COMP 233 Probability and Statistics for Computer Science

Fall 2016, Assignment 4
Due: December 5, 2016

Question 1 In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of .02. Suppose 10 independent measurements yielded the following pH values:

8.18 8.17 8.16 8.15 8.17 8.21 8.22 8.16 8.19 8.18

a) What conclusion can be drawn at the $\alpha_1 = .10$ level of significance?
b) What about at the $\alpha_2 = .05$ level of significance?
c) What is the p-value?

Question 2 The lifetime of special light-bulbs is normally distributed. A sample of 81 bulbs has produced a mean of 738 hours and a standard deviation of 38.2 hours. Test the hypothesis $H_0 : \mu = 747.5$ versus $H_1 : \mu \neq 747.5$, at the significance levels of $\alpha_1 = 0.05$ and $\alpha_2 = 0.01$. Find the p-value of the test, and use the p-value to verify your answers.

Question 3 A car is advertised as having a gas mileage rating of at least 30 miles/gallon in highway driving. If the miles per gallon obtained in 10 independent experiments are

26, 24, 20, 25, 27, 25, 28, 30, 26, 33,

should you believe the advertisement?

Question 4 All pacemakers generate a pulse of current to depolarize a bit of the heart, the wave then spontaneously spreads to the rest of that part of the heart. Pulse duration is programmable in all currently manufactured permanent pacemakers. The shorter the pulse, the greater the necessary current. The length of the pulse for a particular setting will vary due to random fluctuations such that the actual pulse length is normally distributed with a mean equal to the setting and with an unknown variance σ^2. A type of pacemaker is claimed to be reliable since its standard deviation σ is less than 0.10. If a random sample of 20 pacemakers of that type resulted in a sample standard deviation of 0.08, should this claim be accepted? Suppose the level of significance is $\alpha = 0.05$.

a) State the appropriate choice of the null hypothesis, and the alternate hypothesis.
b) Compute the value of the test statistic, χ_0^2 .
c) Determine its p-value.
d) Based on this evidence, should the claim be accepted? Also, comment on the appropriate choice of a significance level for this problem.

Question 5 The following data indicate the relationship between x, the specific gravity of a wood sample, and Y, its maximum crushing strength in compression parallel to the grain.

x_i	y_i (psi)	x_i	y_i (psi)
.41	1,850	.39	1,760
.46	2,620	.41	2,500

.44	2,340	.44	2,750
.47	2,690	.43	2,730
.42	2,160	.44	3,120

a) Plot a scatter diagram. Does a linear relationship seem reasonable?
b) Estimate the regression coefficients.
c) Predict the maximum crushing strength of a wood sample whose specific gravity is .43.

Question 6 The determination of the shear strength of spot welds is relatively difficult, whereas measuring the weld diameter of spot welds is relatively simple. As a result, it would be advantageous if shear strength could be predicted from a measurement of weld diameter. The data are as follows:

Shear Strength (psi)	Weld Diameter (.0001 in.)
370	400
780	800
1,210	1,250
1,560	1,600
1,980	2,000
2,450	2,500
3,070	3,100
3,550	3,600
3,940	4,000
3,950	4,000

a) Find the least squares estimates of the regression coefficients.
b) Test the hypothesis that the slope of the regression line is equal to 1 at the 0.05 level of significance.
c) Find a prediction interval such that, with 95 percent confidence, the value of shear strength corresponding to a weld diameter of .2250 inch will be contained in it.

Question 7 A new drug was tested on mice to determine its effectiveness in reducing cancerous tumors. Tests were run on 10 mice, each having a tumor of size 4 grams, by varying the amount of the drug used and then determining the resulting reduction in the weight of the tumor. The data were as follows:

Coded Amount of Drug	Tumor Weight Reduction
1	.50
2	.90
3	1.20
4	1.35
5	1.50
6	1.60
7	1.53
8	1.38
9	1.21

10	.65

Estimate the maximum expected tumor reduction and the amount of the drug that attains it by fitting a quadratic regression equation of the form

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2 + e$$

Question 8 If $x_0 = 3$, and $x_n = (3x_{n-1} + 13) \bmod 53$

a) find $x_i, i = 1, \ldots, 10$.
b) Using $r_i = x_i/53$, find the first 11 random variates, using $i = 0, \ldots, 10$, for the Exponential probability density function using $\lambda = 3$.
c) Compute the sample mean and sample variance of the values found in part b), and compare the results with population mean and population variance for the Exponential PDF.