions, and the steps with which to solve this problem. Please show all work in the boxes provided and then choose the correct answer at the bottom from the choices provided.

1) Explain briefly, in words, how one can construct the electric field on axis of a uniformly charged annulus using the electric field on axis of a charged ring. You will eventually need to set up an integral; what is the variable you will need to integrate over? (2 marks)

The annulus can be divided into concentric thin rings of thickness dr. The electric field magnitude for a thin ring is given: $E_{ring} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+r^2)^{3/2}}$. The contributions to the electric field at point P from each ring can be added up by integrating over the radius r from R to 2R.

2) In the figure there is a thin grey circle of thickness dr. What is the equation for the area of this thin grey circle, call it dA, in terms of dr and r? (1 mark)

A thin ring can be approximated as a rectangle, One side = circumference ($2\pi r$) the other side equal to the thickness of the ring.

$2\pi r$

dr $\boxed{dA = 2\pi r\, dr}$

dA

3) How much charge, call it dq, is contained in this small area dA? (1 marks)

$\sigma = \frac{Q}{A}$, $q = \sigma \cdot A$

$dq = \sigma \cdot dA = \sigma \cdot 2\pi r \, dr$

4) There is a small contribution to the total electric field at point P, dE, from the thin ring. What is the equation for the magnitude of dE in terms of r, dr, ε_o, and σ? and z (2 marks) Using an argument based on symmetry, what is the direction of dE? (1 mark)

$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq\, z}{(z^2+r^2)^{3/2}}$ → contribution from a ring

Due to symmetry all the components except the one in the z - direction cancell, $d\vec{E}$ is in the $\hat{k}$ direction.

$dE = \frac{1}{2\cancel{4\pi}\varepsilon_0} \frac{\sigma\, \cancel{2\pi} r \cdot z}{(z^2+r^2)^{3/2}} dr = \frac{\sigma z}{2\varepsilon_0} \frac{r}{(z^2+r^2)^{3/2}} dr$

5) Set up an integral expression for the magnitude of the net electric field at point P. Be sure to include the proper limits of integration. (2 marks)

$$E = \int_R^{2R} \frac{\sigma z}{2\varepsilon_0} \frac{r}{(z^2+r^2)^{3/2}} dr$$

constant

$$E = \frac{\sigma z}{2\varepsilon_0} \int_R^{2R} \frac{r}{(z^2+r^2)^{3/2}} dr$$

Chose the integral which describes the total electric field vector at point P due to a uniform annulus of charge. (1 mark)

A. $\mathbf{E} = \frac{\sigma z}{2\epsilon_o} \int_R^{2R} \frac{r}{(z^2+r^2)^{3/2}} dr\hat{r}$ B. $\mathbf{E} = \frac{\sigma z}{2\epsilon_o} \int_R^{2R} \frac{r}{(z^2+r^2)^{3/2}} dr\hat{k}$

C. $\mathbf{E} = \frac{\sigma z}{2\epsilon_o} \int_0^{R} \frac{r}{(z^2+r^2)^{3/2}} dr\hat{k}$ D. $\mathbf{E} = \frac{\sigma}{2\epsilon_o} \int_R^{2R} \frac{rz}{(z^2+r^2)^{3/2}} dz\hat{k}$